

Summer 7-1-1964

## A Longitudinal Program of Individualized Arithmetic Instruction in Grades Four, Five and Six

Charles Gordon Libby  
*Central Washington University*

Follow this and additional works at: [https://digitalcommons.cwu.edu/all\\_gradpapers](https://digitalcommons.cwu.edu/all_gradpapers)



Part of the [Educational Assessment, Evaluation, and Research Commons](#), [Educational Methods Commons](#), and the [Science and Mathematics Education Commons](#)

---

### Recommended Citation

Libby, Charles Gordon, "A Longitudinal Program of Individualized Arithmetic Instruction in Grades Four, Five and Six" (1964). *Graduate Student Research Papers*. 147.  
[https://digitalcommons.cwu.edu/all\\_gradpapers/147](https://digitalcommons.cwu.edu/all_gradpapers/147)

This Thesis is brought to you for free and open access by the Student Scholarship and Creative Works at ScholarWorks@CWU. It has been accepted for inclusion in Graduate Student Research Papers by an authorized administrator of ScholarWorks@CWU. For more information, please contact [scholarworks@cwu.edu](mailto:scholarworks@cwu.edu).

A LONGITUDINAL PROGRAM OF INDIVIDUALIZED ARITHMETIC  
INSTRUCTION IN GRADES FOUR, FIVE AND SIX

---

A Research Paper  
Presented to  
the Graduate Faculty  
Central Washington State College

---

In Partial Fulfillment  
of the Requirements for the Degree  
Master of Education

---

by  
Charles Gordon Libby  
July 1964

THIS PAPER IS APPROVED AS MEETING THE PLAN II  
REQUIREMENT FOR THE COMPLETION OF A RESEARCH  
PAPER.

---

Jettye Fern Grant  
FOR THE GRADUATE FACULTY

## ACKNOWLEDGMENT

I wish to express my sincere appreciation to Dr. Jettye Fern Grant for her genuinely friendly guidance and assistance in the writing of this paper.

## TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION . . . . .	1
The problem . . . . .	1
Background information . . . . .	1
Importance of the problem . . . . .	4
Limitations of the study . . . . .	6
II. REVIEW OF THE LITERATURE . . . . .	8
III. LOCAL SITUATION . . . . .	10
IV. THE INDIVIDUALIZED PROGRAM OF INSTRUCTION . . . . .	14
Procedures . . . . .	14
Sequence of instruction . . . . .	24
Instructional materials . . . . .	29
Evaluation . . . . .	29
BIBLIOGRAPHY . . . . .	32
APPENDIX . . . . .	34

## CHAPTER I

### INTRODUCTION

The problem. It was the purpose of this research paper to develop a longitudinal program of individualized instruction in arithmetic that (1) would meet the individual needs of children in grades four, five and six; (2) could be used in a graded elementary school situation; and (3) could be adapted to the Scott Foresman program of mathematics.

Background. Individual differences is a term that has meaning in everything that is living. In the field of education there is increasing emphasis being placed on the need to fit the instruction of each child to his individual needs, abilities and interests. It is important, therefore, that an instructor in today's schools knows how to recognize, properly evaluate and meet the individual differences found in any classroom.

The instructor must realize that these differences do exist from birth. These differences may be physical, mental or psychological in nature and may be derived from both heredity and environment. Generally, individual differences are exemplified by behavioristic and physical characteristics. Our long term educational goals in America

include, among other things, statements concerning the development of the child's self concept and the promoting of each child's growth to the best of his ability in all areas. These aims are concerned with the recognition of individual differences and their development in the classroom. To realize fulfillment of these educational aims the teachers of America must be concerned with individual differences in the students.

. . . The difference of form which is due only to the different dispositions of matter causes not a specific but a numerical difference; for different individuals have different forms, diversified according to the differences of matter. (8:83)

Several studies have been done regarding the range of differences found in the classroom. It is important that we examine some of these in order to be aware of their impact on the education of children.

