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Dots Explode in Hawai‘i

Veny Liu
University of Hawaii, West Oahu, venyliu@hawaii.edu

Laurie James
University of Hawaii, West Oahu, ljames7@hawaii.edu

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Veny Liu *[a] and Laurie James[b]

[a]University of Hawai‘i West O‘ahu
[b]University of Hawai‘i West O‘ahu

Teachers are wonderful advocates of mathematics for future generations, and they are continually looking for ways to get students more engaged in mathematics. Through visuals and hands-on activities, the Exploding Dots concept can help teachers and students understand many elementary arithmetic and algebra topics. The implemented tasks promote problem-solving by allowing multiple entry points and varied solution strategies. This paper explored this idea beyond drawing clusters of dots by Locking Legos activity. With a thorough understanding of math content, participants in multiple Math Teachers' Circle of Hawai‘i (MaTCH) meetings expressed confidence in creating and developing meaningful and relevant differentiated learning opportunities, which include teacher candidates through classroom activities and demonstrations. This paper presents these participants' experiences of a simple concept that grew into a mathematical story.

Keywords: Exploding Dots, Global Math Project, Math Teachers' Circle, Place Value

1 Introduction

Place value is often perceived as a difficult concept to grasp and to teach [7][12]. Research shows that many of the errors students make appear in Algebra and Arithmetic to be due to a lack of place value understanding [15][6]. For example, students not connecting the fact that 3 tens equal 30, instead the students often have a misconception that 30 tens represent 30. Other common examples for elementary students were the errors in multiplying multi-digit algorithms

*Please address all correspondence to Veny Liu, venyliu@hawaii.edu.

and decimals. Understanding this concept is therefore fundamental to mathematics performance, and it is positively correlated to variations in mathematics achievement [17][3][6]. Without a grasp of this concept, it becomes challenging to have students learning mathematics in a meaningful way.

As the best advocates of mathematics for the next generation, teachers can help to dispel biased views and perceptions surrounding mathematics. Teachers can change the teaching and learning mathematics to help liberate students from fixed ideas through growth mindset messages and a multi-dimensional approach [2]. Implementing interactive activities is a powerful way to inspire mathematical thinking and discourse [16]. In reality, teaching mathematics content the same way year after year can become mundane, and even experienced teachers can find themselves lacking creativity in the classroom. Furthermore, due to Hawai'i's geographic location, access to professional development workshops can be limited. Concurrently, the statewide average for teachers' retention at the same school for five or more years was 52.3% [11], which illustrated a high staff turnover problem.

One way for teachers to reinvigorate their creativity into their instruction is to put themselves back in the learner's role with peers [22]. On the island of O'ahu, this can be done by participating in the Math Teachers' Circle of Hawai'i (MaTCH) meetings with other educators. MaTCH is a part of the national Math Teachers' Circle (MTC) [14] network that emphasizes ways to create meaningful mathematical experiences that can be implemented in any classroom setting. In 2011, MaTCH was started at the University of Hawai'i at Mānoa, located in the southern part of the island of O'ahu. The fundamental beliefs of MaTCH involve the development of personal mathematical and pedagogical knowledge through ongoing dialogue. The MaTCH meetings focus on promoting hands-on activities to embrace, engage, and empower learning through productive challenges that ignite participation in mathematical problem-solving activities. The goals for the MaTCH meetings are:

(1) To gain a deeper understanding of mathematical processes through the direct experience of doing mathematics,

(2) To promote the development of teachers' content knowledge by engaging in professional discussions and collaborative activities focused on the teaching of mathematics, and

(3) To promote open-ended problem-solving as a way of learning and practicing mathematics in the classrooms by providing teachers with guidance consistent with the Common Core State Standards (CCSS) [4] and Standards for Mathematical Practice (SMP).

The MaTCH meetings are advertised via the University of Hawai'i MATCH Listserv and MTC website. It provided a unique opportunity for K-12 teacher candidates, classroom teachers, post-secondary mathematics educators, and education researchers to experience math problems. These mathematical sessions began with concrete activities, which allowed participants to use their prior knowledge to work through each part of a problem. This style of learning placed all participants back into the role of a mathematics learner by engaging in discussions with other educators and exploring how they can recreate these kinds of experiences for their students.

The MaTCH leaders wanted to help all participants become more aware of their math identities and to motivate them to engage in diverse math activities. Each MaTCH meeting agenda was designed to instill a productive and positive learning environment where educators at all grade levels were comfortable taking risks, asking questions, and making conjectures through inquiry. By implementing hands-on math activities and games, the focus was on cognitive strategies designed to improve analytical problem-solving and critical-thinking skills, moving away from doing math problems in a traditional lecture-style setting. Overall, the math sessions emphasized effective pedagogy, enhanced teachers' content understanding, and aligned each topic to at least one of the SMPs.

By assisting the need for quality mathematics teachers across all grade levels in the state, MaTCH has evolved to become a professional learning community of teachers, mathematicians, and mathematics educators, where they meet regularly to engage in mathematical problem-solving activities and pedagogical discussions [5] [13]. In the past four years, MaTCH has expanded to the Leeward (western) side of O'ahu, a region that serves a higher percentage of Native Hawaiian and lower-income populations. Various math sessions have been held during the summer break and on Saturday mornings at the University of Hawai'i at West O'ahu or local public and private schools throughout the island. The meetings ranged from two to four hours, with the first part devoted to working on mathematics and then transitioned to pedagogical discussion to meet the teachers' changing needs.

1.1 The Journey of Dots

Exploding Dots was developed by Dr. James Tanton, formerly a high school teacher for a decade and now the Mathematician in Residence at the Mathematical Association of America (MAA). He is a mathematician who is genuinely interested in bridging the gap between the school mathematics experiences and the creative mathematics practiced and explored by mathemati-

cians [20]. His Exploding Dots was inspired by a "chip firing" model developed by German educationalist, Arthur Engel, in the 1970s, which also matches the workings of an Asian abacus with beads and rods [21]. Tanton used the abacus models back to reconnect with the K-12 curriculum by playing with dots placed in a row of boxes to illustrate the mechanics of place value. His concept moved beyond place value and school arithmetic to algebra and can extend to undergraduate work such as Fibonacci numbers and infinite series.

The first session of Exploding Dots in Hawai'i was presented to the authors in Fall 2016. After attending the initial session, two professional educators were trained to become MTC leaders. The MaTCH facilitators coordinated the October 10, 2017 launch of the Global Math Project [8] with the September math circle meeting. Global Math Week introduced the Global Math Project, which was known for bringing Exploding Dots to the world [21]. This worldwide event helped to connect millions of students through a shared experience of mathematics. Exploding Dots begins with the known mathematics and transforms into more thought-provoking concepts. Besides being used in the classrooms and math clubs or circles, the Exploding Dots web app is played by over 5.5 million students from over 170 countries and territories [19][18].

Since participating in the Global Math Project in 2017, four Math Teachers' Circle sessions and five local professional development sessions in Hawai'i have been organized around the Exploding Dots content. Therefore, this article aims to show how MaTCH sessions surrounding the Exploding Dots theme have been implemented into multiple activities in various educational settings and analyze to what extent these sessions have inspired educators.

2 The Extension of Dots at MaTCH

Inspired by the Global Math Project, the first fall MaTCH meeting on the Leeward side of the island was intended to create low threshold high ceiling tasks allowing each participant to work at their level of engagement. Each session allowed participants who felt less confident to stay close to the original task and consolidate their understanding, while those who felt more confident had a chance to explore. The MaTCH facilitators began with an ice-breaker activity that allowed participants to work at different paces without the worry of judgment. Throughout the session, participants discussed aspects of the action, were asked probing open-ended questions, and discovered mathematical knowledge.

