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MathAmigos: A Community Mathematics Initiative

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MathAmigos: A Community Mathematics Initiative

Cover Page Footnote

We would like to thank the rest of the members of the MathAmigos managing group and coaches, especially Founder and Chair Lynn Bickley and co-Chair Gary Clendenen. We could not have made such progress with the schools without the considerable support of Santa Fe Public Schools district Curriculum and Instruction Executive Director Melissa Lomax, Assistant Director Peter McWain, and Math Coordinator Jenifer Hooten. Other members of our MathAmigos founding team we cannot go without thanking include Dean Gerber and Chari Kaufman. Financial support for our work came from the Santa Fe Community Foundation, the City of Santa Fe, and private donors. Many thanks to family and friends for reviewing and commenting on drafts, most notably critic, old friend, and mentor Reuben Hersh, Coco Rae, Pam Homer, and Rick Hoefler. Finally, we give special thanks to our Santa Fe Community Foundation VISTAs, especially Penny Holcomb, for above and beyond support in making our events go so smoothly.

MathAmigos: A Community Mathematics Initiative

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We present a broad, and we think novel, community mathematics initiative in its early stages in Santa Fe, New Mexico. At every level, the program embraces community-wide collaboration – from the leadership team, to the elements of the mathematics being implemented (primarily Math Circles and the Global Math Project’s Exploding Dots), to the funding model. Our MathAmigos program falls within two categories of Math Circle-related programs: outreach and professional development (PD). We will touch on one of the more novel components of our PD: the use of retired master teachers as classroom coaches.

Keywords: Professional development, Coaching, Exploding Dots, Elementary School

1 Introduction

MathAmigos, a community mathematics initiative in Santa Fe, New Mexico, was developed with the goal of sparking innovative, even radical, improvements in mathematics education in local public schools. It began with a small grant proposal to a local foundation for conducting Saturday professional workshops aimed at fourth through sixth grade teachers in the spring semester of 2018. The curriculum was based upon Math Circles, the Global Math Project’s Exploding Dots curriculum, and “off-label” use of Cuisenaire Rods. In this article, we will focus on only the Math Circle and Exploding Dots curriculum.

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Using this work as a prototype, we were able to transition into a two-year, city- and grant-funded initiative.

The current program includes teacher stipends for workshop attendance, paid in-class coaching by retired master teachers, and a series of family math festivals. The five targeted schools are geographically grouped and identified as requiring considerable additional support by the state's Public Education Department. The area has a large immigrant population, with many English language learners and low-income families. Our city contract supports programs for grades three and four, but we have augmented this group with other funding to include some grades one, two, and five teachers at the same schools. There are 35 teachers in the 2018-19 cohort, which includes 65% of the third and fourth grade teachers for those schools. Teachers were recruited at presentations to the majority of the faculty and some administrators at each of the five schools at the start of the 2019-20 school year. The presentations all included a Math Circle demonstration.

A fundamental theme of our work is that of treating teachers as awakening mathematical thinkers and doers. We feel that this is not often a principle in mathematical professional development, and this is reflected in the practice of "training" teachers to deliver methods rather than to grow as mathematician-teachers. Often, such trainings offer little follow-up and support. We address this by offering an introduction to the productive struggle of the mathematician via Math Circles combined with a professional partnership with our coaches.

This article describes the MathAmigos program, how it has been implemented in the Santa Fe schools, and how participants have reacted to it. We begin with a timeline of the program for reference ([Table 1](#)).

2 Origins

For this model of a community math initiative to be useful to the wider Math Circle community, we consider it necessary to give some history of the group. This background will help give the reader a sense of the small early steps, recruiting of the larger MathAmigos team, and the development of our direction, philosophy, goals, and curriculum.

2.1 Forming a Collaborative Working Group

What was to become MathAmigos began in 2017 with a meeting of public school math tutors and education leaders who sought to gain a better understanding of the problems they encountered in the schools. Another subsequent

Table 1

MathAmigos Program Development Timeline.

Stage	Year	Period	Event
Origins: Formation	2017	Spring	Santa Fe Public Schools (SFPS) math tutors meeting.
		Spring-Fall	Formation of Collaborative Working Group on Math, Santa Fe Community Foundation. Grant application.
		Fall	Founding of MathAmigos and development of programming design. Planning for spring 2018 workshops.
Origins: Pilot workshops. Festivals under foundation funding.	2018	Jan-Feb	First two grades 4-6 workshops. Test of curriculum concepts.
		May	Family math night. Apply for City of Santa Fe funding.
		June	Notification of City funding award.
Year 1: Formal MathAmigos program	2018	June-Sept	Planning for 2018-2019 sessions. Recruiting coaches.
		Oct	1 st Saturday workshop.
		Nov	2 nd Saturday workshop.
	2019	Jan	3 rd Saturday workshop.
		Feb	Family Math Night Festivals 1, 2.
		May	MathAmigos integration (29 of 44 sessions) within 3-day end-of-year SFPS district math professional development workshop.
Year 2: Formal MathAmigos program	2019	Sept	Begin first formal yearlong student Math Circle at large SFPS K-8 school within 21 st Century Program. (Grades 5-7) Additional teacher funded by grant through MathAmigos.
		Oct	1 st Saturday workshop.
		Nov	2 nd Saturday workshop.
	2020	Jan	3 rd Saturday workshop.
		Spring	Family Math Night Festival 1, 2.

Note. The formal MathAmigos program has five cohort schools, grades three and four with some grades one, two, and five.

meeting was held by a local foundation to apprise people of a new community grant program. Some of the attendees of the two meetings began to correspond in order to brainstorm a K-12 mathematics education project that would fit within one of the types of grants, which was intended to improve the level of eighth grade math education and test scores. The group was led by retired physician Lynn Bickley, co-leader of the Santa Fe Interfaith Coalition for Public Education,¹ who brought many years of experience working with public school math tutoring and related programs.

