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Developing Number Concepts in Grade One

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DEVELOPING NUMBER CONCEPTS

IN

GRADE ONE

by

Hazel Haynes Elfbrandt

A paper submitted in partial fulfillment of the requirements for the degree of Master of Education, in the Graduate School of the Central Washington College of Education

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of Education

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Chapter I

Philosophy of Number Concepts

Introduction

The purpose of this paper is twofold: (1) to review current research materials as they pertain to a first grade number program, and (2) to develop a number program for first grade dependent upon a logical progression through children's understanding and meaning. The sources used in the writing of this paper consist of secondary materials--educational books, pamphlets and periodicals.

In reviewing the library sources on number work it is difficult to discern how number concepts can best be taught in the classroom. A need for a clear, concise program based on a well-founded philosophy becomes evident. The ingredients for this program are complex; but there is a need for a program that is ready for a teacher's immediate use. The philosophy of this writer is in agreement with the thought that "arithmetic should be informal but not haphazard; planned but apparently casual."¹

NEED FOR NUMBER WORK IN GRADE I

An elementary school child needs to use arithmetic in the solution of everyday situations. Numerous studies have been made to determine the

1. Whittenberg, Clarice, Making Arithmetic Meaningful for the Young School Child. Laramie, Wyoming: Bureau of Educational Research and Service, 1949, 17.

knowledge and use of number by young children. A comprehensive survey of about eighty of these investigations was recently made by Brownell. His conclusion was that "school entrants already know much about number; the inference is that they can learn more; nothing is gained, and much more may be lost, if the school delays to later grades the discharge of its obligation."² Since number is ever-present in children's environment, they constantly use it and need it.

Brownell states that the results of this study "do not prove that systematic teaching of arithmetic should be started in Grade One"...rather it means "not only children in Grades One and Two can learn arithmetic, but that with properly graded learning activities they will learn arithmetic and learn it happily and satisfactorily."³

MACLATCHY'S STUDY

Several years ago a simple number test was given to twenty-three hundred children who entered the first grades of eleven Ohio cities to determine the number abilities of first grade children. Since the children were unable to write, each child was questioned individually by his teacher. Each teacher tested six pupils. To obtain an unselected

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2. Brownell, W. A., Arithmetic in Grades I and II. Durham, North Carolina: Duke University Press, 1941, from Brueckner, L.J. and F.E. Grossnickle, How to Make Arithmetic Meaningful. New York: John C. Winston Company, 1947, 55.
 3. Brownell, W. A., Arithmetic in Grades I and II. Durham, North Carolina: Duke University Press, 1941, from Association for Supervision and Curriculum Development, The Three R's in the Elementary School. Washington, D.C.: National Education Association, 1952, 19.

group the teacher chose the first six children in her alphabetical listing whose birth dates fell between March and September. Some of these children had kindergarten experience, others did not; however, none of the children had been in first grade before.⁴

The abilities of children in an unselected first grade to count by rote is something like this:

One child will not know how to count
 One child will count to five
 Three children will count to ten
 Nine children will count to fifteen
 Nine children will count to twenty
 Two children will count to thirty
 Three children will count to forty
 Two children will count to fifty
 Three children will be uncertain counters from 50
 to 99
 Three children will count to 100.

Variations in ability and experience, however, may affect the number knowledge of the group.⁵

The Ohio children's knowledge of addition did not seem to be systematic. Two trends were evident in the results of the children's addition: (1) their familiarity with addition was closely related to counting; (2) the percentages of familiarity for the combinations having the same sum to fall rather close together.⁶

"This numerical picture of the familiarity with number which is characteristic of a group of six-year-olds puts individual differences in

4. MacLatchy, Josephine, "Number Abilities of First Grade Children," Childhood Education, 11: 344-347, (May, 1935).

5. Ibid., 345.

6. Ibid., 346.

the foreground. The evidence is no argument for the beginning of formal instruction in arithmetic but it shows the necessity of great care and wisdom in bringing these heterogeneous familiarities to a place of common understanding.⁷

WOODY'S STUDY

In a study similar to MacLatchy's, Woody⁸ shows that (1) children have considerable knowledge of counting before formal instruction begins, (2) the exercises involving counting 20 circles and counting them in order prove easier by 1's or 10's, (3) the exercise involving rote counting to 100 by 10's are easier than that involving counting to 100 by 1's. Usually the knowledge possessed by children is not limited to counting and adding simple combinations but includes elementary knowledge of fractions, United States money, units of various types of measurement, and the understanding of the processes demanded in simple verbal problems.

VALUE OF NUMBERS FOR YOUNG CHILDREN

A recent study by Culver illustrates the number experiences young children have in their out-of-school life. Parents of elementary school children were requested to write down over a period of time all the questions, problems and comments regarding numeration which their children voluntarily expressed at home. Materials were distributed and explained

7. Ibid., 346.

8. Woody, Clifford, "The Arithmetical Background of Young Children," Journal of Educational Research, 24: 188-201, (October, 1931).

at Parent-Teacher Association meetings held in the respective school buildings. In addition to these group contacts, about one-hundred home calls were made to request the cooperation of the parents.⁹

In this investigation it was established "that interest in time is at its peak between the ages of six and ten. At this same age there appears to be a steady interest in the measurement centering chiefly around length, weight and liquid measure. The subject of money begins to be important at about age six with questions centering around values of coins, counting change and how much money it will take to buy the things they desire."¹⁰

"Counting objects is a very important activity from age five through eight. They count food, toys, piano keys, chairs, eggs, telephone poles, steps, children, etc. Much of the counting is seemingly just for fun. Two of the most functional applications are to set the table and to divide food."¹¹

TRENDS IN NUMBER INSTRUCTION

"Teachers today are faced with conflicting opinions on matters of what arithmetic to teach, the selection of appropriate instructional procedures, how number meanings are developed, whether social situations

9. Culver, M. M., "A Study of Children's Interest in Arithmetic," Children's Interests, Elementary School Level. Twelfth Yearbook, California Elementary School Principals Association, Los Angeles: California Elementary School Principals Association, 1940, 40-45.

10. Ibid., 40-60.

11. Ibid., 27.

alone offer sufficient number learning for elementary school children and what use to make of drill."¹² There is wide divergence of opinion, however, in the choice of teaching procedures for the attainment of these understandings.

There are three theories of instruction which are defined in Spitzer's The Teaching of Arithmetic:¹³ the "drill theory", the "incidental-learning theory" and the "meaning theory." In addition to these three theories, Brownell discusses a fourth theory, that of the social approach.¹⁴

The oldest of the three theories is the drill theory. The proponents of this theory believe that number facts are most easily learned by repetition. The fact is said and/or thought over and over again; repetition here fixates the fact. While the initial instruction demonstrates the fact and explains each process, drill is used to attain mastery. Thus the tendency is to focus on isolated phases of number facts.¹⁵ According to this theory a knowledge of number facts is the all-important responsibility of the school requisite to entrance in the third grade.¹⁶

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12. Association for Supervision and Curriculum Development, The Three R's in the Elementary School. Washington, D.C.: National Education Association, 1952, 99.
 13. Spitzer, Herbert F., The Teaching of Arithmetic. Boston: Houghton Mifflin Company, 1948, 6.
 14. Brownell, W.A., Arithmetic in Grades I and II. Durham, North Carolina: Duke University Press, 1941, from Brueckner, L. J. and F. E. Grossnickle, How to Make Arithmetic Meaningful. New York: John C. Winston Company, 1947, 156.
 15. Spitzer, op. cit., 7.
 16. Brownell, op. cit., 157.

Knowledge is defined here as the ability to give a response to a number fact. While an important tool in the drill program is the textbook, other materials frequently used are "flash cards", drills, tests, number games, and worksheets. This theory is widely practiced throughout elementary schools, as well as, junior and senior high schools.

The incidental-learning theory states that number is more effective if presented only when the child has need for a fact or process. The need, according to this theory, will produce understanding as well as retention.¹⁷ This approach is used more extensively in the lower primary grades. One of the chief criticisms of this approach is expounded by Brownell who suggests that the incidental approach to number is a non-existent program since under this plan there is no systematic arrangement.¹⁸ Number experiences are explored when a classroom situation presents itself. These experiences depend altogether on chance. This approach was formulated as the direct result of a reaction against the formal drill program.¹⁹

The social approach according to Brownell, provides planned activities rich in arithmetic which are actual child experiences similar to those met by adults in life-like situations.²⁰ The group may operate a

17. Spitzer, op. cit., 8.

18. Brownell, op. cit., 156.

19. Brueckner and Grossnickle, op. cit., 156.

20. Brownell, op. cit., 157.

store, post-office, or bakery. Only the social phase of numbers is explored; however, the program is planned. The mathematical phase of numbers is not approached through this type of program. Thus the child lacks knowledge concerning the number system itself and the mathematical meaning of number is neglected since isolated number facts may be learned but not as a result of directed learning.²¹

Major publicity was given to a fourth theory, the meaning theory, in 1935 when the National Council of Teachers of Mathematics presented the Tenth Yearbook. "The meaning theory is characterized by the viewpoint that arithmetic can be learned most easily if children see sense in what they do and if arithmetic is taught as a closely knit system of related ideas, facts and principles."²² While this theory places more emphasis on meaning than the drill or incidental-learning theory, the approach utilizes good features of all three previous theories of learning, but organizes them in a different way.

Meaning, according to the National Council Committee on Arithmetic, is characterized by the child's seeing sense in what he does. Meaning involves seeing not only that a number statement or procedure is correct but also why one way of getting a result is better than another way.²³ Understanding through the meaning theory is gained by letting the learner see the reason for studying the fact or process and by emphasizing the

21. Brueckner and Grossnickle, op. cit., 156.

22. Spitzer, Herbert F., The Teaching of Arithmetic. Boston: Houghton Mifflin Company, 1948, 9.

23. Ibid., 13.

relationships between various aspects of the number system.²⁴ The chief characteristic of this program is that the children see sense in what they do. Number is taught as a progression of facts dependent upon previous learnings. Children are encouraged to discover facts and relationships for themselves.²⁵ In these ways then the number program is sequential and arranged. A planned organized program for teaching beginning arithmetic does not imply that it is formal. A formal program is narrowly based on drill. In a systematic program emphasizing meaning children have many concrete experiences to aid in the development of meaning. Meaning is developed as the child gradually expands his concepts through these concrete experiences which he works with before dealing with abstract facts and processes. By varying these experiences there is repeated use of a fact or a process; meaningful practice takes the place of repetitious drill.²⁶

Brownell has given four reasons why meaning is important in the teaching of arithmetic. First, arithmetic is functional only when it is understood. Secondly, meanings facilitate learning through insights and relationships. Third, meaning increases transfer. Fourth, meaningful arithmetic is more easily retained than is mechanically learned arithmetic.²⁷

24. Ibid., 13.

25. Brueckner and Grossnickle, op. cit., 158.

26. Ibid., 158.

27. Brownell, W. A., "When is Arithmetic Meaningful?" Journal of Educational Research, 38: 481-98; (March, 1945).

The meaning theory stresses both the social applications and the mathematical relationships of number. Brueckner and Grossnickle urge the use of both the social application of number and the mathematical phase. This theory is supported by Horn, who, in The Teaching of Arithmetic, says that a well designed program of instruction in arithmetic is essential, and such a program should include not only provision for systematic and meaningful learning in the arithmetic class but also careful attention to the mathematical needs and contribution of other areas.²⁸ Thus, the number experiences are not left to chance because the program is planned, systematic and sequential. The systematic and sequential relationships among numbers is emphasized as well as their ordinary everyday uses.