After the welcome and introduction, a Locking Legos (Appendix A) ice-breaker activity provided time for participants to collaborate in promoting a positive learning space. The Locking Legos activity was designed to use

a pile of colored Legos and the Locking Legos handout (Figure 1) to solve patterns using three rules. First, when moving the Legos from right to left, the value changes according to the code provided. Second, if moving from left to right, the "opposite" code must be applied. Third, whenever a Lego is moved from one space to another, the Lego must match the color on the handout. The facilitators modeled these actions by moving Legos from right to left with the value of the Lego diminishing by 1/2 each time. Next, the pieces were moved from left to right using the rule of doubling each time. The example included placing seven red Legos in the red space on the handout. Two red Legos were picked up and "locked together" trading for one yellow Lego that was placed in the yellow area. Once all the possible red Legos were moved, the pattern repeated by locking together every two yellow Legos trading for a blue Lego and moving to the blue area. The pattern continued to the left until no more Legos could lock together.

Legos.JPG

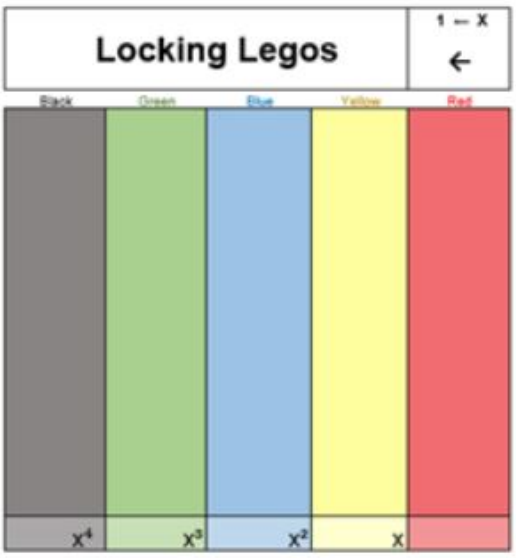

Figure 1. Locking Legos Handout

Having introduced the task, participants worked with a partner and used the Locking Legos handout to solve the next code by starting with 10 red Legos. After the code was identified, they challenged each other to solve more codes with different quantities of Legos. This warm-up activity was designed to start a dialogue while identifying patterns in math. This led to the main portion of the MaTCH meeting focusing on the Exploding Dots.

The Exploding Dots activity began by having participants watch a short video on the Exploding Dots Puzzler [8]. Afterward, participants were asked, "What did you notice and discover from the short video?" The most popular responses were: "two bubbles disappeared into one", "binary numbers", and "place value." A connection was made from the Locking Legos ice-breaker to the video. Alternatively, the Exploding Dots Puzzler may be presented before the Locking Legos.

Next, the facilitators modeled how to bridge concepts from a place value number system with hands-on learning using an array of horizontal boxes that contain several dots. The purpose was to move the dots between the boxes to solve the math problem using specific codes. A demonstration for the

Exploding Dots Machine was provided using the large whiteboard. After that, participants used small individual whiteboards to solve other codes for a $1 \leftarrow 2$ machine, $1 \leftarrow 3$ machine, and $1 \leftarrow 10$ machine. At this point, participants at each table discussed what they learned and shared their method for breaking the codes.

When the participants were asked to come up with codes for others to solve, the activity moved beyond the initial Exploding Dots Machine. After exploring, the participants noted that addition was grouping dots together like traditional addition. Furthermore, with each explosion of dots, a code needed to be implemented. For subtraction, participants mentioned the concept of regrouping, which meant the dots needed to be "unexploded." The idea of Exploding Dots offers flexibility in operating within any base system. The Exploding Dots session inspired mathematical thinking while developing and replicating experiences that built a solid foundation for enhancing math skills. The emphasis was placed on identifying multiple ways to learn, which builds a classroom culture requiring engagement and productive participation. To create more opportunities for higher-order thinking, participants made connections to the Standards for Mathematical Practice (SMP). The SMPs describe types of proficiency and processes designed to ensure that students connect and apply concepts. The National Council of Teachers of Mathematics [16] identified each of the eight SMPs to strengthen mathematics teaching and learning.

The SMPs were discussed and expanded upon to develop appropriate instructional strategies for all grade-level bands. The Exploding Dots activity ignited learning through productive challenges that include: 1) looking for patterns, 2) collaborating as mathematical thinkers, and 3) exploring new approaches for solving problems that correlate with the SMPs. The MaTCH meeting highlighted SMP#4 to model with mathematics, SMP#7 to look for and make use of structure, and SMP#8 to look for and express regularity in repeated reason. Participants walked away with practical activities that shifted the focus in the classroom from direct instruction to a supportive learning environment through active participation.

It was essential to establish a positive learning atmosphere in every MaTCH meeting where participants could make mistakes and persevere while building a strong foundation in the mathematical practices of active learning. The sessions evoked diverse feelings and emotions as participants were asked to share out and have conversations that highlighted their takeaways. Some participants struggled or felt uneasy during particular portions of the MaTCH activities. These feelings would be similar to how their students feel encountering challenges in the classroom. With a supportive environment, they began

to find inspiration through their various interactions and hearing how other educators process math differently.

3 The Extension of Dots beyond MaTCH

To reach more individuals to join the Global Math Week in 2017, the MaTCH facilitators presented this topic to students at the University of Hawai'i at West O'ahu (UHWO) through the Math+Science+X Seminar and class lectures. In promoting 2018 Global Math Week, the MaTCH facilitators presented *Exploding Dots* and *Going Beyond Exploding Dots* at the Hawai'i Council of Teachers of Mathematics (HCTM) Conference. Additionally, at the 2019 Pacific-Basin STEAM Teaching Conference on the Big Island of Hawai'i, the MaTCH facilitators lead two professional development sessions titled *Locking Legos Machine to Lifesize Manipulatives: Engaging Hands-on Activities for Elementary Students* and *A Mathematical Journey of Dots*.

3.1 Teacher Candidates

Sari & Olkun [17] suggested that addressing and developing meaningful and appropriate interventions for learning place value concepts early in students' development is essential. Since a professional learning community is an effective method for changing teaching practice within a school or university [22], MaTCH meetings provided opportunities for classroom teachers and teacher candidates to further their math knowledge and discuss innovative pedagogical approaches, to be life-long learners, and to be a part of a professional learning community.

Each semester, the elementary and middle-level teacher candidates took an active role in their learning by attending the first MaTCH meeting focusing on place value using the Locking Legos and the Exploding Dots Machine. This allowed the candidates to expand their professional development as well as observe their university professors modeling a similar math lesson focusing on Global Math Week. Even though some activities were familiar to the candidates, they were expected to think like their students when working through the problems.

3.1.1 Modeling Lessons for Teacher Candidates

Modeling lessons for teacher candidates consisted of the following. First, the professor created a detailed lesson plan, so the teacher candidates could follow along and participate during the lesson demonstration. The lesson plan

(Appendix B) identified the goal, objective, learning intentions, success criteria, and mathematical practices. The lesson plan and manipulatives were distributed ahead of time. At the beginning of the demonstration, directions were given for the expectations of the lesson. Then, the professor explained that the math lesson would be a review of the MaTCH meeting and continue to build upon the topics of place value and algebra. Finally, to model best practices throughout the lesson, the professor identified scaffolding techniques, checked for understanding, explained modifications for diverse learners, and provided frequent feedback. At times, the professor shared professional experience to draw their attention to a specific teaching strategy or to explain why something was taught a certain way.