As former Associate Dean for Curriculum at Texas Tech University School of Medicine, Dr. Bickley also brought experience with curriculum reform and with navigating complex educational organizations. This founding group, which continues to be the core MathAmigos team, included retired IBM mathematician Dean Gerber, retired math and computer science teacher James Taylor, Santa Fe Public School (SFPS) Curriculum and Instruction Director Melissa Lomax, and SFPS Science and Math Coordinator Chari Kaufman. Also participating in early discussions were representatives from local colleges, foundations, cross-cultural organizations, and STEM programs. Eventually we added a co-lead, Gary Clendenen, a retired business professor. Dr. Bickley recognized from the beginning the need to be as inclusive across the community as possible.

The MathAmigos team's mix of skills and willingness to take on all the major roles for conducting workshops and family festivals have been essential components to our success. Another key to our success was embedding our program in the school district itself, so that our mostly volunteer steering group would grow, from the beginning, to include many Santa Fe Public Schools administrators. According to Dr. Bickley:

This strategy has allowed this program to give virtually on-the-spot feedback and continually take the temperature of how much support we are actually getting. It also allows us to provide feedback to school administrators that teachers would be reluctant to give directly. It also prioritizes our unique on-the-ground knowledge of teacher concerns. [personal communication, April 2, 2019]

This involvement by many levels of the school district, from classroom to front office, sets this program apart from typical professional development, and we believe it has led to more extensive adoption in classrooms. It has also led to greater district-level interest in spreading the model and the curriculum

¹<http://icpesantafe.org/>

to other schools in the district. It has been necessary to have members attend school board meetings, design and assemble the workshop mathematics activities, teach the courses, work with school administration and staff to run a festival event, train festival table leaders, write surveys and other measurement instruments, and then assemble and summarize the data, write grant proposals, work with the school district administration, meet with granting organizations, and other tasks. For the group's efforts to be sustainable, no one person can have too much to do.

2.2 Pilot Workshops and Math Festivals

The founding group, which was to become MathAmigos, decided to propose, for the winter 2018 semester, two Saturday professional development math workshops for up to 35 fourth through sixth grade teachers from local schools. In addition, the team decided to run a family math night in May 2018 at a K-5 school (one of our current program's cohort of five schools), open to parents and children at that school. The night's program was dominated by Julia Robinson Math Festival (JRMF) activity [4] tables, run mostly by teachers from our two preceding professional development (PD) workshops.

2.3 Funding for the Formal MathAmigos Professional Development Program

Following on this pilot program's successes and favorable responses from the school district, we decided to apply for greater, contract-type funding through the City of Santa Fe. In preparation for this larger proposal, the MathAmigos team also met with several grade six teachers at a local elementary school to learn what teachers needed most, in the way of support. This conversation helped us understand better the challenges teachers face, more than giving us specific direction and ideas for curriculum. We got the sense that teachers and schools were regularly having new curriculum ideas thrown at them, and we concluded that something deeper was required. In April 2018, we decided to apply for a \$164,000 contract to address math education in grades three and four over two years, using the spring semester prototype as a model. We were fortunate to be awarded half of the applied-for funds by the end of May, giving us a more modest project to scale up to from our minimal beginnings. The two-year program would include three Saturday PD workshops with follow-up coaching, a three-day end-of-year PD workshop, and two family math festival nights for cohort schools.

Before the 2018-19 school year started, we worked with all five of our cohort schools to present our program at opening faculty meetings in order to recruit

teachers for our program. At each school we led a Math Circle demonstration for the entire faculty and discussed what they could expect and be expected to do as participants in the new program.

3 The MathAmigos Mathematics Teacher Professional Development Program

During the 2018-19 school year, MathAmigos held three Saturday workshops during which the teachers were exposed to (1) Exploding Dots, (2) formal Math Circle and related Julia Robinson Math Festival-style problems, and (3) Cuisenaire Rods. In the first two workshops, teachers were required to participate in activities coinciding with all of these workshop elements. In the third workshop, they chose one or more areas from a larger set of course choices (still within the three core elements) they wanted to learn more about. During the workshops, teachers also met with their in-class coaches to plan classroom visits. The teachers were surveyed at the end of each workshop. The school year ended with a three-day mathematics PD workshop at the end of May 2019.

3.1 The Three Workshop Elements, Rationale and Discussion

The workshops focused on the following three elements:

1. Exploding Dots/Global Math Project (GMP)

This element is by far the easiest for teachers to implement, and nearly our entire cohort has been using this extensively. Using the GMP and Exploding Dots websites and materials [3], local fifth grade teacher and article co-author Delara Sharma led sessions teaching addition, multiplication, subtraction, and division using this model. She has provided teachers with templates in plastic sleeves for use with their classes. Many teachers have reported dramatic improvements in student understanding, as have our in-class coaches.

2. Math Circle and Math Festival Activities

We see in Math Circles short-term and long-term strategies. These are more difficult than Exploding Dots for most teachers to feel comfortable introducing. In the short term, though, teachers appreciate being treated as professionals and as mathematician-teachers. This is a substantial departure from typical PD, where teachers are being trained

to deliver content or techniques rather than encouraged to be mathematical thinkers and investigators. Some of our coaches have observed teachers not only using the Math Circles we've taught in workshops, but also adapting their math teaching style to the "be less helpful" strategy favored in Math Circles. We see this component as nurturing mathematical maturity and confidence in the teachers, as well as creating greater joy in teaching mathematics.