"The major effects that this theory has had on teaching are that it has (1) increased the emphasis given to concept-building; (2) increased the use of concrete and semi-concrete materials; (3) stimulated the recognition of the value of relationships in arithmetic; (4) stimulated attempts to teach the system of number rather than separate elements; and (5) emphasized having children see reasons for the work they do in arithmetic."²⁹

The meaning theorists declare that meanings or understandings are not "rules" to be studied as such by children but rather are formulated

28. Horn, Ernest, "Arithmetic in the Elementary School Curriculum," The Teaching of Arithmetic. Fiftieth Yearbook, Part II, National Society for the Study of Education. Chicago: University of Chicago Press, 1951, 6-21.

29. Spitzer, op. cit., 10.

as a result of meaningful learning, not learned through imposed "memory exercises." Understandings grow slowly and come as a result of many experiences.

The writer endorses the fourth, or meaning, approach as the basis for Grade One number work. The social as well as the mathematical phase of number is stressed in the program.

Chapter II

UNDERSTANDINGS AND MEANINGS TO BE TAUGHT IN GRADE I

Children's first arithmetical ideas are relatively crude. As children have need for precision their ideas of number and measure gradually develop. Children develop these ideas best in real-life situations. A teacher may help children clarify and expand their ideas as well as enlarge their number concepts.¹ She may aid by being cognizant of other opportunities which will stimulate children's interest in number.

Opportunities to use number are unlimited. Mott² has categorized these opportunities under three topics: (1) those which may arise in the daily routine of school life; (2) those which pop up in unexpected moments and places; (3) those which are definitely incorporated in the teacher's plans for the group at its level of development.

A desirable arithmetic program for Grade One should (1) capitalize and extend pre-school learnings, (2) provide a good balance of learning experiences between the social and mathematical phases of arithmetic; and (3) provide learning experiences through mastery of the grouping

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1. California State Department of Education, Guide to the Teaching of Arithmetic in Kindergarten and Grades One and Two. Sacramento, California: California State Department of Education, XVIII, No. 8, (November, 1949), 4.
 2. Mott, S. M., "Letting Arithmetic Function in the Primary Grades." Arithmetic, 1949. Supplementary Educational Monograph No. 70, Chicago: University of Chicago Press, 1949, 21-25.

idea that will aid in the development of readiness for addition and subtraction facts. Since the development of basic number understandings is a long and gradual process, some of the concerns of the teacher as she guides children through this process are given consideration in this section. Among these are how children are guided in discovering the number system, how experimentation results in building basic understandings, and how children are aided in learning the processes.

A subject which needs recognition and employment in a well-rounded number program, is that of cooperative (pupil-teacher) planning. The primary classroom provides many opportunities for cooperative planning of learning experiences. The role of the teacher as a mature member of the group aids in "helping the children make better decisions, provides learning materials of all kinds, develops a healthy mental and emotional learning situation, aids the children in learning to work together, suggests rich and rewarding learning experiences, and helps individuals and groups in evaluating their learnings."³ Cooperative planning does not mean that there should be no pre-planning by the teacher. Modern classroom practice requires the teacher to plan the goals and general direction of learning carefully; too, pre-planning allows the teacher to contribute to classroom planning more effectively. However, pre-planning must not be mistaken for doing the thinking

3. Association for Supervision and Curriculum Development, What Does Research Say About Arithmetic? Washington, D.C.: National Education Association, 1952, 7.

for the children and imposing it on them during the class planning periods.

RECOMMENDED PROGRAM FOR GRADE ONE⁴

1. Developing understanding of numbers 1 through 9

To understand that each number in a series is one more than the number before it and one less than the one following it.

2. Developing understanding of tens as a basis of the number system
3. Developing understanding of 10's to 100

2 tens (20), 3 tens (30), etc.
4. Counting by 1's to 100, using real-life situations as they arise
5. Reading numbers to 10--beyond 10 as they are needed for e.g., pages in books, calendars, etc.
6. Writing numbers to 10--beyond 10 as needed, e.g., numbering pages in booklets, making calendars, recording attendance.
7. Telling time--hour, half hour
8. Learning value of and identifying coins, penny, nickel, dime
9. Solving orally simple story problems from situations in children's daily living
10. Developing an adequate vocabulary for expressing the arithmetical ideas which the children use. An adequate first grade number vocabulary is shown on page fifteen.

4. Adapted from: Brueckner, L. J. and F. E. Grossnickle, How to Make Arithmetic Meaningful. Philadelphia: John C. Winston Company, 1947; Glanfield, Mary, Mastery Program in Arithmetic for Elementary Schools. Longview, Washington: Longview Public Schools, 1952; and Rosenquist, L. L., Young Children Learn to Use Arithmetic. Boston: Ginn and Company, 1949.

A FIRST GRADE NUMBER VOCABULARY

about	flock	ever—under
above—below	for	pair
add	from—to	part—whole
again	full—empty	quick—slow
ahead—behind	group	quickly—slowly
all—none	half	rough—smooth
altogether	halfway	round
always	hard—soft	row
amount	heavy—light	same
another	height	several
any—not any	herd	side
around	here—there	single
as...as...	high—low	size
at	hot—cold	soon
back—front	hurry	square
backward—forward	in—out	straight—curved
before—after	inside—outside	take away
begin—end	into—out of	tall—short
behind	just	than
beside	large—small	thick—thin
between	left—right	this—that
big—little	length	time
both	long—short	tiny—huge
bottom—top	loud—soft	together
by	many—few	too
center	measure	triangle
circle	middle	twice
clockwise	more—less	twin
close—far	most—least	upper—lower
compare	much	wait
corner	near—far	warm—cool
crowd	never	weigh
couple	new—old	weight
deep—shallow	no	when
distance	noisy—quiet	where
double	none—some	while
down—up	north—south	wide—narrow
each	nothing—something	width
east—west	now—then	with
enough	off	young—old
even—uneven	on	very
every	once	fast—slow
flat	other	only
first—	--next	--last

California State Department of Education, Guide to the Teaching of Arithmetic in Kindergarten and Grades One and Two. Sacramento, California: California State Department of Education, 1949, 7.

Library

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NUMBER READINESS

In the past there has been much discussion and some research on what constitutes readiness for learning arithmetic and how such readiness should be measured. There is common agreement among authorities that this readiness is not a general readiness but a readiness for specific problems in arithmetic. Each problem requires specific preparation.⁵ Woody summarizes this point of view in his definition of educational readiness as "...the preparation which the teacher consciously makes in getting the child ready to learn the things to be taught."⁶ Readiness, then, is a continuing process of becoming more ready to learn. This kind of a developmental process is the teacher's responsibility which only begins with an appraisal of the individual and group readiness for instruction in any particular phase. Swenson says,

What a child or a group of children should be taught at any particular time is not so much a matter of teaching a certain piece of subject matter when the children are a certain age or in a certain grade. It is much more the matter of planning subject matter and teaching method so that the learning experience follows naturally upon preceding learning experiences and results. Arithmetic is a systematic area of knowledge with very clear lines of sequential development.⁷

5. Rosenquist, L. L., Young Children Learn to Use Arithmetic. Boston: Ginn and Company, 1949, 97.
6. Woody, Clifford, "A General Educator Looks at Arithmetic Readiness." The Mathematics Teacher, 30: 314-321, (November, 1937.)
7. Swenson, Esther L., "Arithmetic for Preschool and Primary-Grade Children." The Teaching of Arithmetic. Fiftieth Yearbook, Part II, National Society for the Study of Education. Chicago: University of Chicago Press, 1951, 60.

Readiness is not an "either-or" state nor is it something with which the primary grade teacher alone needs to be concerned. Swenson discusses the first premise by saying:

Only to the uninitiated can readiness appear to be a simple matter of reaching some mythical, magical point preceding which the learner is clearly not ready and following which he is clearly and unequivocally ready to learn. Only by the psychologically naive is readiness conceived as being a boundary line across which the learner steps at a clearly defined time from 'Unreadiness Land' to 'Readiness Land.' The attainment of readiness is a continuing process of becoming more ready than one was previously. Adults will understand the learning of children much better if they will think in terms of their being more ready or less ready rather than ready or unready.⁸

COUNTING

"Counting in its simplest form is mere rote counting, but counting may be so developed as to include grouping. There are six stages in the complete process of counting: rote counting, enumeration, identification, reproduction, comparison, and grouping."⁹

Often counting receives little emphasis in arithmetical instruction. One reason may be that few teachers recognize its importance and the reasons why it is important. Materials provided teachers have treated counting only lightly or have advocated counting to find how many.¹⁰

Counting is "the" fundamental process. Spitzer considers counting to find the answer the foundation of all methods of solution. Problems

8. Swenson, op. cit., 54-55.

9. Brueckner, L. J., and Grossnickle, F. E., How to Make Arithmetic Meaningful. Philadelphia: John C. Winston Company, 1947, 170.

10. Spitzer, Herbert F., The Teaching of Arithmetic. Boston: Houghton Mifflin Company, 1948, 67.

used in introducing fundamental processes can be solved by counting.

"The child who can count is thus assured of having at least one solution."¹¹

Rote counting. Rote counting is the repeating of number names without meaning. Children may repeat the number names in nursery rhymes or in other verbal activities but they seldom know the mathematical meaning of the numbers.¹²

Rhymes such as the ones below are often used:

One for the money,
Two for the show,
Three to make ready,
Four to go.

One, Two, three, four, five;
I caught a hare alive
Six, seven, eight, nine, ten;
I let him go again.

One, two; buckle my shoe.
Three, four; shut the door.
Five, six; pick up sticks.
Seven, eight; lay them straight.
Nine, ten; a big, fat hen.

One little, two little, three little Indians,
Four little, five little, six little Indians,
Seven little, eight little, nine little Indians,
Ten little Indian boys.

The rhyme has little value beyond acquainting the children with the number names and aiding in the memorization of these number names in their proper order. Children enjoy rhymes and repeating number names

11. Spitzer, op. cit., 75.

12. Brueckner and Grossnickle, op. cit., 170.

after adults. Rote counting is an early significant experience which every child needs to practice. It is a necessary beginning; children do need to know the names of numbers with which they will deal in arithmetical situations.¹³

These activities will give children an opportunity to have counting experiences:¹⁴

1. Counting chairs for the reading class
2. Passing out materials, as books, pencils and paper, for a given group to use
3. Counting the empty seats to see how many children are absent
4. Counting balls and other play equipment
5. Playing with blocks
6. Engaging in other experiences such as dramatic play, walks, gardening, cooking, experimenting.