Before moving to the next section of the demonstration, candidates were provided with an opportunity to reflect on their learning, ask questions, and discuss the teaching strategies. This allowed for in-depth exploration of modifying perceptions surrounding the difficulty of learning math regardless of prior experiences. Furthermore, it allowed candidates to continue to build the knowledge, skills, and dispositions necessary to become effective math teachers. The following was observed during the lesson modeling: elementary teacher candidates appeared to struggle more with the algebra concepts toward the end of the lesson, while the middle-level candidates had difficulty breaking down the skills at the beginning of the lesson. Overall, modeling this lesson created a positive shift in how math problems were viewed as patterns emerged from one activity to the next.

3.1.2 Teacher Candidate Practice Lesson

Throughout each semester, the teacher candidates learned various teaching strategies to help reinforce their creativity, improve communication skills, and improve collaboration with other educators. Participants engaged in in-depth conversations to explore differences and commonalities regarding how they arrived at their answers. Also, to help reduce math anxiety and create a positive learning atmosphere, the participants were provided with an ample amount of time to absorb and process mathematical information. In summary, participants were allotted numerous opportunities to deepen their connections to mathematical procedures and to help build their confidence before teaching independently in the field.

After participating in the MaTCH meeting and observing the lesson demonstration, a crucial component of knowledge integration was to have the teacher candidates teach a math lesson to their peers and university professor. Choosing from multiple lesson plan templates to create their lesson, the candidates

modified the warm-up and Exploding Dots activities because some sections were not age-appropriate. For those candidates in the primary classrooms, they were able to visualize and adjust to meet the needs of the younger students by using different manipulatives and phrases such as (1) cubes will cluster, (2) Play-Doh will dissolve, (3) stones will secure, (4) magnets will stick, (5) kids will connect, and (6) chalk will combine. These changes personalized the activity allowing for math lessons to be fun. Furthermore, by teaching their lesson in the university setting, the candidates received immediate feedback from their peers and professor. They were able to refine the lesson objective, adjust the pacing of each activity, and determine the best times to use informal assessment to check for understanding.

For the middle-level teacher candidates, they were given one extra learning opportunity to practice teaching Exploding Dots. This was done by using James Tanton's Middle-School Version of Mathematical Thinking materials [8] as a guide for an extended MaTCH breakout session. This additional lesson demonstration included background information needed to build a strong foundation in the mathematical learning process for middle school students. This lesson also demonstrates how to extend place value to arithmetic division and polynomial division by drawing clusters of dots with simple procedures without memorizing how to perform long division (Figure 2). This model can also extend to more concepts with a fraction or negative bases, Fibonacci sequences, and unsolved research problems in mathematics [8].

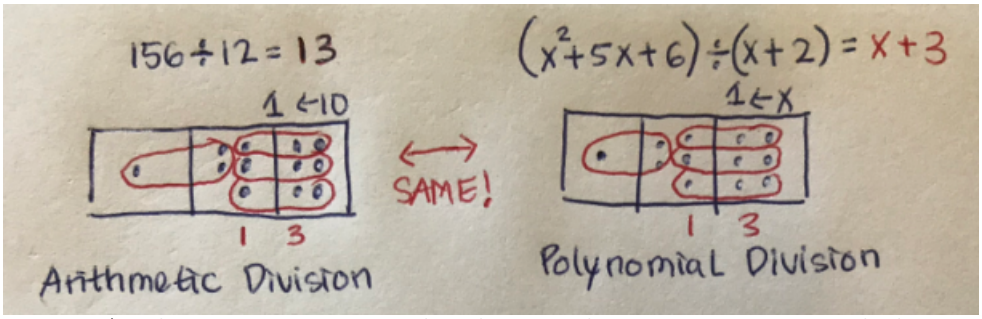

Figure 2. Arithmetic Division and Polynomial Division using Exploding Dots machines

4 Teachers' Responses to the Exploding Dots Approach

Analysis of the discussion of this paper stems primarily from the open-ended free writing responses administered online at every MaTCH meeting (pre and post-session). The collection of this free writing began to answer in what ways the participants benefited from the MaTCH meetings surrounding the

Exploding Dots theme. Over the last two years, the Exploding Dots MaTCH meetings generated 58 pre-session free writes and 86 post-session free writes. At the beginning of each session, participants wrote their thoughts and ideas regarding the math topic. Then, at the end of the session, they elaborated on their learning of new mathematical knowledge and the pedagogical piece of the activity.

Although place value is included in Common Core State Standards, the pre-session free writing responses from the MaTCH meetings revealed that 40% (n=58) of the participants reported their students struggled to grasp the place value concept. As stated from the Section 1, examples include students not connecting the fact that 3 tens equal 30, instead the students often have a misconception that 30 tens represent 30. This leads to errors in multiplying multi-digit algorithms and decimals. Overall, the importance of place value was addressed by 26% (n=58) of the participants who stated that it was a critical aspect and crucial developmental skill needed in the elementary classroom in addition to understanding the value of a digit that supports upcoming mathematics concepts.

4.1 Mathematics Content Knowledge

There is reason to believe that improving teachers' knowledge in mathematics will translate into increased student achievement so that raising mathematical content knowledge is a critical component of teacher professional development. [1]. The Standards for Mathematical Practice (SMP) specify the criteria to engage students in making connections among mathematical representations, deepen understanding of mathematics concepts and procedures, and use tools for problem solving that build procedural fluency from conceptual understanding [16]. The MaTCH Exploding Dots sessions aimed to strengthen participants' mathematical content knowledge and provide new insights about the mathematics topic of place value. According to the feedback, over 60% (n=86) of participants explicitly stated that the Exploding Dots sessions increased their understanding and that they are more confident to engage in dialogue about place value with students and other educators.

Participants further reported that the MaTCH sessions helped them recognize that the decimal system is base 10 and the binary system is base two. Participants also learned numeration systems from other cultures, such as Babylonians use base 60, French use base 20, and Hawaiians use base four. This allowed participants to incorporate a multicultural perspective into their lesson planning and expressed their enlightenment. As a participant commented, "It was eye-opening to find out bases could be decimal, rational,

irrational, negative, and imaginary." Others expressed how easy it would be to implement Exploding Dots into their classroom setting by using dots and boxes to model math operations with any base. Some teachers stated, "It would be a great representation that everyone could understand", "It's basic math but can be applied for higher-level math as well", and "It could demonstrate mastery of counting forward and backward."

The Dots activity triggered excitement from the participants as they wanted to practice a few problems after each session. The deepened understanding of place value gained from these sessions inspired teachers to implement a "new game" into their math lessons. One teacher remarked, "I always hated learning about the different bases, but this was fun, challenging, and made some things finally 'click' for me. I can't wait to try an adapted version with my class!"

4.2 Pedagogical Piece

A precursor to introducing of any math activity is having a well-thought-out pedagogical lesson plan, which will improve the way students learn both independently and collaboratively. The process of gaining a deeper grasp of fundamental material builds on scaffolding prior learning leading to students having greater confidence in their capacity to make sense of mathematics. As suggested by Harris & Sass and Goe [10][9], MaTCH's primary objective is to improve pedagogical approaches and content knowledge through professional development meetings.

A pedagogical component was built into each MaTCH activity, taking into consideration possible mathematical misconceptions. Themes that resulted from the MaTCH meetings included *Joyful and Fun Mathematics*, *Using Academic Discourse with All Students*, and *The Common Core State Standards*. One teacher believed Exploding Dots to be "one of the most important topics to help students understand any mathematical content." Other teachers struggled to keep math classrooms engaging for their diverse students as one participant stated, "fun activities help students understand the mathematical topics at the conceptual level."