Further, by our teachers integrating Math Circle activities into their classrooms, we avoid the common Math Circle problem of ending up with a self- or parent-selected group of already well-enriched students. We adjusted our content and approach for each workshop based upon teacher feedback from the previous workshop. For example, teachers in our second workshop asked for simpler activities that could be dropped into the school day from time to time, so in workshop three we added sessions that introduced a variety of JRMF activities [4]. These activities are relatively easy and fast for a teacher to study, practice, and lead a classroom through – in contrast to the often more difficult material of a traditional Math Circle. Learning these activities in the workshop had the added benefit of preparing all of our teachers to be table leaders in our family math festivals. The leaders of these sessions were co-authors of this article, MathAmigos founding member James Taylor and Shannon Rogers.

3. Cuisenaire Rods

These sessions introduced teachers to some unusual or "off-label" uses of the rods to help students (especially at the younger end of our cohort) with their number sense. We will not go into any detail about this element of our program here, as it does not relate directly to either Math Circles or the Global Math Project/Exploding Dots. The leader of these sessions was Dean Gerber, a founding member of MathAmigos.

Each workshop had sessions in all three of these elements. The first two workshops, held in the fall of 2018, had sessions of each element that groups of the teachers rotated through during the day. The second and third Saturday workshops also ran a single session of Exploding Dots for beginners, for those teachers who missed earlier workshops.

3.2 The Exploding Dots and Math Circle Workshops

Saturday workshops were held in October and November 2018 and January 2019, and the year's program culminated in a three-day workshop at the end of

May 2019. This final workshop was fully integrated into the school district's standard end-of-year math PD workshop for all math teachers. It included many non-MathAmigos sessions, along with our 29 sessions, 12 of ours being Math Circles or Exploding Dots (sample schedule in [Appendix A](#)). The activities of each type are listed below chronologically. In the first two workshops, teachers were required to attend each of three types of activities over the three sessions of the day: Exploding Dots, Math Circles, and Cuisenaire Rods. In the third workshop, teachers could choose any set of three sessions they liked. This could mean exclusive focus on Exploding Dots or Math Circles if they chose, with most participants choosing some combination of the two. The last 10 or 15 minutes of each session was reserved for group discussions of pedagogy and necessary differentiation for diverse student abilities and classrooms.

3.2.1 The Exploding Dots Workshop Sessions

Four sets of Exploding Dots workshop sessions were given in the 2018-2019 school year (three were conducted prior to the writing of this article and one after).

October 2018. Teachers were introduced to the model and to using the online video materials. Each teacher was given a template to reproduce for use in the classroom. They were taught how to do addition in this first workshop. We assume our readers' familiarity with this material.

November 2018. Teachers not at the first workshop were introduced to the model and to using the online video materials for addition. The experienced teachers from the first workshop learned about multiplication and subtraction.

January 2019. Teachers here could get a review of using the model for addition, multiplication, and subtraction or dive more deeply into subtraction, multi-digit multiplication, and division.

May 2019. This three-day workshop was held post-submission of this article, and an abbreviated schedule is presented in the Appendix A.

3.2.2 The Exploding Dots Sessions Leader Commentary

Over this year's first three sessions, all teachers were exposed to the Exploding Dots model and its use with bases two, three, and ten, as well as addition, multiplication, subtraction, and division. In the third workshop, teachers still

struggling with aspects of teaching the model in one or more of the basic operations were allowed to select review sessions of earlier material. We gave teachers time to discuss pedagogy with the session leaders, a local fifth-grade teacher who has used Exploding Dots in her classes and has taught it in a Math Circle workshop at Northern New Mexico College, and one of our coaches. Those session leaders, Delara Sharma and Barbara Bianchi, made the following observations:

Based on feedback from teachers, I began slowing down the pace during workshops, spending more time on helping teachers grasp concepts for cultivating number sense in their students. There were a lot of aha! moments for teachers when we were able to make connections between the standard algorithms (which are so deeply ingrained in our educational system) and Exploding Dots. Several lower elementary teachers commented that they are now able to introduce regrouping with much more ease using Exploding Dots. More than once teachers commented, “I wish someone had taught me subtraction using Exploding Dots!” The success of Exploding Dots with the MathAmigos workshops can be attributed to several things. Minimum prep and resources are required to implement the concept. A whiteboard and dry-erase pens or, alternatively, a sheet protector with a template is all that is required to teach the lessons. This fits well with our goals to ease and support implementation to whatever degree possible. [D. Sharma, personal communication, March 13, 2019 and May 22, 2019]

Our cohort teachers expressed concerns to session leaders Sharma and Bianchi about teaching Exploding Dots. They thought that it would be very challenging to actually integrate concepts like Exploding Dots into their curriculum. It was at times a struggle for teachers to understand the importance of building number sense and engaging in “number talks.” Some initial questions were, “When do I teach or introduce it?” “How do I find time to incorporate one more thing into my already tight schedule?” “Why should I teach Exploding Dots?” “How do I explain what I’m doing if my administrator walks in?” “How does this fit into my curriculum?” “How do I explain what I am teaching to my administrators?” “How does [a teacher] grade activities like Exploding Dots?” “What if parents are concerned about why students would be taught an alternative method of how to solve addition or subtraction by going left-to-right as well as right-to-left?” [B. Bianchi, personal communication, May 22, 2019]

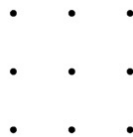
These are all reasonable concerns. After all, teachers are under many pressures including compliance with standards, school and district administrative directives, testing, and having students at far too many grade levels to teach effectively in a single classroom, among others. Some teachers may be concerned about having an inadequate background in mathematics (alternative certification does not require additional mathematics classes, for example). And after having so many new approaches imposed on them every few years, it is understandable that teachers would question yet another one. And what is the real importance of number sense, especially when most teachers learned how to work with numbers through algorithms and not through a sustained focus on the structure of a positional notation system and its benefits? Parents, too, may struggle with the notion that it really does not matter whether you multiply right-to-left or left-to-right! We believe that once our cohort teachers experience supported classroom use (via coaches), trust in us and our methods will grow; we have seen evidence of this in how rapidly this model has spread at our cohort schools.