Below is an illustrative lesson in counting adapted from Spitzer's book, The Teaching of Arithmetic.

To begin this lesson the teacher may say: "Susan tells me that she has eight dolls. I wonder how she knows she has that many dolls." Either some child or the teacher will eventually suggest that Susan can find out how many she has by counting. "How do you count? Can anyone show me?" the teacher may ask. If no child tells how, the teacher says, "I count this way: one, two, three, four, five, six, seven, eight." She touches fingers as she says the number names. "Let's all count fingers. Touch a different finger each time you say a number word. Count with me." The teacher then touches the fingers on one hand with the index finger of the other, saying one, two, three, four, five, as she does so. She does the same with the other hand beginning with the number six. After several tries at this, the teacher remarks: "The last number name that you use tells you how many fingers you have touched. Let's try it again to see if that's true." Teacher and children then touch fingers, stopping at three. The teacher asks someone what number name was used when the last finger was touched; then asks how many fingers were touched.

13. Swenson, op. cit., 61.

14. Wheat, H. G., How to Teach Arithmetic. Evanston, Illinois: Row, Peterson and Company, 1951, 26.

Enumeration. Enumeration means counting to find the number of things in a group. The simplest way to find the number is to touch each object as it is counted, which establishes a one-to-one relationship between the number name and the object.¹⁵ Here the number names themselves are the basic means of discriminating the objects in a group. As a child touches the object, points to it, or nods his head toward it, he applies the appropriate number name in its proper place in the series of names: one to the first object, two to the second object, etc.; in this way his attention is centered on the object he is counting.¹⁶

During his earliest experiences with counting the pupil should pick up an object from a group and set it aside as he says, "One". Then he should pick up another object and set it aside as he says, "Two". In a similar manner he proceeds to move the blocks one by one as he says the number names in their proper sequence.¹⁷

"Children should recall and associate the various number names with experiences in their daily lives.

1. Each of us has one head.
2. Each of us has two feet, two ears, two hands.
3. A bicycle or a scooter has two wheels.
4. A clock has two hands.
5. A tricycle has three wheels.
6. An airplane has three wheels.
7. A wagon or a car has four wheels.
8. Some candy bars cost five cents.
9. We have ten fingers and ten toes."¹⁸

15. Brueckner and Grossnickle, op. cit., 171.

16. Wheat, op. cit., 24.

17. Ibid., 24.


18. Ibid., 31.

This is an illustrative lesson in enumeration adapted from Wheat:¹⁹

The teacher whispers to a child, "Raise your arm five times." Then to the other children he says, "Watch Jack and tell me how many times he raises his arm." The children observe and tell the number of times that Jack performs the prescribed act.

After considerable experience with this type of activity, the pupil may be able to go through a group of objects quite accurately by pointing to each object one by one, and finally he will apply the proper number name as he looks at each object. When the pupil does this with ease he has learned to count or has progressed to the third stage of counting, that of identification.

Identification. Identification means the pupil identifies the number of objects in a group. An activity which best illustrates this is giving the children more than one group of objects and having the child choose the group which has four in it, or five, or ten.

For example, pictured is a group of blocks: 

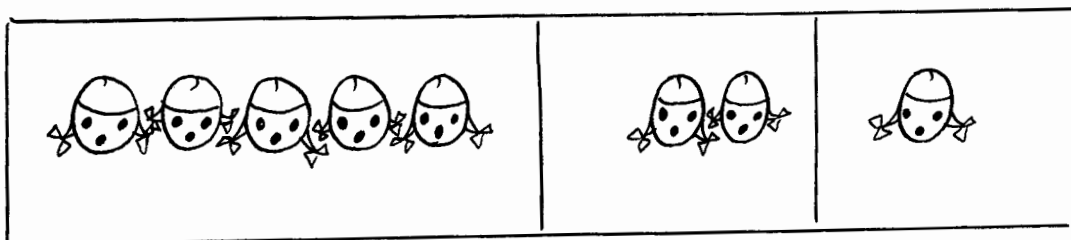
Identification answers the question, "In which group are there five blocks?" When the pupil names the number of black blocks in the group, he identifies the numbers.²⁰

These are illustrations of the type of activity children at this level of counting may be expected to do in a group situation:

19. Ibid., 30.

20. Brueckner and Grossnickle, op. cit., 171.

1. Find the group that shows five girls:



2. Which box has 6 monkeys in it:



Reproduction. Reproduction is the selection of a certain number of objects from a group.²¹ In order to reproduce a number, the child must understand the quantity the symbol represents.

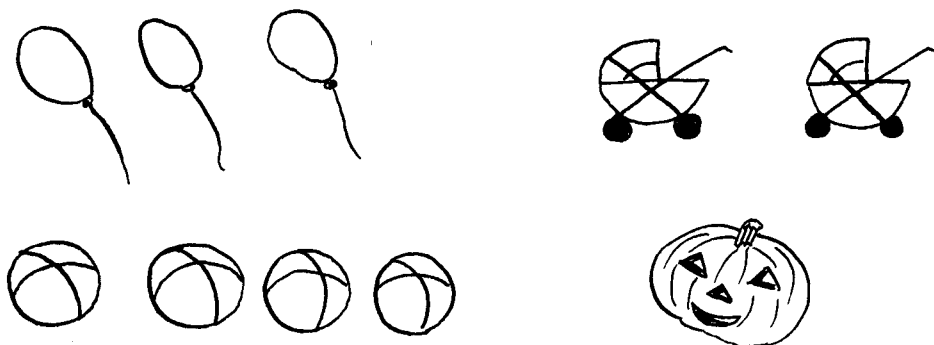
Look at Dick's rabbits. How many rabbits has he?
Dick wants to give each rabbit a carrot.
Draw a carrot for each rabbit.



21. Brueckner and Grossnickle, op. cit., 171.

This type of exercise is excellent for chalkboard or flannelboard type of activity; the pupils enjoy the manipulation and later will make up their own number stories. While working with materials such as kindergarten sticks the child, in reproducing, number, gains understanding in quantity; he learns what makes a number.

Comparison. "Comparison answers the question of the type, "Of which kind...are there more? (or less) How many more?"²²



For first group and later individual practice, this type of work shows how comparison may be taught with concrete objects. Cooperative work with a chart such as this demands that the child determine whether there are more balloons or balls; he must compare the numbers in each square.

When the pupil counts and names the number of pencils in a group, he identifies the number. When a pupil uses counting to bring five pencils from a box or to get a specified number from a larger group, he selects the number. When he draws five apples on the board or makes five marks to show how many apples he has,

22. Ibid., 171.

he reproduces that number. Exercises which involve the identification, selection, and reproduction of groups of given sizes may be given orally or in mimeographed form at this time.²³

Grouping. Grouping is the final stage of counting. "Children who have had many experiences with counting begin to recognize small groups without counting them. A child may see a group of two, three, or four within a larger group. He starts counting with the group and continues counting by ones to find the total. For instance, he says 3, 4, 5, 6 in counting a group of six objects. He may recognize two or more groups within a larger group and not count by ones at all. For instance he may think 'three and two' to get a total of five objects."²⁴

So that children may learn more about groups, some instruction in the grouping of objects according to arrangement patterns should be given. Wheat²⁵ suggests using configuration patterns as a means of review. The teacher may introduce the pupils to the various arrangements or patterns the objects in a group may take. A few of the children may not recognize certain arrangements but this is not all-important since these patterns are merely a means, not an end. The pupils in working with configuration patterns should think of these arrangement exercises as, "Arranging Objects to Show How Many."

It is suggested that this work begin with the possible arrange-

23. Wheat, op. cit., 32.

24. California State Department of Education, op. cit., 22.

25. Wheat, op. cit., 33.

ments for the figure two. It can be shown this way:



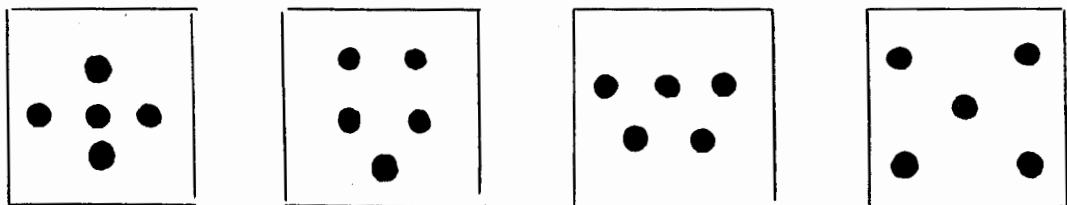
and this way: ● ● The pupils will then be ready for other groups, three, four, five and so on.

Wheat²⁶ uses five for this illustrative lesson:

"Let us make pictures of five," says the teacher as she makes five circles on the board,



and adds, "This is one way to arrange five." Then she directs the pupils to place buttons on a card to show other patterns of five, assisting a few until they can proceed with confidence. Finally the pupils exhibit their arrangements:



Practice in making various number patterns or various arrangements of objects in a group is practice in the study of groups, for the pupil must count and check as he works out an arrangement, and he should count and check when he has completed it. In the beginning children may have to resort to counting to find how many there are in a group, but gradually through such practice in group recognition, most pupils in the first grade should be able to recognize at a glance and without counting groups as large as five when arranged in one or more of the common patterns.²⁷

Since counting is basic to advanced numerical concepts the results

26. Wheat, op. cit., 34.

27. Wheat, op. cit., 34.

of counting are pertinent. Through their daily experiences the children have developed understandings of numbers through 10. They have studied individual numbers and numbers as a group. Their concepts concerning groupings has gradually developed to a point where they are ready for the generalization: "We count to find how many there are. We say number names to tell how many there are. We write figures to show how many there are."²⁸ Pupils receive pleasure and excitement from being able to know how many there are by counting.

Through the exercise of counting, the pupils have begun to learn to rely upon themselves for the answers they seek. The teacher has never said, "See, I have six blocks." The teacher has always asked, "How many blocks do I have?" Each pupil has counted and found the answer for himself. No one has told him. The answer when he has found it has been his answer.