The findings from the free writing responses suggested that participants' perspectives changed on the methods of teaching mathematics. As one teacher said, "Explaining how you got your answer is just as important as having the answer." Another teacher pointed out that, "Students not only need to know the content, but they also need to be able to have strategies under their belt to be able to solve math problems." Asking learners to think about what they already know will help guide them when working through the problem,

followed by discussing techniques or strategies with others deepens their enduring understanding. As their viewpoints began to shift, their self-confidence increased.

Many participants indicated that teaching diverse students can be challenging. As some students struggle to grasp concepts, it is necessary to create hands-on activities to differentiate learning. By being knowledgeable of the SMPs, teachers can create experiences that encourage students to build on previously learned knowledge. Two teachers pointed out that unpacking the SMPs was valuable because they "got a deeper understanding of them" and shared how "It was evident how practices can be incorporated and embedded in daily lessons and plans." Another teacher reported, "It was really helpful to break down all the big words into ideas that are easier to digest and understand. The Frayer models were especially helpful with the more abstract ideas." Several comments reflected the participants' appreciation that the math sessions set aside time to break down the SMPs.

4.3 MaTCH Participants' Benefit from Exploding Dots Sessions

This section focuses on determining the positive outcomes that participants ascribed via the free writing responses to MaTCH Exploding Dots involvement. Responses were analyzed, and five themes were identified (Table 1). Furthermore, Appendix C provides a snapshot of the highlighted comments about the benefits of participating in the MaTCH Exploding Dots sessions.

Table 1

The five themes and number of highlighted comments.

Symbol	Themes	# of highlighted comments
A	Place Value Mathematical Understanding Increased	26
B	Knowledge of SMP Increased	25
C	Perspectives Changed	10
D	Beliefs in Teaching Increased	11
E	Confidence in Place Value Discourse with Peers Increased	6
F	Confidence in Place Value Discourse with Students Increased	5

4.4 Teachers' Questions

For the post-session free writing responses, the participants responded to the new insights gained from the math topic of the day. An analysis of those responses indicated that 13 questions were correlated to the teaching pedagogy of Exploding Dots, eight questions were about Exploding Dots mathematics content, eight questions were related to general teaching pedagogy, and four questions on SMP. Furthermore, participants also made six personal reflection comments, and additional three miscellaneous comments were accolades about the sessions. For more detailed information on these themes and comments, please refer to Appendix D.

The theme correlated to the teaching pedagogy of Exploding Dots was further analyzed, and it revealed that participants reported confusion about the appropriate age to introduce the concept as well as within various bases and classroom activities for different grade levels. The Exploding Dots mathematical questions covered real-world applications and algorithms for multiplication and division. The questions that were categorized as general teaching pedagogy ranged from establishing a positive learning environment to empowering students to share their thoughts and provide relevant engaging lessons beyond a math class.

In summary, the hands-on activities helped the participants to think outside the box and bring the joy back into learning. According to one participant:

> I learned that math is a process and that giving students a chance to struggle and succeed makes learning that much more powerful. Also, that it is our job as professional educators to bring a sense of fun and joy to math through our attitudes and experiences that we share with our students.

Another participant stated that "doing the hands-on activity gave everyone a concrete visual for an abstract concept that may be hard for students to understand." An additional participant commented, "Exploding Dots helped reinforce my belief that simple hands-on lessons have the potential to reach higher math concepts. It helped remind me to bring the joy of math to students." Overall, the majority of the comments reiterated their appreciation for being exposed to different ways to think about math.

5 Conclusion

Exploding Dots is a low threshold high ceiling activity that any age learner can partake in and experience success. This place value concept can pertain to

early number sense as well as higher levels of mathematics. Based on the feedback analysis, the professional development sessions improved participants' content knowledge and built their confidence to teach the Common Core State Standards while gaining a better understanding of the SMPs. Furthermore, the MaTCH facilitators are continuing to seek answers for the participant's questions through Journal Math Circle articles and participating in other Exploding Dots sessions.

The Global Math Project inspired participants by engaging them in challenging mathematics problems, exposing them to a novel approach to teach mathematics, and allowing educators to collaborate. The confidence level to integrate hands-on activities was noted throughout each session. One participant stated, "I can visualize how to differentiate learning for diverse learners as I gained a new appreciation for teaching math. I recognize how to make math class fun and engaging instead of overly focused on students passing the test." Regarding the effectiveness of these sessions, one participant voiced appreciation for the professional development sessions and said:

> I really enjoyed the topic of today and the concept of a math circle. I like the idea and would like to learn more about how we might foster math circles at our schools to encourage students to love math and find challenge and excitement.

Many participants further expressed gratitude for the fun, informative, and meaningful experiences that the activities provided, represented by the comments below:

> Further exploring Exploding Dots was fun. I have introduced (this to) my students and we will be jigsawing the "experiences" during Global Math Week. So, going over the topic, reinforced and gave me more ideas that I can use with my students.

> I definitely want to get my students registered for global math week. I want to share with my colleagues; however I know if I don't experience it, it's hard to have a connection to it but hoping they can be inspired to come with me next time.

After receiving positive feedback from the MaTCH meetings and other professional development sessions with Exploding Dots, the MaTCH facilitators were encouraged that the participants would conduct similar learning opportunities with their students.