Transitioning into our description of the Math Circle sessions, a question naturally arises: How can Exploding Dots help prepare our teachers (and ultimately their students) for Math Circles? While we consider this an open question, consider the low-threshold/high-ceiling aspect of Math Circles. Most of our Math Circle problems can first be approached at a much lower level than where they may lead. Sometimes a question accessible to an elementary school student will lead to areas of current research by mathematicians. Similarly, where at first glance Exploding Dots appears to be about place value and basic arithmetic operations, it leads into high school math with polynomials and infinite series, and well beyond that into college mathematics.

3.2.3 The Math Circle Workshop Sessions

Four Math Circle workshop sessions were conducted in the 2018-2019 school year (three were conducted prior to the writing of this article and one after).

October 2018. Teachers were asked to find how many triangles could be drawn in a square grid of nine points:



They were given time for exploration and were led through various problem-solving strategies. These strategies, in order, were:

- Guessing without exploration
- “Messing around” or “do something!” on dry-erase boards, leading to messy and difficult-to-follow results for a problem of this size
- Scaling the problem down to four dots and working your way up to nine
- Classifying and counting triangle types under rotation, reflection, and translation
- A combinatorial approach

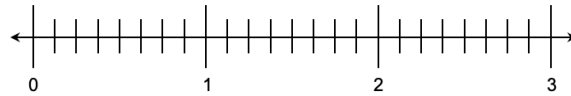
Teachers were given enough time in small groups to struggle through each of these approaches and then discuss them in the larger group. The advantages and limitations of each of the approaches were discussed. The process of leading towards those of greater generality was also part of the discussions.

Teachers expressed joy in these sessions. They reveled in the opportunity to take a problem apart with colleagues and to learn and use problem-solving techniques instead of just learning more math teaching methods. In the survey, teachers remarked how they could use this approach in differentiated instruction and in making sense of word problems. Many described how excited they were to try these approaches with students at the extremes (advanced students and struggling students). A teacher explained that she was now more inclined to help students grasp the underlying concepts of math practices rather than simply teaching formulas. It was clear in the post-Math Circle discussion that teachers recognized how unusual it was for teachers to have the opportunity to use strategies like those above – much less how unusual it is for students to get to do so.

We also had the great fortune to have district directors of curriculum and instruction attend this session, and they also came away with great enthusiasm for the Math Circle approach and for its effect on district teachers.

November 2018. Teachers investigated fractions using games and other activities to help develop number sense. Examples of activities include Fraction Nim and Variations, Fraction Comparisons, and Expression Puzzles:

Fraction Nim and Variations: Draw a number line from 0 to 3, marking every eighth of a unit. Starting at 0, two players take turns adding $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ to the previous result. The winner is the person who adds the final amount to get to the “goal sum” of 3.



Fraction Comparisons: Arrange the digits 3, 4, 5, and 6 in the four blanks below to make a true statement.

$$\frac{\square}{7} < \frac{2}{\square} < \frac{\square}{\square}$$

Expression Puzzles: Arrange 3, 5, 8, and 9 in the blanks of the expression below to get the greatest possible result.

$$\square - \left(\square \times \frac{\square}{\square} \right)$$

Following this Math Circle session, teachers expressed a more sophisticated sense of the uses of Math Circles in their classrooms. Some comments: “I already am integrating these strategies in my classroom – they add excitement and fun to Math time and deepen foundational understanding! I can integrate it in all my math lessons, starting with small groups and then with the whole group.” “I am now more open to outside-the-box thinking and games. I am more comfortable with allowing/encouraging student discovery.” “I just appreciate exposure to ideas of using games to teach math.”

January 2019. In this set of sessions, the choices for activities included a formal Math Circle about the mathematics of the SET card game and a round robin in which teachers formed small groups and explored JRMF activities [4] over the 80-minute session. The three sessions, in order, were:

1. SET

Teachers formed groups and learned to play the richly mathematical card game. We explained that the full game is played in four dimensions and that we could understand the game better in just two. Some questions we explored: “How many sets are possible in 2D?” “What game does 2D

SET resemble?” “How is 2D SET not like regular Tic-Tac-Toe?” We noticed that teachers had become animated and enthusiastic investigators and showed far less reticence than in the first workshop. They had become, as a group, far more willing to take risks.

2. JRMF Round Robin

Teacher groups selected from ten bins of JRMF activities [4], including ConHex, Color Triangle Challenge, Cookie Monster, Criss-Cross, Difference Engine, Estimation Games, Polyhedra Challenge, and Tower of Hanoi. Each group was able to investigate two to three activities. This session was the direct result of teacher suggestions elicited in the previous workshop and a familiar part of many workshops taught by author Taylor. We believe that these festival activities give teachers without extensive mathematics backgrounds an opportunity to learn an activity. With colleagues’ help, they can study a bit more and take this right into the classroom. One of our goals is to empower teachers to become more independent of experienced Math Circle leaders.

3. Teacher’s Choice

Teachers chose either the formal SET session from above or a JRMF round robin. This required two Math Circle leaders to run independent sessions.

May 2019. This three-day workshop was held post-submission of this article, and an abbreviated schedule is presented in the Appendix A. This workshop was run by the Santa Fe Public Schools, and the MathAmigos portion was integrated into its program. Our goals for this final workshop were to deepen our cohort teachers’ knowledge of Math Circles and Exploding Dots. Most of the Math Circle sessions offered new material to our teachers. We had another goal for this workshop as it was attended mostly by teachers from schools not a part of our cohort, but from all of Santa Fe Public Schools. MathAmigos and the district administration saw this as a way to spread our approach more widely. Two of our sessions were directed at only secondary teachers, one session having 21 participants and the other 29. Of the 44 total sessions held, 29 were led by MathAmigos and the rest (on more conventional topics) by paid contractors to the district.