Through the exercise of counting, the pupils have started on the road of learning arithmetic: (1) They have developed and clarified their number ideas to ten. (2) They have learned a method of study and through its use to rely upon their own efforts for the knowledge they seek. They have at least made a beginning. They are ready to move ahead to new studies. Through the study of groups by counting them and by comparing them, the pupils in the space of half a year become familiar with groups. They are now able to use their arithmetic to meet the social and personal demands imposed upon them by the world of numbers in which they live. They have a method of answering the question, "How many?" and they can use with ease, ability, and confidence this newly acquired method to meet their immediate needs. To them and for them arithmetic becomes functional, something they can and will use in their daily activities both in and out of school.²⁹

Aside from the sequence of introduction of ideas, there is much over-

28. Ibid., 35-36.

29. Ibid., 47.

lapping in the development of these phases of counting. One topic is not completed before another is begun. Rote counting is not completed before enumeration and identification are begun. A child may first count to 10 by rote, next count 10 or fewer objects in order to designate which one or how many in all and later extend his counting to larger numbers. Each of the earlier concepts is continuously expanded and extended along with and in relation to the ideas which are introduced later. Often several ideas are so inseparable if a number experience is to be satisfactorily understood by the learner. The same type of interrelationship applies to number processes, among themselves and in relation to number meanings.³⁰

GIVING MEANING TO THE NUMBERS FROM ONE TO TEN

"A pupil may be able to do rote counting and yet have no meaning of the numbers."³¹ By showing these numbers in many different ways the pupil learns the meaning of each number. One way to aid children in learning these concepts is by use of charts and posters.

A basic chart recommended by Brueckner and Grossnickle in How to Make Arithmetic Meaningful³² consists of ten sheets pasted on a large sheet of newsprint. Each sheet, about 8" x 10", contains the name of a single number from 1 to 10, a picture showing the value of the number, and figure. Chart I, on page 28, is a sample sheet of this basic chart. The sheet may be pasted on the chart as the concept is developed

30. Swenson, op. cit., 64.

31. Brueckner and Grossnickle, op. cit., 172.

32. Ibid., 173.

CHART I*

one



1

*Adapted from Brueckner and Grossnickle, How to Make Arithmetic Meaningful. Philadelphia: John C. Winston, 1947, 173.

in class. The pictures may be drawn by the teacher or cut from magazines.

Supplemental charts are recommended to accompany the preceding chart. The first in this series, about 24" x 36", shows the numbers from 1 to 10 in vertical position with the numbers written on the left-hand side, corresponding names in the center and the pictures of objects to show the value of the numbers on the right.³³ This chart is illustrated on page thirty.

The second chart in this series shows the figures from 1 to 10 arranged horizontally with semi-concrete materials, such as circles, triangles, squares, stars and other geometric forms placed above the number name. An example of this is illustrative chart number III. "If different colors are used for each row of designs the geometric figures will stand out more effectively than if one color is used."³⁴

Another chart, the third, modeled after the preceding chart is suggested for familiarizing children with number in various patterns; it is composed of semi-concrete materials in vertical arrangement.³⁵ This chart is pictured on page thirty-two.

The fourth supplemental chart is used in the teaching of grouping and identifying the numbers by grouping. Here the figure is on the left,

33. Ibid., 173.

34. Ibid., 173.

35. Ibid., 174.

CHART II*














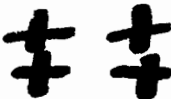

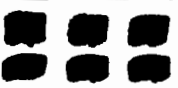

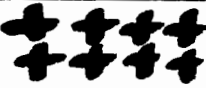


1		one
2		two
3		three
4		four
5		five
6		six
7		seven
8		eight
9		nine
10		ten










CHART III*

				
one	two	three	four	five
1	2	3	4	5

				
six	seven	eight	nine	ten
6	7	8	9	10











*Adapted from Brueckner and Grossnickle, *op. cit.*, 174.

CHART IV*

1	one	
2	two	
3	three	
4	four	
5	five	
6	six	
7	seven	
8	eight	
9	nine	

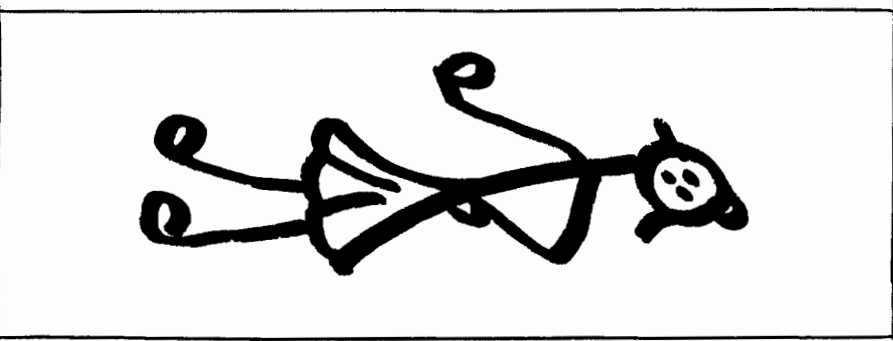
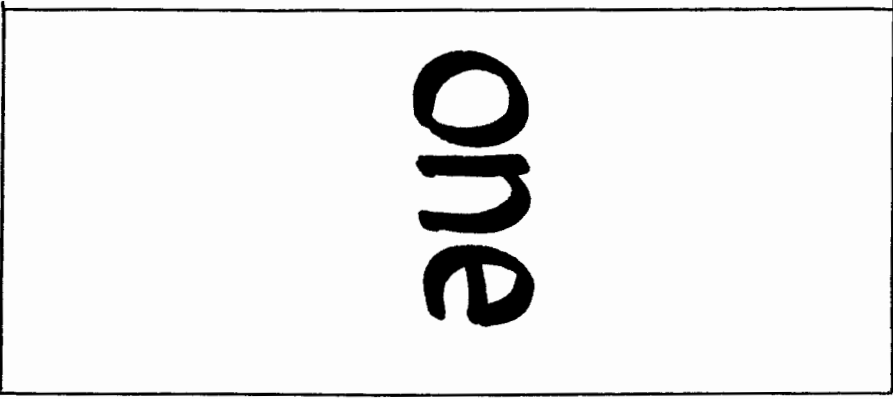
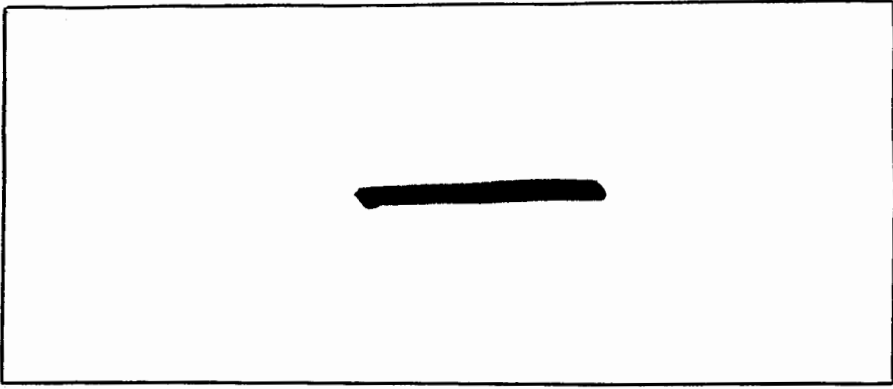
*Adapted from Brueckner and Grossnickle, *op. cit.*, 174.

CHART V*

1	one	
2	two	
3	three	
4	four	
5	five	
6	six	
7	seven	
8	eight	
9	nine	
10	ten	

*Adapted from Brueckner and Grossnickle, op. cit., 174.

CHARTS VI - VII - VIII*



*Adapted from Brueckner and Grossnickle, op. cit., 174.

the number name in the center, and the configuration pattern on the right; the arrangement is vertical.³⁶ Chart V is an example of this supplemental chart.

Three sets of 10 oaktag cards, each about 4" x 6", are suggested by Brueckner and Grossnickle for matching. The first set shows a different number name on each card; corresponding numbers are on each card of set 2; pictures of corresponding number objects on each card of the third set. The children will then match the three sets of cards as a test for their understanding of the three ways of representing number from 1 to 10.³⁷ A sample of these cards are shown on page thirty-four.

Since first grade children have many associations with the number two, it is reasonable that they will be interested in learning to count by 2's. This procedure given here is an excellent one for developing this fact:

*The pupils should learn to count groups by first using groups of concrete things. Counting children by twos is a concrete experience. Children can form in pairs as in marching. Two children march to the front of the room and two more join them. The class then should count the number in the two groups by twos. The teacher should point to one group and have the class state the number, or 2, and then the other group, and then have the class count by twos to get the number, or 4. Then additional pairs can be counted by twos. The pupils also can arrange cards or markers in groups of two and count them. In the same way, other concrete objects such as books,

36. Ibid., 174.

37. Ibid., 174.

pencils, or crayons can be used to show 6, 8, and 10. Each pupil should represent the numbers with cards or other markers. Likewise, he should count by twos, on the counting-block and on the abacus.

Reading the even-numbered pages of a book provides an excellent social application of counting by twos. The pupil should discover that the even-numbered pages are on the left-hand side. Other social applications consist in counting pennies, by twos, or the number of feet, hands, eyes, ears,³⁸ shoes, or gloves for a given number of children."

An oaktag chart, about 24 inches by 36 inches should be made to show the numbers by twos through at least 10. It is sufficient to use the figures, up to 10, and the semi-concrete objects in a vertical pattern. Chart IX shows the type of chart which can be made for the study of twos.

In his study concerning the number knowledge of young children, Woody³⁹ discovered that they know something about United States money. One of the best ways to teach the value of coins is to have coins for the children to count. The three most familiar coins to first grade pupils are pennies, nickels, and dimes. The pupil should be given opportunities to count the number of pennies there are in a nickel and in a dime. A teacher-made chart, such as the chart shown on page thirty-eight, on which the coins are pasted is an excellent way of expressing these values.⁴⁰

38. Ibid., 184.

39. Woody, Clifford, "The Arithmetical Background of Young Children." Journal of Educational Research. 24: 188-201, (October, 1931).