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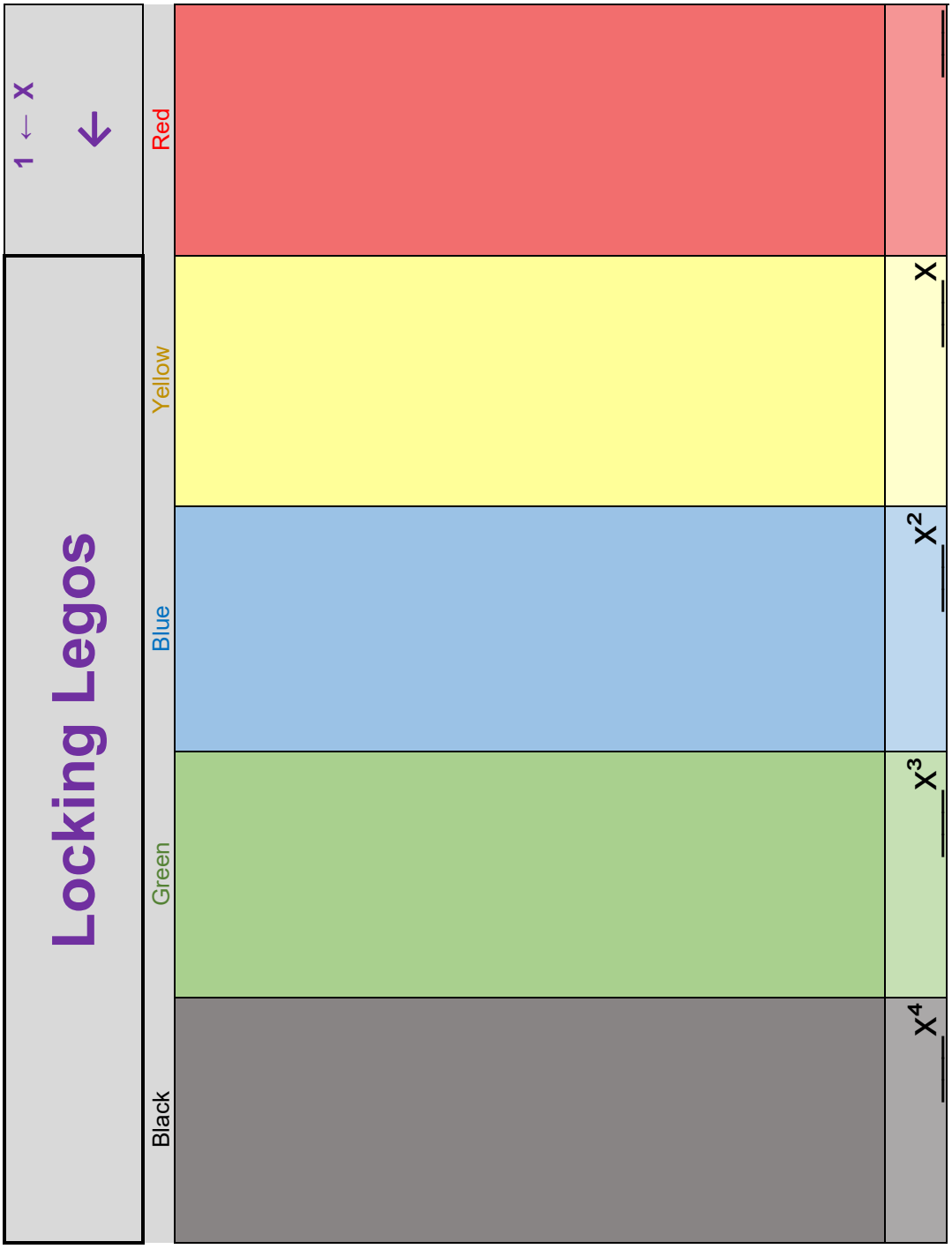
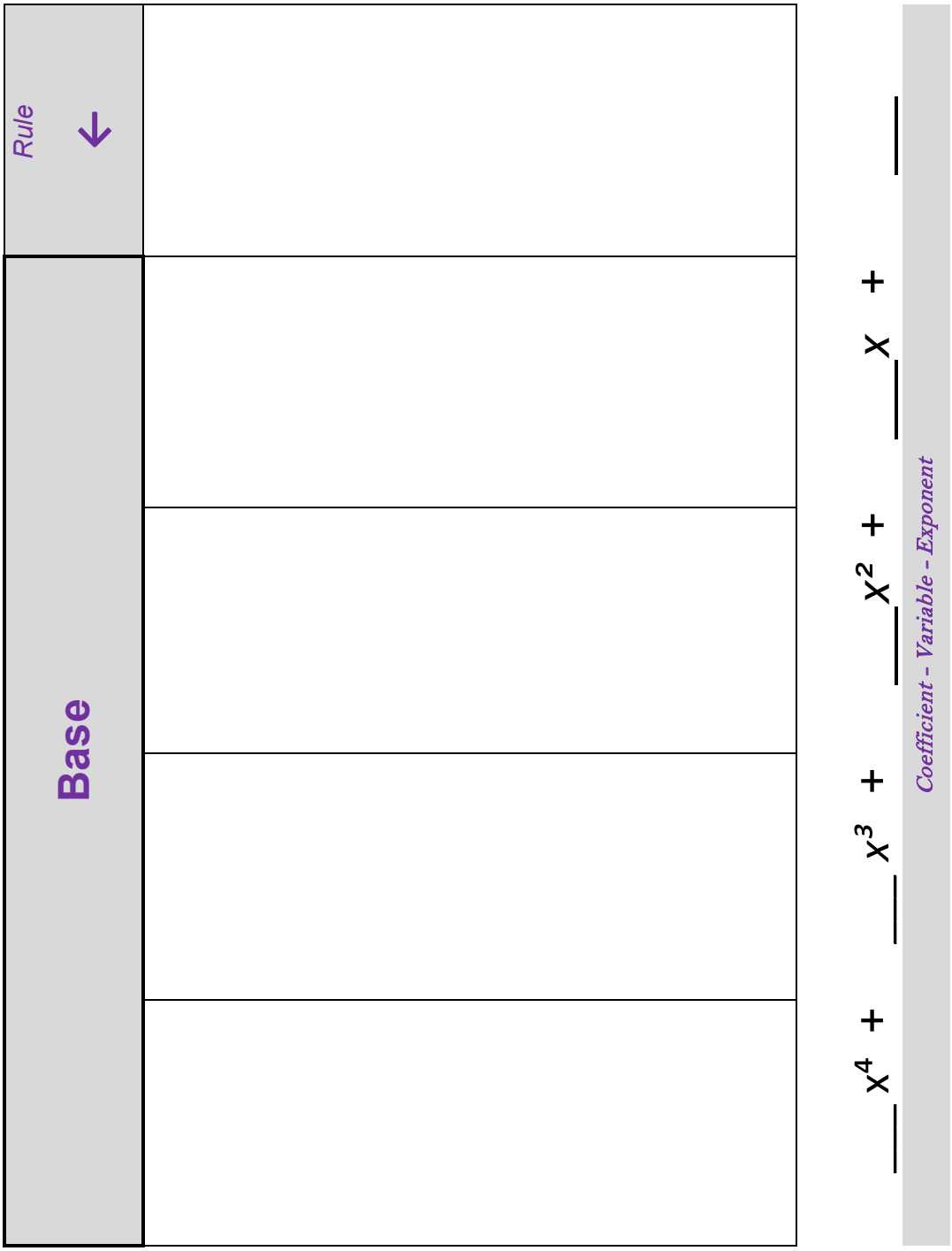
A Locking Legos Handout




Rule
Base
x^4 + ___ x^3 + ___ x^2 + ___ x + ___
Coefficient - Variable - Exponent

B Lesson Plan

LESSON PLAN: Exploding Dots
75 minutes

Goal:
- ✓ To create ways to make an algebra lesson fun by deepen students' understanding of math concepts through hands-on activities and technology.

Objective(s):
- ✓ Use manipulatives to model mathematical content and communicate effectively.
- ✓ Demonstrate multiple ways to solve for an unknown base, x.

Learning Intentions: *I can…*
- ✓ use different strategies to solve math problems accurately
- ✓ model how to convert numbers to different bases
- ✓ make connections and build enduring understanding through hands-on activities and interactive computer activities

Success Criteria: *I will be successful when I…*
- ✓ check my work by using manipulatives
- ✓ convert specified numbers into different bases
- ✓ explain to my partner how I solved the problem
- ✓ identify the coefficient, variable, and exponent in a problem
- ✓ represent expanded form on paper and solve for variables

(InTASC) Standard 1:
- ❖ Learner Development

NCTM Standard 2:
- ❖ Process Standard - Connections

Mathematical Practice 4:
- ❖ Model with Mathematics

Mathematical Practice 7:
- ❖ Look for and make use of structure

Resources/Materials:
- Document Camera
- Legos
- Small "Base" sheets (*copies*)
- Locking Legos sheets (*copies*)
- C-V-E sheets (*copies*)
- Slates (white boards)
- Color dry erase markers
- Exit Slips
- Exploding Dots (*site*)
- Computers or tablets

Student Preparation:
- ➢ Tablet/computer
- ➢ Pencil
- ➢ Paper
- ➢ Dry erase marker
- ➢ Math Journal

Instructor Preparation:
- ➢ Student sign-in sheet
- ➢ Manipulatives
- ➢ Small Paper Tables for Bases
- ➢ Internet access – Math Video
  - https://www.explodingdots.org/
  - http://gdaymath.com/courses/exploding-dots/

Background Information:

There is a need to build a strong foundation in the mathematical learning process by identifying multiple ways for students to learn math concepts. I want to provide opportunities for students to feel comfortable sharing their thought process with their peers. Part of this lesson will model a student-centered approach using hands-on manipulatives to enhance engagement.
(For example: visually represent expanded form, exponential notation, and solving for variables.) By using James Tanton's Exploding Dots activities to support algebra content, the students are more engaged and willing to collaborate with each other.