MathAmigos-led Math Circle sessions included Grid Power [7] and Liar’s Bingo [5]. We also led a special Exploding Dots session on using the model to the right of the decimal point, including a bit of history about place value and the introduction of zero. Some of the MathAmigos sessions were identical to

ones in our earlier workshops, since many attendees were from schools not in our five-school cohort.

3.2.4 The Math Circle Workshop Sessions Commentary

The goal of this component was to cultivate cohort teachers as mathematical thinkers and to inspire in them ways to offer similar opportunities for their students. This was an unusual experience for many teachers, who in PD are often given only new curriculum to implement, rather than new ways to think. Problem solving, discussion, and unfamiliar areas such as combinatorics were introduced in the context of an “attractive puzzle.” That puzzle could even be a game with a lovely underlying mathematical structure, such as the SET card game or Brussels Sprouts [2], or one of the activities investigated in our workshops. Sometimes five or six problem-solving strategies were introduced along the way. Since learning the Math Circle approach takes time, personal investigation, and practice, and more than a little courage, Math Circles have penetrated more slowly than Exploding Dots.

Feedback from teachers has indicated that their views of themselves as math teachers are changing. Teachers have told us that they are trying out Math Circles independently, with our coaches assisting during their visits, and in after-school programs. Some teachers are even employing Math Circles when groups of students must stay after school in their classrooms to wait for rides home. (This was quite a surprise when relayed by the special education teacher at one of our cohort schools, and she said that she was not the only one at her school doing this.) One of our coaches noted that a teacher was giving more time to students to work things out, being less helpful in Dan Meyer’s [6] words. At one of our final MathAmigos committee meetings, one of our coaches told us that the gifted and talented program teacher at the school, also one of our cohort, had these students (grades 3-5) so enthralled with Math Circle problems that he had the entire class prepare for and enter a district math competition. That school not only won the competition, but three of these students placed first, second, and third.

One of our third-grade teachers has led Math Circle or festival activities five times in her classes, as well as introducing Exploding Dots. Towards the end of the 2018-19 school year she shared some comments:

I recall that when we were working on finding triangles in a 9-dot array² students were respectfully disagreeing with one another and defending their

²The [Math Circle Problem](#) taught in our first workshop

thinking about what a triangle is according to the constraints of the problem. [A. Ayala, personal communication, May 23, 2019]

Math Circles have helped me plan engaging lessons that prompt both higher order thinking and application of a wide range of mathematical concepts that directly align to Common Core State Standards (CCSS) [1]. They have opened a door to a new way of thinking and relating to math that makes mathematical concepts attainable to all students. I continue to be impressed by the realizations made by students who have an identified learning disability in math. Math Circles have transformed math in my classroom into an active learning experience that students enjoy and look forward to because it challenges them to think about mathematical concepts in ways beyond what has traditionally been taught in the classroom.

In the late spring of 2019 author Taylor led a Math Circle activity in this teacher's class, investigating strategies to win the game Brussels Sprouts [2] after playing several games. He found the students to be engaged to a high degree, and early in the analysis, one third grade girl suggested, "I think it matters who goes first and who second and whether you have even or odd numbers of Xs." True! And this seemed to make sense to many of the students. Clearly these students, and especially this girl, were already used to thinking mathematically about games.

3.3 Common Core Alignment

New Mexico is a Common Core state, and thus all of our Math Circle and Exploding Dots workshop materials included a list of alignments specific to the session being presented and to the grade levels of the teachers. This was desired by the school district administration and helpful to teachers in justifying introducing our materials into their classrooms. And while Common Core Standards for Mathematical Practice have many worthy aspirations, in implementation they often have a regrettably less desirable effect. As a colleague put it, "Common Core is meant to encourage this [more mathematical] type of thinking yet has had the opposite effect and led to more rigid curricula, thus the teacher and administrative fear of anything 'new' " [C. Rae, personal communication, May 22, 2019]. We believe that our program better supports Common Core's well-stated Standards for Mathematical Practice³ than more conventional approaches.

³<http://www.corestandards.org/Math/Practice/>

4 In-Class Coaching

In-class coaching is the most unusual component of our professional development model, and the component we see as most responsible for early and repeated implementation. In MathAmigos' first City of Santa Fe grant proposal discussions, James Taylor raised his concern about post-workshop follow-through. In Math Teachers' Circle and computer science workshops he had led in the past, teachers who were enthusiastic in the workshops nonetheless had low rates of implementation when they returned to the classroom. It is easy for even excellent and teacher-valued PD to go unimplemented in the face of teachers' competing demands. Taylor suggested that we include coaching follow-up to our PD workshops in the grant proposal. MathAmigos has thus recruited, trained, and compensated five retired teachers to attend our workshops, adopt one of our five schools, and schedule up to two classroom visits per workshop with each cohort teacher. We reached out through retired teachers known to the MathAmigos team in order to locate educators to whom we could present our vision and approaches to math education. Coaches received some initial training, and then attended each of the PD workshops alongside the teachers. Subsequent classroom visits included some mix of observation, assistance teaching new material, and post-class discussion with the teacher. The coaches recorded their experiences on a form and met to discuss them with the other coaches and the MathAmigos management team. Thus, coaching ensures that the PD is implemented. It gives teachers someone to lean on if they are unclear about the material. The coaches can start modeling the lesson or even take over instruction to help teachers with material about which they are uncertain. After class, the coach meets with the teacher to provide immediate, non-assessment feedback⁴ to the teacher. Finally, the coach can assist the teacher with non-mathematics issues such as classroom management, drawing upon the coach's decades of experience.