40. Brueckner and Grossnickle, op. cit., 185.

CHART IX*

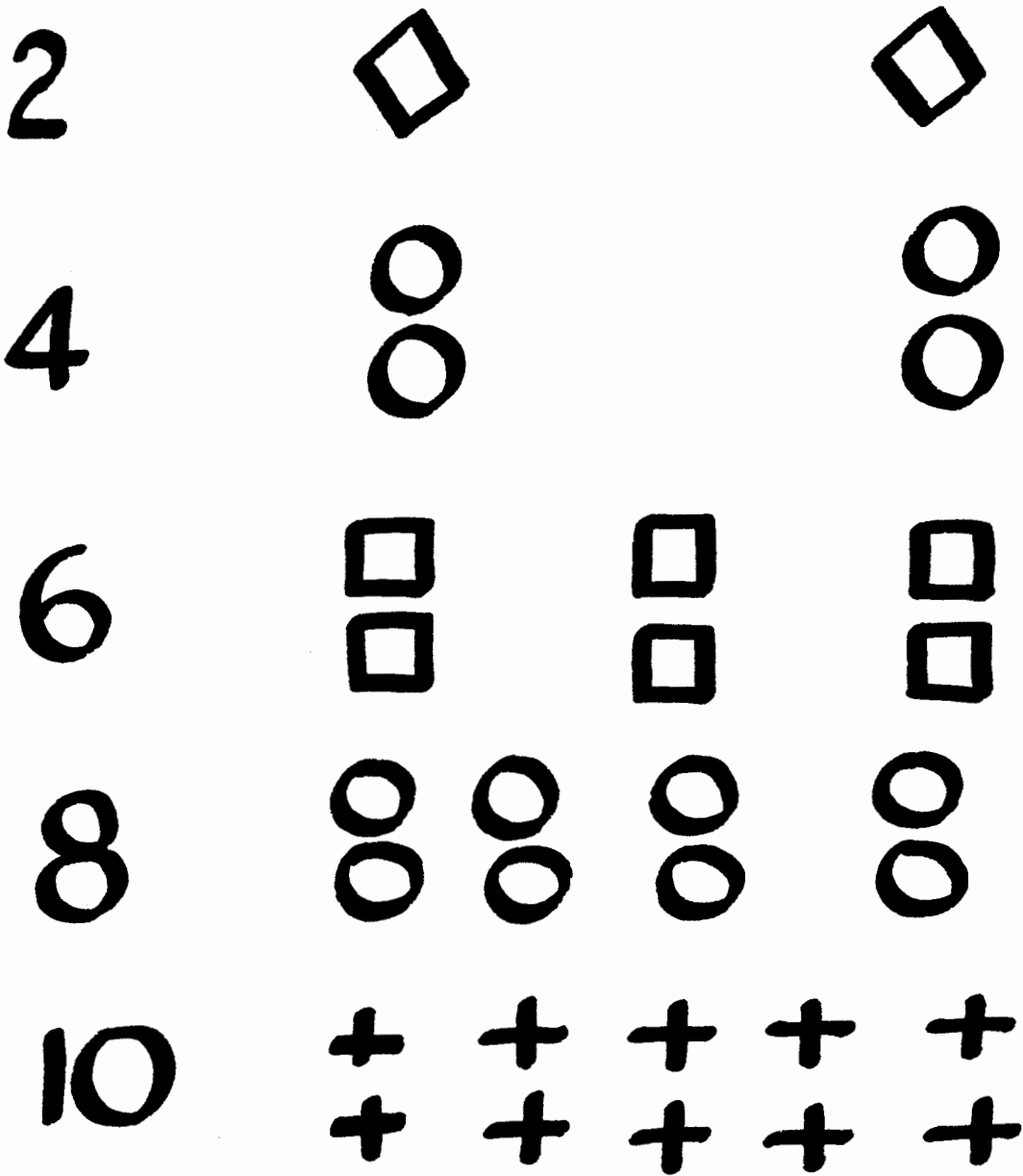
*Adapted from Brueckner and Grossnickle, op. cit., 184.

CHART X*

10 cents



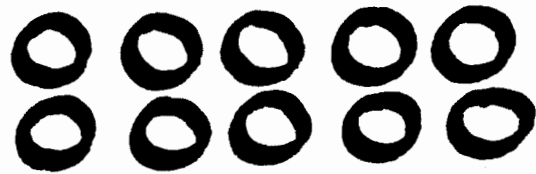
1 dime



2 nickels



10 pennies



1 nickel



and

5 pennies



TEACHING THE WRITING OF FIGURES

Learning what the number symbols represent and how to write them legibly and with ease requires time and effort. At first the pupil does not need the figures, however, many children recognize some of the written symbols for numbers, and some children can write some of them. Reading and writing figures are skills which have to be taught, but they are most effectively taught in connection with activities so the children will have real purpose for learning and using these skills.

There are nine numerals and zero which children need to learn to write. All other numbers are different arrangements of these ten symbols. Children need to be shown how to make these symbols, because, if they are merely given a sample and left to copy it according to their own devices, they develop awkward and uneconomical methods of writing numbers. Children readily learn to identify the number name with the symbol as they are using numbers in their work and play in the schoolroom. Classroom activities in which children read and write numbers are:

1. Reading and writing directions for activities which include numbers.
2. Finding pages by number
3. Making records of attendance, weather, temperature, and scores in games
4. Making a calendar
5. Reading recipes, copying recipes in booklets.⁴¹

41. Rosenquist, op. cit., 121.

Wheat⁴² gives the following illustrative lesson for beginning the writing of figures informally:

The teacher presents the figures as ways of telling the answer to the question, "How many?" For example, the teacher holds up five pencils and asks, "How many pencils do I have?"

The pupils count and answer, "Five."

The teacher says, "I will make the chalk say five," and writes the figure 5 on the board. "This (5) is the figure five. The figure tells how many pencils I have."

The pupil must learn that we use both the word five and the figure five to answer the question, "How many?"

Later, when the pupils can write the figures, the teacher directs, "Write the figure that tells how many pencils I have." Still later when the word number comes into use, the children may be directed, "Write the figure that tells the number of pencils I have."

"Inability to write figures correctly results from a failure of the pupil to know where to begin the stroke and incorrect concepts of format because of poor perception techniques."⁴³ Printing each figure on a large chart with the beginning point marked in color shows the pupil where to start the stroke, as shown on chart XI, page forty-two. The beginning point of the ten digits should be shown.

While authorities are agreed on the way to make the figure formations they differ in their methods of presentation. Brueckner and Grossnickle in How to Make Arithmetic Meaningful use a logical step-by-step progression which can be developed nicely in a classroom, for it provides multi-sensory activity. First the pupil traces the digit with

42. Wheat, op. cit., 27.

43. Brueckner and Grossnickle, op. cit., 175.

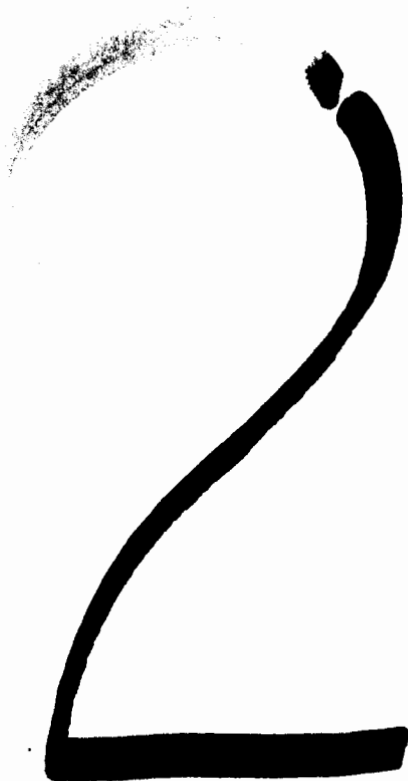
with his finger to get the form. Large-sized calendar numbers may be used to give him a kinaesthetic feeling of the format of the figure. A practice sheet in which part of the outline of a figure is given is an effective help to small children.⁴⁴ A sample of a practice sheet is given on page 43. As with handwriting there should be instruction in free-arm movement. This movement is necessary for clear, legible figures. Small, irregular figures result when the pencil or crayon is held too firmly.

The digits 3, 6 and 9 are frequently reversed; 2, 4, 5 and 7 are less often reversed. "Reversals occur usually when children are immature; the initial steps in writing figures need to be carefully supervised. Pupils who are left-handed have a greater tendency to reverse numbers than right-handed pupils. Frequently showing children how to write numbers will prevent reversals, as well as other bad habits."⁴⁵

44. Ibid., 175-176.

45. Brueckner and Grossnickle, op. cit., 176.

CHART XI*



*Adapted from Brueckner and Grossnickle, op. cit., 175.

10		
10		
10		

*Adapted from Brueckner and Grossnickle, op. cit., 175.

EVALUATING PROGRESS AND ABILITY

Evaluation is closely related to teaching and learning. It is a cooperative process, carried on by teacher and children in terms of the objectives of instruction and the experiences of pupils. "How valuable evaluation is as an aid to education, and classroom teachers in particular, depends on how well it fulfills the functions claimed for it."⁴⁶

The teacher carefully observes what the children do and say. By noting growth in understanding and readiness, interests and needs are discernable. By informal questions, comments, and discussion a teacher helps children become aware of their progress. The teacher constantly endeavors to aid pupils in developing an objective attitude toward their strengths and weaknesses. It is essential to help children see that they are not working for grades. After learning what the children do not know, the teacher will have opportunity to help the children.⁴⁷

Several techniques of evaluation are useful in the primary grades. Observation of children's behavior is the most valuable. Individual tests, interviews and informal teacher-made group tests are also useful. An individual oral readiness test is shown on page forty-eight of this paper. Certain teachers find it helpful to keep anecdotal records; others prefer inventories on which items are checked. An inventory-type evaluation sheet is shown on page forty-seven. Regardless of technique

46. Quillen, I. J., Education for Social Competence. Chicago: Scott, Foresman and Company, 1948, 416.

47. California State Department of Education, op. cit., 46.

the important thing is the regularity of the evaluation. "Since behavior is not static but continually changing, the collection of evidence must be a continuous process and careful recording and reporting of such evidence, must be made so that intelligent appraisal can take place."⁴⁸

While watching and talking with individual children the teacher keeps several questions in mind.

1. Does the child use terms relative to number in spontaneous conversation?
2. Does the child enter into discussions concerning number?
3. Does the child use number in solving problems which arise in his school activities?
4. Does the child use of number give evidence of valid ideas at his level of understanding?

The teacher watches carefully as the children deal with numbers of things. There is a need for the teacher to note whether pupils see differences in groups and understand equality in number. How do the children count? Do they know the number names in order? Is the child aware that the last number said tells how many or which one? Must students touch each item to keep track of their counting? Can pupils recognize a group of a given number? Can the children reproduce a group, giving the teacher the correct number? Is the class beginning to understand the number system as a way of grouping in tens?⁴⁹

48. Quillen, op. cit., 405.

49. California State Department of Education, op. cit., 50.

The first grade teacher may test children's knowledge of number by using objects. It is necessary to work with each child individually or with small groups of children.

While giving the test the teacher watches the way in which each child finds answers to questions. Teachers notice whether pupils judge differences by pattern, by counting or by grouping. The way in which the pupil counts should be checked carefully. As the pupil says the number names does he touch or move each object, or count by fixing his eyes on each object in succession. If the way in which a child works is not obvious the teacher may ask, "How did you know?" or "Why did you think so?"⁵⁰

The teacher uses the test results in planning to meet the children's needs. Children who match to find answers should have many opportunities to match things and to learn the number names and their meanings. Those who count with difficulty and have trouble in recognizing and reproducing groups need further experiences in counting and in attending to groups in functional classroom situations. Children who answer the questions with ease and assurance are ready to count larger numbers of things and to separate and recombine groups of objects.⁵¹

50. Ibid., 51-52.

51. Ibid., 52.

A CHECK-LIST FOR EVALUATING PUPIL PROGRESS

1. Can say the number names to ten
2. Can point to each object in a group
3. Can give each object its name in counting
4. Can tell how many objects in any group to ten
5. Can name each figure
6. Can find and name the figure telling how many
7. Can write the figures
8. Can write and read figures to tell how many
9. Can tell the larger group from the smaller
10. Can count how many more or how many less
11. Can count on fingers how many more
12. Can tell how many more (less) without objects
13. Can count groups away or together
14. Can think groups away or together
15. Can think answers without objects
16. Can think answers to problems
17. Can make problems to work
18. Knows and can tell the story of two
19. Knows and can tell the story of three
20. Knows and can tell the story of four
21. Knows and can tell the story of five
22. Knows and can tell the story of six
23. Knows and can tell the story of seven
24. Knows and can tell the story of eight
25. Knows and can tell the story of nine
26. Knows and can tell the story of ten
27. Can change a group into equal groups
28. Can count equal groups
29. Can work out questions without objects
30. Can work out questions about equal groups

READINESS TEST

Readiness Test in Primary Arithmetic—Mathematical Phase

1. Number sequence: (All oral, read by teacher).
 - When I say 1, 2, 3, 4, what comes next?
 - When I say 5, 10, 15, 20, what comes next?
 - When I say 2, 4, 6, 8, what comes next?
 - What number is left out? 1-2-3-4-5-7-8-9.
 - What number is left out? 100-200-300-400-600.
 - What number is left out? 10-20-30-50-60.