Procedures:

- Model the Universal Math Salute.
- Students will listen to a brief introduction about Exploding Dots.
- After the introduction/background, begin the lesson by reviewing the Goal/Obj/Target/Standards…
- Demonstrate/model place value activities using Locking Legos and Exploding Dots. **(I Do)**
- Provide practice problems to complete together. **(WE Do)**
- Students discuss and complete the problems/activities using slates. **(YOU Do)**
- Demonstrate/model different bases with variables and exponents using the Coefficient, Variable, and Exponent (C-V-E) sheet.
- Students will represent expanded form, use exponential notation, and solve for variables.
- Assessment: students will have an Exit Slip at the end of the lesson.

Time	Students	Instructor
2:00-2:15 15 min	Welcome Sitting at the desks	**<u>Universal Math Salute</u>** Who has heard of Exploding Dots? *Give a basic introduction - background to Exploding Dots* Go over the lesson ***Goal-Objective-Target-Success Criteria-Standards-MPs*** "You will be given an Exit Slip at the end of this lesson."
2:15-2:35 20 min	Listen and participate in the math lesson Can work with a partner Use the slates to solve using the 1←2 & 1←3	**<u>Locking Legos</u>** *Model the activity with Legos & the document camera* **(I Do)** Rule **1** ← **2** *(with Legos)* **(WE Do)** Ex: (7 Red) code111 (8 Red) code1000 *Student Practice: (students pick numbers)* ▪ What did you discover about the Legos? ▪ What digits are in a Base 2 system? *Model Base 2 using a "T" chart* **(WE Do)** Rule **1** ← **3** *(Legos and/or slates)* **(WE Do)** Ex: (11 Red) code102 Ex: "Unlocking" the **Code** of **<u>120</u>** *Student Practice: (students pick the numbers)*

<table>
<tr>
<td></td>
<td>Complete Base 6 table</td>
<td>
Distribute small Base 6 sheet to complete (YOU Do)
<table>
<tr><th>Base 6</th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>10</th></tr>
<tr><td>0</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>10</td></tr>
<tr><td>1</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>10</td><td>11</td></tr>
<tr><td>2</td><td>2</td><td>3</td><td>4</td><td>5</td><td>10</td><td>11</td><td>12</td></tr>
<tr><td>3</td><td>3</td><td>4</td><td>5</td><td>10</td><td>11</td><td>12</td><td>13</td></tr>
<tr><td>4</td><td>4</td><td>5</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td></tr>
<tr><td>5</td><td>5</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td></tr>
<tr><td>10</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>20</td></tr>
</table>
Extension: Have students work together to complete base 5 and multiplication.

Check for understanding
Listen to the group conversations/discourse
Encourage students to “challenge” each other

</td>
</tr>
<tr>
<td>2:35-3:00

25 min</td>
<td>Use Exploding Dots with slates to show work</td>
<td>
Time to switch back to Base 10
Introduce Exploding Dots on the board

<u>Addition</u>: model with “codes” on the board
Ex: <u>163 + 489</u> (answer 5 | 14 | 12)
Read as... “five hundred fourteen-ty twelves”

Is my response okay?

Use Exploding Dots to stack & solve: <u>1,023 + 347</u>
Student Practice

The lesson tomorrow will expand to Multiplication

<hr>

What is the opposite of Matter? (antimatter)
What is the opposite of a Dot? (anti-dot)
empty <u>dot</u> = <u>tod</u> <u>dark dot</u> + <u>tod</u> = <u>“nothing”</u>

<u>Subtraction</u>: use “anti-dots” model problem on the board
Ex: <u>441 – 254</u> (answer 2 | -1 | -3)

unexplode = 187
show standard algorithm
200 – (10) – (3) = 187

Student Practice: <u>2,222 - 1234</u> (answer 988)
<hr>
<u>Division</u>: model standard algorithm on the board
Ex: <u>12 / 276</u> (answer 23)

Use Exploding Dots *explode & circle 1 | 2 dots

Student Practice: <u>12 / 1,234</u> (answer 102 R10)
<u>14 / 1,122</u> (answer 80 R2)
</td>
</tr>
</table>

	Use the C-V-E sheet to use the Exploding Dots Method	***Coefficient – Variable – Exponent*** *(CVE) handout sheet* **Algebra:** *Polynomials (jump into division)* Ex: (**$2x^2 + 7x + 6) \div (x+2)$** (*answer* 2x+3) *(2x+3) (x+2) FOIL method* Draw out… explode and circle x+2 1 | 2 dots Practice: (**$x^2 + 7x + 12) \div (x+3)$** (*answer* x+4) (**$15x^2 + 26x + 8) \div (5x+2)$** (*answer* 3x+4)
3:00-3:10 10 min	Using computers Internet site: https://www.explodingdots.org/	**Computer Activity:** Exploding Dots * **12 Islands** **Island 1** * start the journey button (2 & 3 easy) Explore **Islands 4-12**
3:10-3:15 5 min	Discuss & share out Complete the Exit slip Write in your Math Journal	**Wrap-up:** Summarize the math activity. • Explain how to use Exploding Dots to model place value? • What patterns did you notice? • What part of the lesson made the most sense to you? • What made you uncomfortable or unsure? • How do you think your students in your field placement will react to this activity? • What do you want me to review tomorrow? *Hand out Exit slips* *Allow time to write/reflect in their Math Journals*

Assessment(s):
- ❖ Watch students communicate effectively and complete practice problems throughout the lesson *(informal assessment)*
- ❖ Base 6 & Base 5 practice sheets
- ❖ Exit Slips
- ❖ Math Journals

Differentiation/Accommodations:
- ❖ Modify problems
- ❖ Scaffold activities
- ❖ Model with math manipulatives
- ❖ Work with a partner or in a group
- ❖ Online tutorials (pre-lesson)

Extension:
- ❖ Write a written reflection
- ❖ Create other problems to “challenge” each other
- ❖ Use different tools to model the math
- ❖ Computer activities/sites

C Appendix: Highlighted Comments

A. Place Value Mathematical Understanding Increased

1. Exciting! Every number has a code and every code has a number. But it's not a secret anymore! Numbers have a place because the places have values that are fixed and based on logic. Randomness is not Math we have to find logic and patterns just like we did today for Place values.

2. Deeper way of understanding the operations with different base numbers and even polynomials by using the conversion process made perfect sense to me and it simplified the understanding of the operation for clearer understanding.

3. I always hated learning about the different bases, but this was fun, challenging and made some things finally "click" for me.

4. Today I learned that dots and boxes for multiplication, division, addition and subtraction in any base is quite awesome!!! Today I learned different methods and tools to assist with quicker and more efficient ways of completing operations with dots and boxes. I still cannot believe how awesome it is!!!

5. Learning place value with games such as the exploding dots is new for me.

6. The place value system can change and by changing the value system, it demonstrates a mastery of counting forward and backward. An understanding of place value is also critical to understand the value of learning fractions in elementary school too.

7. Increased understanding of place value in different bases: ten, 2,3,4,5 and grouping.

8. It is mind blowing how you can apply place value to algebraic expressions! It makes doing algebraic long division so much easier!