4.1 Coaches' Comments

Cohort teacher feedback on post-workshop surveys underscores the value of the coaching. Selected coaches' comments from classroom visits following two of the three Saturday workshops, comments from teachers passed on by coaches, and some additional comments can be found in [Appendix B](#). The coaches' remarks indicate a growing understanding of the material, and the mechanics

⁴The school district asked that our session evaluation not resemble assessment in any way. MathAmigos agrees with this, believing that we will get better cooperation from teachers in our cohort.

of first steps with new material maturing into a focus on pedagogy. One of our coaches contributed the following comments about her broader experiences in her role as a MathAmigos coach:

Many cohort teachers are open to thinking in a new way and open to trying new skills; however, some have great fear and so must be reassured by the coach. Teachers are feeling tremendous pressure to increase test scores. They continue to be inundated with mundane teacher duties. Teachers love this program because they can lean on the coach, with coaches starting the lesson or just being up on what is being taught so that we can intervene when necessary. Some teachers have no idea how to move kids through group work. They are unsure of the noise level, student needs for understanding, etc. “Do I give hints?” “How do I know they are understanding and/or talking about math?” “How do I tie this in with the curriculum needs?” So, so many questions that the coach deals with and can address at that moment.

After exposure to this way of teaching, methods and ideas, students are begging for math games, even for their holiday parties. They are enthused, they bumble through group work, but they are learning by feedback and more importantly, how lucky they are to be exposed to the most amazing tricks and knowledge of mathematics. [J. Ewert, personal communication, March 3, 2019]

These coach comments express something we had not expected: the benefits of having a highly experienced, fellow Santa Fe Public School teacher in your classroom as a coach and a coach for classroom and lesson management as well as for our mathematics approaches. They also show the surprisingly broad interest in math games and activities, something we addressed in the final Saturday workshop in January 2019. We see this interest as a manifestation of our teachers and their students beginning to think of themselves as mathematical investigators, and of seeing themselves as active participants in mathematics rather than passive consumers of decontextualized content and techniques.

5 Implementing Exploding Dots, Post-Workshop

Our teachers are using Exploding Dots far more than the other two approaches. This is clearly the most easily grasped and has the most developed set of supports, from introductory videos to handouts. As far as we have been able

to tell, all or nearly all of our cohort teachers have used this in their classes, and many are using it regularly.

One of the most surprising and striking uses of Exploding Dots has been in first and second grade classrooms. While our program is directed towards grades three and four teachers, we have acquired additional stipend funding for teachers in grades earlier and later in our cohort schools. We were concerned that Exploding Dots may not be as appropriate for earlier grades, based upon material found on the Global Math Project website. However, here are responses from two teachers:

I'm a 2nd grade teacher and have been using Exploding Dots for 2 months. Using the ED [Exploding Dots] method has really helped my students to focus, engage and use logical thinking to solve the problems. My students and I are very excited to practice this method in our math lesson everyday. I find that it is easier for the 2nd graders to use Exploding Dots in counting rather than numbers. The students are happy and have fun doing this activity. Currently we are working on three-digit addition – I have used mental math, place value, and expanded form – however when I teach Exploding Dots, the result is much better in comprehension and correct answers. Thank you for the opportunities for this special training, which has helped me so much also. You are welcome to come to my classroom and see the students working with Exploding Dots – I think you will be very pleased! [X. Krasniqi, personal communication, Feb 25, 2019]

I am a first-grade teacher and just started introducing Exploding Dots with my class during our Place Value unit. I feel the students have grabbed onto the concept much faster than the traditional method with the place value cubes and rods. This gives them a very visual representation of what is happening when we regroup. For years, I have had students physically regroup 10 ones as 1 ten but exploding the dots makes it very clear that the dots have changed places, not gone away. We can take those dots back and forth and the kids seem to just get it this way. It is very exciting because now we are moving into two-digit addition and subtraction and I am expecting this to be a much simpler concept for the students with this new method. I expect that my high students will be unlimited in their addition abilities with whole numbers by the end of the year. And I can already tell that this concept is extremely helpful for my struggling students and is putting grade level expectations in a presentation they can master. Unfortunately, I have just started introducing this method so I don't have a significant amount

of feedback, but I am very pleased with results thus far and excited about teaching concepts I would normally be a little hesitant about with young children. [A. Vateitei, personal communication, Feb 26, 2019]

Such enthusiasm from these early elementary teachers is both gratifying and a boost to the teachers in our core grades 3-4 group who will be teaching these students in the following school years. We also see how the Exploding Dots model may be more likely to be revisited in more advanced classes for topics involving polynomials and beyond. This teacher/student feedback is invaluable to us as we refine our approaches to teaching Math Circles and Exploding Dots.

6 Family Math Festivals: Teacher and Community Reinforcement

Our group has run three evening family Math Circles over the past year, with nearly all of the activities being JRMF activities [4] (including ConHex, Color Triangle Challenge, Cookie Monster, Criss-Cross, Difference Engine, Estimation Games, Polyhedra Challenge, and Tower of Hanoi) or games such as SET, Swish, and Blockus. We believe that this will help families support their children's math education and encourage them to use these activities at home, as well as to build community enthusiasm for mathematics.

The cohort second-grade teacher quoted in the previous section ran an Exploding Dots table at one of our family math festival evenings, using laminated, legal-sized box frames and handing out plain paper versions for families to take home. We observed that the table was always busy with a mix of adults and children.

We have evidence from cohort teacher reports that the festivals build upon MathAmigos PD efforts already in place in our five cohort schools and, in fact, expand our reach to teachers and administrators not in our program's cohort. Teachers at our cohort schools, both participants in our program and non-participating teachers, visited and studied table activities and asked if they could take home handouts and materials to try in their classes. Cohort schools themselves host the festivals and become partners in running them, thus demonstrating the strength of our partnership and their acceptance of our approach.