2. Reading numbers:
 - Read these numbers for me: 5 8 9
 - Read these numbers for me: 21 34 47

3. Fractions: (Oral; read by the teacher)
 - What part of this pie is eaten?
 - What part of the pie is not eaten?
 - How many halves make a whole pie?
 - How many eggs in a half-dozen?
 - How many quarters does it take to make a half-dollar?

4. Use of number in solving problems: (Oral; read by the teacher)
 - Bob had 5 pennies. He spent 1 penny. How many pennies did he have left?
 - Mary had 2 cents. Her aunt gave her 3 cents. How many cents did she have in all?
 - How many 5-cent ice cream cones can you buy for a dime?
 - Jimmy is 4 years old. His sister is 1 year younger. How old is she?
 - John has 20 pennies. He wants twice as many. How many does he want?
 - Carol was invited to a 3 o'clock party. She got there at 3:30. How late was she?
 - If I divide 9 books among 3 children, how many will each get?

Readiness Test in Primary Arithmetic—Social Phase

1. Uses of precision instruments (Oral)
 - What is the thing we use to tell what time it is?
 - What is the thing we use to find how long a room is?
 - What is the thing we use to tell the day of the month?
 - What is the thing we use to tell how warm or cold a room is?
 - What is the thing we use to see the stars better?

What is the thing we use to tell how heavy a child is?
 What is the thing that the man reads in your home to find out
 how much electricity you are using?

2. Units of measure (Oral)

How many cents in a nickel?
 How many cents in a dime?
 How many cents in a dollar?
 How many nickels make a dime?
 How many days does it take to make a week?
 How many quarts make a gallon?
 How many hours is it from noon today to noon tomorrow?

3. General uses of number (Oral)

How many sides has a square?
 How many things is a pair of things?
 What is the name of the last day of the week?
 What is the date of your birthday? (month and date)
 We buy tea by the pound. How do we buy eggs?
 We buy sugar by the pound. How do we buy milk?⁵³

53. Brueckner, L. J., and Grossnickle, F. E., How to Make Arithmetic Meaningful. Philadelphia: John C. Winston Company, 1947, 56-58.

Chapter III

VALUE OF INSTRUCTIONAL MATERIALS OF LEARNING

"The success of a meaningful program in arithmetic depends, in a large degree, upon methods and materials of instruction. There is no one method or any single type of instructional material which will suffice in all situations. The skillful teacher selects methods and materials in terms of the outcomes to be achieved and of the needs and the interests of the children. If instruction in arithmetic is to insure a steady growth in understanding number relationships, a wide variety of instructional materials must be used to enrich and supplement the learner's experiences."¹

Instructional materials are defined as anything which contributes to the learning processes; these include pictures, models, books, real activities and teaching aids which provide multi-sensory experience for the pupils. These materials may be used for (1) introducing and enriching number concepts, (2) developing desirable attitudes toward number work, and (3) stimulating interest and activity in number.² Later in this chapter listings of and illustrations for the use of instructional materials are given.

Instructional materials encourage the type of learning which is generated by discovery and experimentation.³ This concept of learning

1. Grossnickle, F.E., Junge, Charlotte, and Metzner, William, "Instructional Materials for Teaching Arithmetic." The Teaching of Arithmetic. Fiftieth Yearbook, Part II, National Society for the Study of Education. Chicago: University of Chicago Press, 1951, 155.
2. Ibid., 155.
3. Gates, A. I., Jersild, A.T., McConnell, T.R., Challman, R.C., Educational Psychology. New York: Macmillan Company, 1942, 344.

is a process of growth; learning is a meaningful process based on understanding. Learning of this type does not occur quickly; many different kinds of experiences are needed for discovery and experimentation.

Grossnickle, Junge, and Metzner⁴ state that learning number consists in an orderly series of experiences which begins with concrete objects and progresses toward abstractions. Instead of definite lines of demarcation among the progressive stages there may be an overlapping between these stages.

Readiness, the first step in learning number work, "has been considered synonymous with mental maturity and thus related to grade placement."⁵ It is now recognized that readiness in arithmetic is a function not only of mental maturity and inner growth but also of previous experience, methods of learning, interests, attitudes, and purposes.⁶ Because of the complexity of readiness it can not be circumscribed as to time or grade. Readiness in arithmetic should be regarded as that period when the child's background for learning a new concept is "appraised, the foundational experiences are provided, and a purpose for the new learning is established in the mind of the learner. One of the main tasks of a teacher during the period of readiness is to create a problem situation which will challenge the

4. Grossnickle, et. al., op. cit., 156.

5. Ibid., 156.

6. Gates, et al., op. cit., 305.

child's interest and which will provide the felt need basic to discovery and experimentation."⁷

There is no one way to create readiness. The successful teacher is able to tap many different facets of children's interest which lead to effective work in the initial study of a topic or process. The initial stimulus is more important in the early grades than at the advanced grades. Readiness builds up like a rolling snowball providing the work is meaningful to the pupil. In this case the inner urge comes from innate curiosity and the teacher does not have to provide situations which will call for answers to problems which satisfy an immediate felt need. Therefore, it is highly important that all beginning work in number is meaningful. When this work is not understood, the teacher has to rely on drill and artificial means for stimulating enough interest to have the pupil continue his study of a given process or operation."⁸

In developing a meaningful number program for young children there is necessity for a period of experimentation and discovery. "A developmental discovery period should provide for two things: First, a class demonstration and second, a period for individual discovery. A class demonstration performs the same function in arithmetic that it does in science. In each case it gives the pattern of activity to follow so that the pupil may discover for himself a particular fact or principle. Laboratory materials are essential for a demonstration in both arithmetic and science."⁹ During this period of discovery and experimenta-

7. Grossnickle, et. al., op. cit., 157.

8. Grossnickle, F.E., Metzner, William, The Use of Visual Aids in the Teaching of Arithmetic. Brooklyn: Rambler Press, 1950, 9.

9. Ibid., 12.

tion the pupil works with a variety of instructional materials to obtain possible solutions to a problem.

"The laboratory period is not an activity period for the sake of having an activity."¹⁰ The teacher is a director who guides, questions and stimulates the pupil to make discoveries by helping him use materials to portray a given situation. Individual discovery is only one phase of this experimentation. Group discovery is just as important; for in group situations the class works as a group to make discoveries with manipulative or visual materials or both.

A meaningful program in arithmetic implies that the classroom must be equipped with and arranged for different instructional materials. Without many manipulative materials it is not possible to teach arithmetic meaningfully to many pupils in the primary grades.

The statement that the classroom is to become a laboratory does not imply that all arithmetic is laboratory work any more than a study of a science is all given to laboratory work. After the pupil makes one or more discoveries about a fact, he must use it and apply it. Now the laboratory becomes a workshop or a conventional classroom in which the pupil practices with materials to become efficient and to achieve mastery of the process. The classroom becomes the place for a period of discovery instead of a place for the teacher to tell the pupil what to do or to learn. The laboratory technique provides the optimum condition for the pupil to learn by doing and discovery rather than by limitations of meaningless operations."¹¹

There are four classifications for instructional materials: (1)

10. Grossnickle, et. al., op. cit., 157.

11. Grossnickle and Metzner, op. cit., 6.

real experiences, (2) manipulative materials, (3) pictorial materials, and (4) symbolic materials. "The reader must view instructional material as existing on a continuum in which various materials appear in increasing abstraction as one proceeds from direct experiences to symbolic materials. The lowest quantitative thinking results from dealing with real objects, whereas the highest level results from dealing with abstract symbols."¹²

For many years teachers have been primarily concerned with abstract symbolic materials. The results have not been wholly satisfactory. In recent years a great interest has been brought about in introducing visual materials as part of the instructional program in arithmetic. There is danger in using instructional materials if these materials are not conceived as part of a comprehensive number program; failure with a narrowly arranged program is possible with visual and manipulatory material as well as with symbolic materials. Each kind of material has a function to perform.¹³

Real experiences in themselves are not instructional material. They are treated here as instructional material because "they make the real situation the learning situation in a functional setting."¹⁴ Therefore, the things used or met in the experience, and not the experience itself, are the instructional materials. Whether or not

12. Grossnickle, et. al., op. cit., 162.

13. Grossnickle and Melzner, op. cit., 1.

14. Grossnickle, et. al., op. cit., 162.

the experience contributes to the child's number understanding is determined by the use made of the experience. The basis of meaningful learning is purposeful, real experience.

Real experience, then, means tangible, direct, first-hand experience. Such experiences are counting money, for some school activity, using a measuring cup to find how much $\frac{1}{2}$ or $\frac{1}{4}$ is, recording the time taken for recess, etc. "Real experience means active participation in a real-life situation with responsibility for the outcome."¹⁵

Manipulative materials are materials which the pupil is able to feel, touch, handle, and move. They may be real objects which have social applications in our everyday affairs, or they may be objects which are used to represent an idea or characteristic of number or the number system. Objects like the measuring cup, ruler, other measurement devices, and money are used in our daily lives; materials designed specifically for number work are counters, kindergarten sticks, flannel board and colored circles, etc.

Pictorial materials include such things as charts, posters and materials which may be projected on the screen. Slides, films, and filmstrips are also represented in this classification.

Symbolic materials are written or printed material such as textbooks, workbooks, and instructional tests.

15. Ibid., 162.

There should be opportunity for excursions and field trips, visual, manipulative and symbolic materials in the instructional program.

Grossnickle¹⁶ recommended that the classroom become a laboratory where the pupil can manipulate things to discover number themselves. This laboratory method is applied to learning by various techniques. The nature of the instructional program determines how it can be used in the classroom. Each experience demands a different approach to the problem of learning number.

16. Grossnickle, F. E., "The Use of Multi-Sensory Aids in Developing Arithmetical Meanings," Arithmetic, 1948. Supplementary Educational Monograph, Number 66, Chicago: The University of Chicago Press, 1948, 1-14.