9. It was very eye-opening to convert different bases. I didn't know there is a decimal in different base and it is called radix point.

10. It's amazing to see numbers in this way. I've only ever thought about numbers in base ten, and even then, I didn't think about the reason why our digits went from 0 - 9. With the amount of difficulty I had doing numbers in base two or base three.

11. New ways to add, subtract, multiply, and divide using place value. I really like that although it sees taking a long time to build the understanding for the topic, but we end up learning a simpler and more intuitive way to do math operations.

12. I learned about the value of place values. Helped me look at our number system in a new light.

13. Boxes and dots is a useful way of visually representing place values, even to representing polynomials. Playing with bases really isn't formidable as long as the place values are held firm with boxes.

14. I loved the place value activity that we did this morning. It made me think deeply about looking at the structure of numbers (in the different bases).

15. I am happy that I realized why we subtract the what we do that is why borrowing 1 from the neighbor makes 3 a 13 (base 10 logic) and not 1 + 3 as 4. It was nice to realize the reason why we do what we do.

16. Didn't really think about using the dots to solve these kinds of problems.

17. A new way to group numbers to numbers to combine and divide.

18. Drawing pictures really helps to visualize what a problem looks like in any base. Starting with easier, workable numbers helps before moving into larger number or different bases to build a stronger base of understanding.

19. The topic for today was place value using exploding dots. It helped me understand place value a little better with using concrete materials and making thinking visible. It made learning fun! The biggest take away was seeing how division and multiplication work with base ten. I understand the concept now.

20. Place value and operations: To solve a problem, it works to write the initial answer according to the operation in base ten, then regroup in the different base for that problem.

21. Appreciated the exploration of working in different number bases. It is good to be able to reinforce my number sense understanding of base ten

22. The Exploding Dots activity is deep. We found explored connections up to calculus and beyond.

23. That most of the "rules" for math sometimes only apply to the base that you are working in. Showing the dots as groups when using division and multiplication helps you understand why division and multiplication works and why the rules are true.

24. Really interesting and loved how it's a visual method to show division multiplication works.

25. It is very interesting how you can translate any number into a code using different bases.

26. We learned what x means. It can be seen as a unknown base. Now I'm seeing how all these items work together to model math.

B. Knowledge of SMP Increased

1. It was a good opportunity to deconstruct the SMP's. I like the rotation of the posters and then the relating of today's experiences to each SMP. I would like suggestions on how to implement using the SMP language in my classroom.

2. I loved that we were able to break down the SMPs. That is something that wasn't done at our school and I am glad that we got to do that. They make so much more sense now and I will feel more comfortable referring to them during my lessons.

3. I really appreciated us going over each practice in an expert group kind of way as well as being able to read and give feedback to the other groups' interpretations. It was really helpful to break down all the big words into ideas that are easier to digest and understand.

The Frayer models were especially helpful with the more abstract ideas.

4. This task especially emphasized several CCSS mathematical practices: MP1, MP3, MP7, and MP8.

5. I liked how we unpacked the SMPs so that we got a deeper understanding of them. I know that I need to be more focused on incorporating the SMPs into my lessons in the future.

6. Mathematical Practice #7!!! I love it!

7. We focused on SMP #2 and SMP #3 and what would you observe if the teacher were fostering the learning environment. It is a classroom culture that has been nurtured by the teacher that would be respectful to any ideas by students, this would encourage everyone to share their ideas.

8. The CCS SMP are relatively new to me, so I am very appreciative that we are going to go over these.

9. This was a great activity to make us familiar with the math practices.

10. Discussing math practices helped me understand the connection of topics between grades.

11. Unpacking the math practices was valuable because I honestly never took the time to understand each practice. It was also evident how practices can be incorporated and embedded in daily lessons and plans.

12. I am glad it is a topic, because in our text series it is its own unit and I skipped it. I understand that these habit/practices are vital - but struggle with the time to "teach" them explicitly. I am certain the answer is making sure that I teach the content units in A WAY that develops the practices simultaneously. However - that seems daunting to me. So I look forward to the discussion.

13. I really enjoyed today's discussion because we were looking at what teachers should be doing instead of what students should be doing when we observe. The discussion we had encourages me to

try harder to exhibit these behaviors so my students will be better practitioners of the CCS SMPs.

14. It was very informative and useful. We learned about how to look at looking at rubrics in a more sensible way.

15. These practices are what drive instruction...they frame all of the content we teach in math. Although I know that any time we do math we engage in these practices, I think we (as teachers) forget that these skills need to be taught to students as well. I think part of the reason I've always had difficulty teaching these is because I don't think I truly understand what each of these mean and what they look like in practice (especially at the lower elementary grades). I'm excited that we are exploring these and are focused on what it looks like to TEACH them!

16. I got a deeper understanding of the 8 Mathematical Practices and what they mean

17. I like the topic that is discussed and think it is important. It is something that is on my mind as I do my lesson planning and I'm always looking for ways to incorporate the standards into my lessons.

18. It was a good exercise. I still love mathematical practices #7 & #8. It took me a really long time to get a better understanding of it and I think that because I love patterns it helps me always look for patterns.

19. Teachers know Mathematical Standards, but maybe do not emphasize it in the classroom. Mathematical Standards are important for students to become problem solvers and productive and effective mathematicians. Teachers need to set up their students to explore and provide inquiry for our lessons. Students need to be challenged and work forward and backwards.

20. Nice reminder to look at the math practices. I tend to forget about that and just look at the standards and stepping stones lesson. item Learned a lot in depth on Math practices.

21. Now, I have a better understanding of the common core standards.

22. I thought it was a thorough refresher of the math practices.

23. It was a good review of all since our school has been focusing on 1,2,3,4, and 6.
24. Focus on the SMP, learning experiences, and antecedent actions.
25. Understanding of the Mathematical practices.

C. Perspective Changes on the Teaching of Mathematics

1. There are multiple ways to show your work.
2. I really liked how we were put in the place of students and given problems to solve. It was great seeing how there were multiple ways to solve a problem.
3. I am reminded how my students may feel in any subject that is difficult for them. They may be missing vocabulary as I feel I am in this activity. This makes it more of a problem when we ask them to explain themselves. Without the proper foundation, how can they feel confident to explore new ideas or concepts? I appreciate how generous and open our teachers have been today. This helps me feel like I am part of the group even though I struggled a bit.
4. Explaining how you got your answer is just as important as having the answer.
5. Math lessons can be fun
6. Doing the hands-on activity gave us a concrete visual for an abstract concept that may be hard for students to understand.
7. I see that we really had to understand how things worked in order to build our understanding and create our own methods of how to get to an answer. I liked that we were presented with a problem first and figured out things on our own. The level of understanding for someone who thought they already knew the answer was increased because they were forced to explain themselves to someone else and the level of understanding for someone who wasn't sure or didn't know was increased from talking to a peer about what things they were thinking. Sometimes the pieces of thinking I had were made

whole by sharing and collaborating with others, regardless of our levels.

8. Students not only need to know content, but they also need to be able to have strategies under their belt to be able to solve math problems.

9. Having kids struggle is a good technique.

10. I liked it. Thoughtful about problem selection for students. I think that is important. I was a bit bothered about the accusation that all elementary teachers tell our students myths about mathematics. I know that a lot of us do that but please don't accuse ALL of us doing it. There are some of us that try to stay current and try to set up young children for success for higher level mathematics.

11. I see that working with math can be challenging to everyone, I see how my students may want to give up, as an adult I needed to use SMP#1 Persevere on solving problems.