Since most of the table leaders at these festivals were teachers in our PD cohort, those teachers had a chance to extend their experience with leading participants through Math Circle-type activities and to learn new activities to try in their classrooms through direct experience doing them. This greatly

increases the likelihood that they will introduce these activities to their students. And their experience at the tables in leading students and their parents through the activities gave them direct practice with their target audience. All table leaders were given copies of the activity handouts a few days prior to the festival so that they could prepare. They arrived early at the festival so that MathAmigos staff could train them further and answer questions about the activity and how to run a table with students and parents.

6.1 Post-Festival Table Leader Responses

We mailed a brief questionnaire to many of the table leaders working the second festival and received the following responses from cohort teachers:

How prepared did you feel to lead your activity? “Completely. I am using the Math Games in my classroom and feel facile with most of them.”

Any thoughts I've not listed you'd like to mention? “I'm so thankful for the support to our community to grow in math knowledge.” From another cohort teacher: “I think the fun aspect of math is contagious, and I think having teachers work together on tables rippled out that MathAmigos spirit to teachers who have not even been coming to our seminars.”

These responses, along with casual visits to the tables by one of the authors, reinforced a general impression of enthusiasm and a deepened understanding of mathematics and the pedagogical value of the activities. Some of the table leaders had actually requested specific activities that they had tried out at our January workshop's festival activity sessions. The first of the two math festivals was held in the gymnasium at a large 800-student, K-8 school with few participating MathAmigos teachers. We were gratified to find many of this school's teachers, not in our cohort, (about a third of the total table leaders) arrive to become table leaders. This after-school participation suggests that the enthusiasm for our program had spread widely at that cohort school. This school will be hosting the first weekly formal student Math Circle in Santa Fe in the 2019-20 school year.

7 MathAmigos Program Measurement

Our program was not originally designed as a research project, but rather as one to help transform a school district performing poorly in mathematics. Yet MathAmigos has conducted surveys from the very beginning of our projects,

even before there was any sort of institutional or grant requirement for data. The summaries and group queries of our coaches and teacher cohort in this article came from these surveys. We are primarily interested in serving the community, and so we need feedback to develop our approach and curriculum. Our surveys have had many consistent questions, but as we refine our work we have had to modify and add questions to these anonymous surveys.

One consistent set of three questions across the workshop surveys was about whether the Math Circle and Exploding Dots sessions were valuable, whether the teachers understood the concepts, and whether the teachers understood how the practice could be integrated into their classrooms. Consistently, the Math Circle and Exploding Dots sessions rated close to five (highest) on a one to five scale. As we developed our surveys for subsequent workshops, questions about attitudes towards mathematics and its teaching have been replaced by questions about changes in pedagogy resulting from the experiences and techniques learned through attendance at our workshops and work with our coaches.

It is clear from this that our teachers are already learning to think differently about mathematics and mathematics teaching. We believe that even our limited surveys and observations show the effectiveness, not only of our three-pronged approach to introducing more mathematical thinking, but of our highly collaborative approach to partnering with the school district and community groups. It is our hope that our admittedly modest accumulation of data and narrative will contribute to the limited literature on implementation of Math Circles and Exploding Dots.

8 Response from the School District

Santa Fe Public Schools director and assistant director of curriculum and instruction and the district math coordinator are part of our MathAmigos managing group. They have expressed interest in integrating Exploding Dots and Math Circles into the district-wide curriculum and into curriculum maps for mathematics. The school board is aware of our work as well and has publicly praised MathAmigos as a model for community-district collaboration. A school board member has regularly attended our workshops and family math nights.

In partnership with the federal 21st Century Community Learning Centers program,⁵ the Santa Fe Public Schools district is also supporting MathAmigos' introduction of a weekly student Math Circle for grades 5-7 in the fall of 2019

⁵https://www.sfps.info/teaching_learning/curriculum...instruction/21st-century_schools/

at a large K-8 school in our MathAmigos school cohort. Grant funding for this was awarded in March 2019.

9 The MathAmigos Community Model

As stated in our abstract, our program is also about outreach, both in the composition of the MathAmigos management team and in the community organizations we partner with and receive funding from. We work with the Santa Fe Public Schools district (administration, teachers, students, and parents) and the City of Santa Fe's Children and Youth Commission⁶ (our funders via a two-year contract), Santa Fe Birth to Career,⁷ and Poder Familiar.⁸ Our MathAmigos group of partners meets weekly. MathAmigos also has multiple members on the district's math advisory committee. In a narrower sense, we also seek to develop and strengthen a community of math teachers in Santa Fe and northern New Mexico in general – up to and including a future formal organization of such.

10 Moving Forward

Since we are only in year one of our current two-year program, we are working to improve and make more uniform the adoption of our practices. Not all schools have embraced these approaches equally well, despite the best efforts of our cohort teachers – and for a variety of reasons. Policy and recommended practices do not always move smoothly from district administration to school principal to faculty. Punitive school and teacher assessment systems⁹ have made some teachers and school administrators wary of implementing new practices. And, of course, natural variation in teacher engagement and a school's teacher morale contribute to adoption. Ongoing conversations between MathAmigos and the school system seek to address these issues. We believe that MathAmigos can help by improving communications from district to classroom through our broad-based leadership team, most of which is not part of the organizational structure of the school system, and thus can bring an independent perspective.

Building upon the groundwork of our curriculum, we would like to see school-centered programs begin to arise, such as the student Math Circle set

⁶https://www.santafenm.gov/children_and_youth_commission/

⁷<https://www.santafeb2c.org/>

⁸https://www.earthcarenm.org/poder_familiar/

⁹Such as the State Public Education Department's former opaque A through F school grading system.

to begin at one of our cohort schools in fall of 2019. Other possibilities include efforts to get students involved in math competitions such as the statewide UNM-PNM contest, the American Mathematics Competition, and the Mandelbrot. We have had conversations with cohort teachers about starting SET matches, or even Math Wrangles¹⁰ (math debates) between schools. While one of us has run a Math Teachers' Circle for a number of years, it is not currently operating, and we will be applying for grants to operate such a Math Circle, one in which we would pay participants stipends. MathAmigos and the district are currently in discussions about making a joint grant application to start such a public school Math Teachers' Circle in school year 2019-20.