ILLUSTRATIONS OF THE USE OF INSTRUCTIONAL MATERIALS

Real ExperiencesConcepts Concerning Weight¹⁷

Grade one class---introductory lesson for a short unit on the topic of weight.

Purpose of lesson: To stimulate the children to formulate and to state their ideas concerning the meaning of weight, procedures for estimating weight, procedures for measuring weight, and some of the ideas underlying balance.

Materials used for lesson: (1) Two paper bags containing sand; one bag was colored red and one bag was colored blue. The bag colored red was larger than the one colored blue and contained more sand. (2) Two paper bags colored green. Each bag was the same size, but one bag contained rocks and one contained cotton.

Lesson procedure

Teacher: (Teacher held up a large paper bag colored red and smaller paper bag colored blue. Each of the bags had sand in it.) Today we are going to talk about weight. These paper bags have sand in them. Which paper bag do you think is the heavier?

Christina: The big one.

Teacher: Why is it heavier?

Laura: Because it is bigger.

Craig: I'll show you. (Craig came up and lifted the two bags and continued with the following statements.) This one (the larger one) is heavier because it is so hard to lift. I can lift this one higher. (He showed what he meant by lifting the lighter one higher than the heavier one.) I can hold this one up just this far, and the light one up this far.

Teacher: That is a good way to find out which bag is heavier.

Joe: That is like a teeter-totter, Craig. The heavy one goes down farther, and the light one come up higher.

Craig: Yes, if Richard and Jim were on the teeter-totter, Richard

17. Spencer, P. L., and Brydegaard, Marguerite, Building Mathematical Concepts in the Elementary School. New York: Henry Holt and Company, 1952, 17-19.

would go down and Jim would go up. (Richard was much heavier than Jim.)

Richard: That wouldn't be true if I moved forward and Jim moved back on the board.

Teacher: Craig, what would need to be true for Jim to go up and for Richard to go down?

Craig: Richard and Jim should sit the same distance from the end of the board. If they do, Richard will go down and Jim will go up.

(Note: Experimentation with concepts of balance was left for another lesson.)

Jim: That is just the way it works with the paper bags.

(Teacher held up two paper bags colored green. The bags looked alike in color, size, and extent to which they were filled.)

Teacher: Here I have two other paper bags. Which one of these do you think is heavier?

Laura: What is in them?

Teacher: That is a good question. Could you be a good judge of which one is the heavier if you didn't know what was in them?

Mary: No. You could guess, but it wouldn't make much sense. You really can't tell which is heavier unless you know what is in them.

Teacher: That is true. I'll tell you what each bag contains. This bag has rocks in it. This one has cotton in it.

Susan: Well, that's easy. You were just trying to catch us. Cotton takes up a lot of room, but it is soft and light. The bag with the rocks is heavier.

Teacher: Without using scales such as are used in a store, when we look at two packages, how can we tell which package is the lighter one and which is the heavier one?

Richard: By lifting the things, you can tell which one is light and which one is heavy.

Frank: You can tell by looking and seeing which bag is bigger if they have the same kind of stuff in them.

Lewis: You can tell which one is heavier if you know what is in them. If one sack had cotton and one sack had rocks, you couldn't tell by just looking at the sacks.

Craig: You could use a teeter-totter. Using a teeter-totter, we could see who is heavier when two of us are on the teeter-totter.

Karen: Our big teeter-totter would work.

Jerry: I think that the big teeter-totter would work.

Teacher: Would you like to make a small teeter-totter?

Dick: Yes. We could make it with some boards. I'll show you how.

(Dick went to the board and sketched a plan for a teeter-totter.)

Teacher: That is a good picture, Dick. Would you like me to get the

boards so that you could put them together for your teeter-totter?
 Dick: Could you bring them tomorrow? Then we could weigh things tomorrow.

Teacher: I shall try to have the boards by tomorrow. At recess, you may want to try finding out more things about heavier and lighter when you play on the teeter-totter.

Manipulative Materials

Real objects

Procedures for Measuring¹⁸

This lesson was developed by a group of children in grade one. The group had done considerable measuring and had evolved a concept of one-half that was well-refined for the level of the given group of children.

Purpose of the lesson: (1) To challenge thinking concerning the nature of measuring and procedures for measuring; to analyze and to refine the ability to estimate. (2) To lead the children to evolve measuring sticks.

Materials used: Four sticks. (Two sticks were straight; one was a piece of doweling about 36" long and the other was a narrow piece of board about 16" long. The two other sticks, different in length, were not straight but were not very crooked.) A ball of string, a piece of string, and pieces of colored paper of the following sizes—18" x 24", 12" x 18", 9" x 12" — were on the table near where the group was seated.

Lesson procedure

Teacher: What things do you see on this table that could be used to help us measure?

General response: A piece of string, sticks, a ball of string, and pieces of paper.

Teacher: If you wanted to measure how long our blackboard is, what would you use to measure it?

Sandra: I would use this stick. (Sandra pointed to the longer straight stick.)

Bill: (Picking up a crooked stick.) This stick is not good. I wouldn't use it. It's too crooked and wouldn't be good to use.

Jim: I wouldn't use this stick, either. (Jim picked up the other

18. Spencer and Brydegaard, op. cit., 40-44.

stick that wasn't straight.)

Teacher: Why wouldn't it be all right to use these sticks?

Bill: It wouldn't be good because you can't stretch it out. (Bill tried to straighten the stick that he had rejected to show that he was right.)

Dick: Let's throw out the two sticks that we won't use to measure with.

Mary Lou: I would use a stick rather than a piece of string because a string might sag.

Teacher: Which of the things on the table would you use to measure how tall you are?

Peg: I'd use this stick. (Peg selected the longer stick of the sticks accepted by the group for measuring.) I'll show you how it works. Will you hold the stick, Jim?

(Jim took the stick and tried to measure Peg by standing the stick against her. He readily saw that Peg was taller than the length of the stick. Together, they evolved the idea of having Peg stand against the wall and placing the stick at right angles to the wall and against the top of Peg's head. They decided upon a piece of chalk to mark Peg's height on the wall. Then Peg stepped away, and they measured the height of the mark from the floor.)

Jim: Peg, you are this tall. You are one stick and this much more of the stick tall. (Jim showed how much more of the stick.)

Tim: That is one stick and almost a half more.

Teacher: How can you tell where half is on the stick?

Page: I know one way that you can find out. You could take the stick, and put it on the end of your finger, and find the middle. It will balance if you have the middle.

(This was about the last suggestion that the teacher anticipated. The teacher experimented to see if she could perform the set task. It worked fairly well!)

Page: Now we can measure to see if it is as far from this end to your finger as it is from the other end. We can use this piece of string.

(Page used the piece of string and measured from one end to the position of the teacher's finger, held the place on the piece of string and compared it with the other side of the stick.)

Page: It worked.

Tom: I know another way that we could have found the middle of the stick. We could use a piece of paper, but the paper we have on the desk isn't long enough.

Susan: I have another way. May I cut a piece of string from the ball of string?

Teacher: You may, Susan.

(Susan took the ball of string, measured a piece that was the length of the stick, and cut the string.)

Susan: I can fold this string in the middle. Then I can measure and put the string on the stick.

(Susan placed the string on the stick and found the center of the stick. She then checked to see that the two sides of the stick were equal in length).

Dick: I'll make a mark in the middle, and we'll see if you come to the half mark.

(Jim measured Peg.)

Jim: Peg, you were as tall as one and this much less than one-half more of the stick.

Sandra: May I be the next one to be measured?

Teacher: You may be the first one to be measured tomorrow, Sandra.

The lesson ended for the time being, but with the measuring that followed the next day, the children evolved the idea that it would be very helpful if they again divided their stick--each half into half again.

Using Objects to Represent an Idea

Number Grouping.¹⁹ Give each child 6 counting discs. Then ask him to see what new and interesting ways he can arrange the discs in groups. Next have each child whisper something which he discovers independently. Select the answers which have something to do with the making of a number fact and ask each child who made the discovery to tell it to the class. Then the teacher has each pupil make the number combination with the discs.

Pictorial Materials

Teaching the Meaning of the Numbers from One to Ten.²⁰

The children in the first grade made a number chart by placing colored paper balls in a pocket chart on the bulletin board. Then the children made a permanent chart of oaktag.

1	one	0
2	two	00
3	three	000
4	four	0000
5	five	00000
6	six	000000
7	seven	0000000

19. Grossnickle, et. al., op. cit., 165.

20. Ibid., 165.

8	eight	00000000
9	nine	000000000
10	ten	0000000000

Projection Materials

Teaching Comparison.²¹

Colorful pictures from magazines and story-books were used to teach the pupil an understanding of the words "big" and "little". The children studied the pictures and indicated understanding by locating things in the picture according to their size. The type of question used is as follows:

1. Find the little girl.
2. Tell what the big boy is doing.
3. What color is the little dog?
4. How many big birds do you see?

Developing Ideas Relating to Time²²

Show pictures of typical daily activities (eating lunch, going to bed, rising, time for school, etc.) together with pictures of clocks showing the time for each of these activities.

VISUAL AND MANIPULATIVE MATERIALS

The visual and manipulative materials are categorized under these topics: (1) manipulative materials, (2) pictorial materials, and (3) projection materials. While there are many materials available, this is merely a representative listing of both manufactured items and inexpensive items collected and/or teacher-made; it is, however, an adequate sampling.

Manipulative materials may be purchased from supply houses or they may be made in the classroom. Primary grade teachers have so many

21. Ibid., 167.

22. Grossnickle, et. al., op. cit., 167.

duties that it may be difficult to make all of the needed supplementary materials.

Materials Available From Supply Houses²³

1. Concrete Materials. Such concrete materials as 1-inch cubes, sticks, beads, clock faces, and toy money are necessary items in an arithmetic laboratory. Materials of this kind are readily secured from most school-supply houses.
2. Teach-a-number Kit. The kit consists of a set of wooden blocks $1\frac{1}{2}'' \times 1\frac{3}{4}''$ in which a different colored block is used for each digit from one to ten. Each block is labeled with the numeral it represents. The blocks vary in thickness. The block to represent 1 is $\frac{1}{4}''$ thick and the block to represent each succeeding number increases $\frac{1}{4}''$ in thickness. Pupils may learn the value and meaning of numbers through comparison of sizes of blocks.
3. Arithmetic Readiness Kit. The kit consists of ten wooden blocks $2'' \times 2'' \times \frac{3}{4}''$, with a hole through the center, a stringing lace, small individual number cards, and a small supply of play money. The materials may be used to develop the serial idea of numbers from 1 to 10 and to show relationships among the basic coins.
4. Primary Number Cutouts. These materials include a portable easel ($19'' \times 28''$) which has a display surface of black velour and an ample supply of cutouts of rabbits, ducks, stars, and circles that are lined on both sides with contrasting colored velour. The coherence of the velour surfaces holds the cutouts in position on the display board even when this board is almost vertical. The material is used to demonstrate different concepts and relationships with small groups.
5. Counting Disks. These disks are made of durable fiber about $1\frac{1}{4}''$ in diameter in a solid color. They lend themselves to counting, discovering number facts and demonstrations.