D. Beliefs in Teaching Increased

1. This helped reinforce my belief that simple hands on lessons have the potential to reach higher math concepts. It helped remind me to bring the joy of math to students.

2. I learned that math is a process and that giving students a chance to struggle and succeed makes learning that much more powerful. Also that it is our job as educators to bring a sense of fun and joy to math through our attitude and experiences that we share with our students.

3. With all the math activities and strategies and old and new ways to make math lessons fun I learned today, I believe it is an important tool to determine students' progress and can adjust to more effective teaching.

4. I loved it! Having to think about it and explain it to others made me have to understand it.

5. When I sit and listen to others, I hear the joy of math in their voices.... I think this is the key to bringing joy of math to our students....it starts with us.

6. I think we can make math joyful incorporating most of the SMPs. It's up to us as teachers to do so.

7. It's the teacher's job to bring the joy into mathematics. If we model our excitement about math, it will translate to our students.

8. As teachers, we need to make a concerted effort to create an environment that is condusive to academic discourse. This is where students will build their conceptual understanding and encourage them to think critically about a problem and their answers. Think-pair-share is a great way to get students who aren't confident to start talking and sharing their thoughts.

9. I think that it is our jobs as educators to expose the fun in learning to our students. Math is hard work and can be very frustrating for children, but if we show them how fun math can be, then we can influence their mindsets.

10. If you enjoy math and make it fun for your kids, your kids will see that and get excited to.

11. The joy of learning mathematics should come from inside. For that reason, it should be a collaborated efforts of teacher and students. It would require a meaningful questions and some sort of motivation within students' mind. The motivation will come with positive experiences and understanding the purpose of mathematics learning.

E. Confidence in Place Value Discourse with Peers Increased

1. It was great to work with a partner and get to discuss the mathematics and hear her thinking.
2. Sometimes the pieces of thinking I had were made whole by sharing and collaborating with others, regardless of our levels.
3. Love seeing the different ways think about math.
4. I see that the struggle that I went through today is similar to the struggles that my kids have, but being able to persevere is what helped me understand. I also think that the collaborative part helped because it let me explain what I was thinking out loud. The focus wasn't on answers, but on a process. As I worked with other teachers, we discussed our process and implications for use in the classroom. We also talked about our challenges and "ahas".
5. I really enjoyed hearing from and learning from the instructors and peers in today's training session.
6. It was fun to see how other "math" people think.

F. Confidence in Place Value Discourse with Students Increased

1. Exploring different bases helps students really understand place value. It also challenges students to explore other bases besides base ten and emphasizes the understanding of place value.
2. This activity reminded me of how easily we take for granted our base 10 system and how the normalization of base 10 often renders it invisible. This is definitely true for our students, as well. By asking us to work in base systems other than 10, it makes visible the process behind our place value system. This can elicit a deeper generalization of the process behind any place value system and can provide the/a justification for such.
3. I realize that teaching kids different bases may help them to have a better theoretical understanding of place value and what it means.

It may also reinforce to them what is really happening when we regroup and "borrow".

4. I'm beginning to truly understand just how difficult it is for students to understand base ten (I always knew this was a difficult topic to understand...but I'm guessing this confusion I've been feeling throughout most of the activity is the same kind of confusion that my students feel...yikes!). This activity is making me think of just how many opportunities (moving from ones to tens, and tens to hundreds, and back down) students need with hands-on manipulative before they will feel comfortable with, and conceptually understand, place value.
5. It's sure fun to use in the classroom for students to enjoy

D Appendix: Highlighted Questions

A. Teaching with Exploding Dots

1. How to integrate this with the kindergarteners?
2. For Kindergarten, would I just focus on base ten? Would I do other bases?
3. How would I use this for first graders?
4. Will my students who are 10 and 9 years old be able to do it?
5. What age is an appropriate developmental age to introduce this idea. Does it work with younger children or does it "muddy" their understanding?
6. How might this be introduced to students who are lower-level?
7. What age level to begin playing with other bases?
8. What base should children learn first?
9. How I can share exploding dots with my students?
10. How does this activity relate to real world or scientific events? What grade level is this activity developed for?

11. No question. However, I would like to see more of how things like exploding dots are used in the classroom. Maybe see students using that strategy and the impact it has on students.
12. I am skeptical about using the exploding dots with various bases because my students are familiar with base ten so if I introduce other numbers, they might get confused.
13. I really enjoyed the problem that was presented (exploding dots), and am wondering if there are other problems that would prompt students to think about place value for elementary (particularly lower elementary)?

B. Content of Exploding Dots

1. How exactly the binary code is used in computer?
2. Are the algorithms for simple operations different & if so, how?
3. How do I connect the dot activity to the standard algorithm for multiplication and division?
4. Would love to see manipulatives with the bases.
5. Is there a couple short cuts or summarized principles that apply to all conversions?
6. What other models can be used to help people find the patterns?
7. What will happen if exploding dots for division of polynomials?
8. What are the application of different base number system? I know some of them are for computer programming purpose. Do we have any good example(s) outside of the computer science field?

C. General Teaching

1. If students are stuck on solving the problem how can we better guide or product their thinking processes?

2. What are online resources/ websites that I can use that I can use with my class as interactive/ hands-on lessons?

3. I use GregTang math for a lot of fun activities, but I am always looking for new resources. What are some popular and new resources that are not commonly known to the public yet?

4. How do we make the idea of application of mathematics becomes how to enjoy itself for its own purpose? What can we do differently to help our students to learn how to enjoy mathematics and have pure fun with mathematics?

5. How do you incorporate more fun into math, while still meeting the curriculum requirements? Also how to get students to not be so sensitive to people questioning or critiquing their learning/answers?

6. I wonder how the DOE can help middle and high school students who are entering our school system with little English and math background because they are coming from places where formal education was different from what it is in the US.

7. My questions are more of a pedagogical nature. Initially, I struggled greatly with the mathematics, so I questioned heterogeneous grouping as I had a colleague who worked faster than I was at first... This almost made it harder for me to figure problems out on my own since I was working at a slower pace. Then again, would I be floundering at cognitive dissonance for too long?

8. What internet sites would be age appropriate for kids to explore?

D. Standards for Mathematical Practice (SMP)

1. How am I going to gather all evidences or monitor progress with the use of SMP and evaluate success using the CCSS?

2. Do we take the standards and apply it to these mathematic practices? Do we need to figure out where the standards fit? What is more important to learn to teach the math standards or the mathematical practices?

3. I see Exploding Dots as a form of manipulative for helping students to better appreciate the major mathematical operations in different bases, with variables, etc. My questions: Will my students see what I see? Will the activity serve as an effective vehicle for the CCSS Mathematical Practices?

4. How would you relate different bases to real life application? What grade would you start exploring other bases? How does this relate to common core standards?

E. Reflections (not a question)

1. The group next to me began discussing bases in negative numbers, decimals and imaginary numbers. This was really interesting to overhear and now I want to practice a few problems.

2. Want to explore further to refine understanding.

3. Just need more practice using dot method for division!

4. Place Values in Binary or Octal or any Value is represented in Powers of that Value. The computer language for programs have an impact in our mathematical world.

5. None right now, maybe when I try to adapt it so that my whole class will be engaged.

6. I probably should've gone to the other session for beginners, because it would've been interesting to see how the dots could be used for younger kids.

F. Accolades (not a question)

1. The different ways that we learned how to create an answer was excellent.

2. I'm excited for our next MaTCH session!

3. Need more real life applications on what we did today other than just understanding place values but that was awesome and exciting!