Since the school district has shown interest in building some of our practices into the wider curriculum, we further hope to see other schools embrace our initiative. Scaling up to a district level will require more of our cohort teachers to become change agents within the system. One or more of our cohort teachers in each of our five cohort schools are such exceptional adopters and proponents that we see them as future Math Circle leaders and teachers of Exploding Dots. Further, we saw the end-of-year three-day math PD workshop in May 2019 as a way to introduce our methods district-wide.

As the district is beginning their mathematics text adoption cycle in 2019, MathAmigos has been asked to participate in the process. Our suggestion that the district consider a pilot adoption of Art of Problem Solving's (AoPS) elementary-level *Beast Academy*¹¹ series and their older secondary-level books is receiving serious consideration. Further, district curriculum administrators are considering introducing a new middle school problem-solving course for accelerated algebra students that would use one of the AoPS problem-solving texts.

To further support our efforts, we will continue to increase the rigor, reach, and consistency of our evaluation efforts. From the start we have engaged the school district's Department of Accountability and Assessment to understand the state of mathematics education here, and we hope to increase cooperation in measuring student, class, and school outcomes in our cohort schools. These efforts are still in their early stages, primarily due to district concerns about confidentiality.

More ambitiously, we would like to see other New Mexico communities find ways, using our model, to build such sustainable community groups, eventually forming a stronger network of math teachers. Each community has its own special people and educational resources and problems, so such efforts must

¹⁰http://sigmaa.maa.org/mcst/documents/math_wrangle_rules_revised_9feb12.pdf

¹¹<https://beastacademy.com/>

be driven from within each community by those who understand those unique assets.

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Appendix A

End of Year 1 (May 2019) MathAmigos Workshop Math Circle and Exploding Dots sessions only

Time	Tuesday	Wednesday	Thursday
10:30 - 12:00		Math Circles I for High School Teachers	Math Circles IIc, "And the Numbers Came in Sixes, Red & Black" grades 3-12
1:00 - 2:20	Exploding Dots Basic: Addition and Multiplication	Exploding Dots Intermediate	Exploding Dots, "Decimals and Beyond"
	Math Circles I, "Introduction"	Math Circles I, "Introduction"	
2:35 - 4:00	Exploding Dots Basic: Addition and Multiplication	Exploding Dots Intermediate	Math Circles IIc, "The Joys of SET"
	Math Circles IIa, "The Versatile Grid"	Math Circles IIb, "Counting Questions" OR "Farm Field Sensor Challenge"	

Appendix B

Selected Coaches' Post-October Workshop Comments

- All four teachers that she observed used Exploding Dots. Coaching is an incentive to apply what they learned. Teachers were excited, so kids were excited. Noted the tie-in to renaming when adding and carrying over, also called “trading up.”
- A second-grade teacher did Exploding Dots. [The coach] took over the lesson since [the] teacher seemed unsure. But there are language and vocabulary barriers. The next time [the teacher] did a Math Circle and did a good job.
- One teacher wanted [the coach] to do the Exploding Dots teaching. [The coach] let two teachers who were observing do all of the [student] helping. In the second, visit the teacher wanted a Math Circle; she started [the lesson] out, then handed it off to [the coach]. She noted that all the kids were riveted.
- [The coach] worked twice with the special education teacher; a small group of kids of mixed levels and learning issues—some lacking English, others with challenges that are cognitive. [The coach] started a lesson and [the teacher] took over. Afterward [the teacher] talked about how to take the combinatorics aspect of the lesson further.
- [The teacher] wanted [the coach] to show how Exploding Dots should be started. [The teacher] then started helping her students with their work and ultimately went to the board and continued the lesson on her own. Here we see our coaches and cohort teachers adjusting to the coaching system and to their initial use of our MathAmigos materials and techniques. The teachers needed help at various points, and our coaches were there to assist or even take over the session. Sometimes the coach encouraged the teachers to assist with the class, even if the coach led the class through the activity. Sometimes the coach started the session, as in the case of the special education teacher, and the teacher quickly understood where to go with it. In this particular case, the teacher did an even better job with the topic than the coach due to her being a native Spanish speaker and understanding her students and their thinking styles, and her impressive grasp of the mathematics. In the case of the teacher who began the Math Circle session, then handed it over to the coach, we saw the pedagogical

and emotional benefits of watching an expert lead and model a Math Circle with her own students.

Selected Coaches' Post-November Workshop Comments

With this set of coaches' observations, we recorded the types and number of sessions observed. They were: Exploding Dots (11 sessions); Math Circle (3 sessions); Cuisenaire Rods (1 session).

- [Ask the teacher to suggest] students write problem in traditional way after doing Exploding Dots.
- Give problems to group while working with individuals; let students talk together on solutions.
- Need more time during workshop with teachers to discuss ideas and more time for each activity. We felt rushed. [Need] lots of processing time for Math Circle sessions.
- [Teachers need] more exposure to Exploding Dots so that teachers become experts, especially with division and double-digit multiplication.
- [Teachers need to get comfortable with] terminology for Exploding Dots, as well as with Math Circles in English for Spanish-speaking teachers.
- The teacher expresses joy and excitement that her students now understand, after only two days of practice, how to do borrowing and 3-digit subtraction!

Selected Comments from Teachers Passed on by Coaches

- She (the teacher) hated learning math, doing math, and teaching it until MathAmigos and now she loves it.
- My students are seeing connections in addition/subtraction using Exploding Dots! Even some of my struggling kids are getting concepts!
- Exploding Dots is a great tool for helping students who might struggle. The students enjoyed it.