23. Grossnickel, F. E., "Instructional Materials for Teaching Arithmetic." The Teaching of Arithmetic. Fiftieth Yearbook, Part II, National Society for the Study of Education. Chicago: University of Chicago Press, 1951, 173-175.

6. Modernized Abacus. This abacus has a solid backing which supports 4 vertical wires with different colored beads to represent the first four places in our number system. Nine beads of the same color are on each wire with a top tenth bead of the next succeeding color. The tenth bead completes the ten and shows by its color that 10 beads are equal to 1 bead of the next higher order.
7. Ten-Ten Counting Frame. This counting fram, $8\frac{1}{2}$ " x 9", is made of wood and it supports ten wires, each of which hold ten $\frac{1}{2}$ " beads. Its principal uses are to provide a counting activity and to emphasize the fact that our number system is a system of tens.
8. The Hundred-Board. The hundred board consists of a framed cardboard 20 inches square equipped with 100 cardboard disks and two cards 20 inches square. The one is a counting card which contains the numbers 1 to 100 printed in sequence, ten numbers to a line. The hundred-board is designed for counting and to develop the basic facts meaningfully in all four processes.

Teacher-Made or Inexpensive Instructional Materials

Grade I: ²⁴

Large lima beans or other counters

Tongue depressors, swab or kindergarten sticks, or similar materials (for making bundles of ten's)

Number cards to build concepts and teach recognition of numbers

1. Pictures (1 through 10) with no figures
2. Cards with number name
3. Cards with figure only
4. Spot patterns (1 through 10) with no figures
5. Set of spot pattern cards with figures
6. Set of picture cards with figures
7. Set or sets of abstract number cards, 1 through 10, to use for matching with 1, 4, 5, and 6
8. Spot pattern cards showing groupings, 1 to 10

Ten strip (10 spots on 10 strips of tag) and 10 individual spot cards for use in pocket chart

One hundred cart

1. Circles or spots, 10 rows of 10 each

24. Clanfield, Mary, Mastery Program in Arithmetic for Elementary Schools. Longview, Washington; Longview Public Schools, 1952, 37.

2. Numbers 1 through 100
 Number frame--10 large beads on wire
 Flannel board and colored circles
 Large clock face
 Large thermometer

PICTORIAL MATERIALS²⁵

1. Number Readiness Charts. The set consists of fourteen separate charts $18\frac{1}{2}" \times 24\frac{1}{2}"$, and one perforated sheet of 67 cutouts. The charts are colored pictures representing experiences familiar to young children. Slots are cut in the charts so that the cut outs may be inserted. The set of charts is designed for use at the beginning of the first year of instruction to provide a series of experiences that should precede the usual systematic program.
2. Class Number Chart. This chart shows pictures, numerals, and names of numbers from 1 to 10 and is printed on a large cardboard for classroom display. The chart is designed to teach the reading and writing of numbers.
3. The 100-chart. The 100 chart consists of 10 strips $3" \times 22"$. A row of 10 red dots is printed on each strip. A wall chart is required to display the materials properly.
4. Counting chart. This chart provides symbols in semi-concrete form which may be used for counting from 1 to 100.
5. One hundred chart. The chart, $25" \times 29"$, is printed on heavy paper and is designed for classroom use. The main purpose of this material is to help teach counting to 100 and visualize the number symbols in the process.
6. Numbers We See. This is a 72-page book of pictures in color and designed to provide a systematic development of number concepts by non-formal, concrete methods. Although this is a bound book, it is included in this listing because it is entirely pictorial. A teacher's edition is available which gives detailed suggestions for using each page.
7. Picture-Symbol Cards. These cards are used primarily to teach the number symbol by associating it with the picture of the group of objects it represents. Sets of cards for numbers 1 to 10 are available in sizes suitable for pupil use and class use.

25. Grossnickle, et. al., op. cit., 176-177.

8. Group Recognition Cards. Cards of various sizes show number groups either by using pictures of real objects or by using dots, squares, triangles, or lines arranged in patterns to represent the number groups. Sets are available for individual pupil and class use.

Books

This list includes only a few books which are especially rich in quantitative language and ideas. They are suggestive of the type of materials useful in helping children develop arithmetical ideas.

Bannon, Laura, Patty Paints a Picture. Chicago: Albert Whitman Company, 1946

Barrows, Marjorie, Ezra the Elephant. New York: Grossett and Dunlap, 1934

Brown, Margaret Wise, Baby Animals. New York: Random House, 1941.

Burton, Virginia Lee, The Little House. Boston: Houghton Mifflin Company, 1942.

Gag, Wanda. Millions of Cats. New York: Coward-McCann, 1928.

Hogan, Inez. The twin series. New York: E. P. Dutton.

Hurd, Clement, Bumble Bugs and Elephants. New York: William R. Scott, 1938.

Jackson, Lora Z., Around the Clock. New York: Grosset and Dunlap, 1937.

Merwin, Decie, Time for Tammie. New York: Oxford University Press, 1946.

Milne, A. A., Now We Are Six. New York: E.P. Dutton, 1927.

Smith, Charlotte Helen (Charims, pseudonym). Surprise. Racine, Wisconsin: Whitman Publishing Company, 1934.

PROJECTION MATERIALS

Specific Uses of Projection Materials

There are four specific things which may be accomplished by the use of projection materials.²⁶ They are:

1. To motivate study of number through showing a life-like usage of number in a meaningful situation.
2. To show the type of procedure to use or to follow in presenting a particular phase of a topic or a process.
3. To introduce activities that provide or suggest means of manipulating materials so as to make learning effective for the pupils.
4. To present an over-all view of a particular topic that may be used as a review or as a form of a test.

Factors which should not be overlooked by the teacher in planning for a demonstration are: (1) frequently materials are not available at the opportune time, since often the school does not have films or filmstrips of its own, and (2) the teacher does not plan for the demonstration adequately. Before showing a film or a filmstrip, the teacher should preview the material and plan the lesson with reference to the materials shown and to suggestions offered in the teacher's guide.²⁷

The advantage of filmstrips over films is the ease of adjusting the timing to meet needs of the pupils. The rapidity with which a

26. Grossnickle and Metzner, op. cit., 16.

27. Ibid., 19.

film goes makes it difficult for young children to follow with the same degree of comprehension. Too, it is much easier to have a discussion on a filmstrip than on a film because of the ease of operating a filmstrip. Another favorable feature of the filmstrip compared to a film is the differential in cost. A sound film costs approximately \$45.00 as compared to about \$3.00 for a filmstrip. The filmstrip is better adapted for instructional purposes.²⁸

Some filmstrips may be too long for one class period. As much time should be given to each frame as seems desirable in view of:²⁹

1. The general maturity of the children.
2. Their previous experiences with the subject matter dealt with or suggested.
3. Their previous experiences with visual materials, and their ability to observe accurately and to gain facts and concepts from what they see.
4. The relative difficulty of the frame shown; the amount of background required for understanding its implications.
5. The number of suggested activities needed to understand the frame. The picture may ask the pupil to point to certain things on the screen. Activities of this kind require more time than do those in which pupils merely observe the sequence of portrayals on the screen.

"One caution should be observed with respect to timing. Children's interests must not be dissipated by delay. A filmstrip is successful only to the extent that children give it willing and active attention.

28. Grossnickle, F.E., "The Use of Multi-Sensory Aids in Developing Arithmetical Meanings," Arithmetic, 1948. Supplementary Educational Monograph. Chicago: The University of Chicago Press, 1948, 8-9.

29. Grossnickle and Metzner, op. cit., 18-19.

The first showing of a filmstrip should do the following things:³⁰

1. Arouse curiosity and create an interest in the subject.
2. Suggest questions and stimulate interest for further study.
3. Give the children a chance, through discussion and adroit questioning, to identify with their own experiences each point brought out.
4. Give the teacher leads concerning the backgrounds of the children, and weaknesses in their information, with the result that the teacher may guide them more wisely in their subsequent investigations.

This appraisal³¹ represents the composite group judgment of a committee of classroom teachers at the grade level for which the film is designated. The evaluation is based on the suitability of the material as a teaching instrument to accomplish the purposes indicated by the producers at a specified grade level.

Sound Films:

1. Let's Count. Coronet Films. This film is valuable to the teacher in suggesting types of counting activities which may be used in the classroom. Evaluation: Good. Collaborator: F. L. Wren.
2. Parts of Nine. Young America Films, Inc., The large number of concepts discussed in this film would make it more suitable for review purposes than for introductory procedures. Evaluation: Very Good. Advisers: W. A. Brownell and L. K. Eads.
3. What Is Four? Young America Films, Inc. This film illustrates four in a number of concrete and semi-concrete situations and then proceeds to an abstract presentation for four. Part I should be used for beginning pupils who have some number experiences but who are not ready for the use of symbols to

30. Ibid., 19.

31. Grossnickle, et. al., op. cit., 177-185;
Grossnickle and Metzner, op. cit., 36-58.

indicate addition and subtraction facts. Evaluation: Good.
Advisors: W. A. Brownell and L. K. Eads.

Filmstrips:

1. Encyclopedia Britannica Films, Inc. 35 mm filmstrips

A series of sixteen filmstrips to be used in developing an understanding of the meaning, sequence, and use of numbers, was prepared in collaboration with J. R. Clark and C. H. Clark. The photographs are clear, simple drawings, arranged to permit use in class instruction. These filmstrips are also excellent for review purposes. Evaluation: Good.

Counting to 5

Counting to 10

Reading Numbers to 10

Writing Numbers to 10

Counting by 10's to 30

Counting by 10's to 50

Counting by 10's to 80

Counting by 10's to 100

Counting from 10 to 15

Counting from 15 to 20

Counting from 20 to 40

Counting from 40 to 100

Reading Numbers to 50

Reading Numbers to 100

Working with Numbers to 100

Writing Numbers to 100

SUMMARY

The materials and methods described have proven their worth under the test of classroom use. These have been described in response to a belief that, properly taught, arithmetic should be to the primary child a series of happy, fascinating, yet educative experiences.

First-grade teachers have always known that some children are familiar with a few number facts when they enter first grade. Now research has defined the extent of this familiarity and its prevalence. Authorities present evidence that simple number facts may be understood by children at an early age, and that this knowledge steadily increases. The realization of the nature of number comes slowly to the child through his activity in counting. While counting the child will gain not only the abstract idea of number but will gain concepts in addition, subtraction, multiplication, and division for these are all implicit in counting.

The first-grade teacher will need to appreciate the tiny increments of understanding by which a young child's knowledge of number grows. By answering questions, by providing experiences, by arranging activities and by offering suggestions of additional facts to be learned in a situation the teacher may help most.

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