Goodlad (4:1-28) worked with a group of twenty-seven children. The children were tested at the end of their first year in public school, and the following results were noted. There was a battery test range of over one grade minimum. The test showed that each child had differences within his own battery of over one grade, and in some cases, more than two grades. The study, when carried over the years the children attended school, showed that as schooling progressed the range of differences became larger. By the time the children were in the fourth grade,

each child had a developmental pattern completely different from any other child. The IQ, as tested by mental age, showed a difference of fifty points. This approximated the range of achievement and readiness, except for the upper and lower ends of the range. At the extremities of the scale were students who showed exceptional talent, or lack of talent, on one or more areas of the test.

Another study by Biber (1:179-190) showed the IQ range of a particular group of pupils to be from 105 to 161, with a median IQ of 130.5. These children were involved in tests relating to their superior intelligence. They showed a wide range of attitudes, diversified talents in their areas of expression, as well as pronounced physical and psychological differences. In the main, they were found to be more radical in their differences than a group of less intelligent students.

. . . The differences among children are great and since the differences cannot be substantially modified, school structure must facilitate the continuous educational progress of each pupil. Some pupils therefore, will require a longer period of time than others for achieving certain learnings and attaining certain developmental levels. (54:52-53)

In a graded elementary school where a predetermined area of arithmetic is assigned to be taught many problems arise. These range from the more intelligent child's boredom created by the seemingly slow progression through the arithmetical concepts and skills to the anxieties of



the less intelligent child caused by not having enough time to understand or learn these same arithmetic concepts and skills. These are problems caused when individual differences are not completely taken into consideration as is necessary if one is to educate the individual to his maximum capacity.

To meet the needs of each individual child in the learning of arithmetical concepts and skills, it is necessary to eradicate, as completely as possible, the problems caused by predetermining what is taught and when it is taught. This can only be accomplished by letting each child progress at his own speed in a program of individualized instruction.

Thoreau said:

If a man does not keep pace with his companions, perhaps it is because he hears a different drummer. Let him step to the music which he hears, however measured or far away.

Importance of the problem. It is clearly evident that children differ in many ways. In the understanding and mastery of arithmetical concepts and skills, it is no different. It is a fallacy to believe that any two children will learn exactly the same things or progress at the same speed in a given area of curriculum.

In a graded elementary school where children are held to strict grade standards, a teacher is expected to cover the same textual material and teach all of the

children the same concepts and skills within the school year. As is commonly known and accepted, some children are incapable of grasping the prescribed material within this span of time while others could grasp much more if it were to be presented.

Children differ in their arithmetic abilities from their very first year in school and even before. As has been stated previously, differences are derived from both heredity and environment. Since no two individuals, even those in a specific family, inherit the same traits from their parents or share the same experiences, one would not expect them to be alike in all ways.

These differences in children must be accepted and worked with if maximum learning is to take place. This means that the teacher must diversify teaching to meet the needs of each individual. A simple presentation of a skill may prove adequate for the competent child while much more detail and actual experience may be needed to teach the less able student the same skill.

Differentiating the type of instruction for individuals is not enough, however. Differentiated activities and assignments also will be necessary if all students are to sufficiently master the material presented. As soon as a child is ready to proceed to a new concept or skill, instruction is given to him as needed. When the

teacher feels that the child has had sufficient instruction to begin work then the individual is given a basic assignment to be completed independently. After the basic assignment is completed, the more competent child may work more difficult and/or complicated examples or prepare extra work pertaining to some pertinent point. The less able student may not master the concept or process even after completing the assigned work. Then it becomes necessary for more instruction and possibly some review work, depending on the nature of his difficulty after diagnosis by the teacher.

This research study will attempt to establish a program in which each child may work to his own capacity and at his own speed in a vertical sequence of arithmetic instruction. The program will be designed to meet the individual needs of children in the intermediate grades of a graded elementary school situation.

. . . To care for individual differences, grouping is inadequate, so if the teacher prepares adequate materials to care for the individual, she helps children to enjoy arithmetic and learning seems to blossom. The instructor teaches the skills, abilities, appreciations and attitudes to all--only--when it is the first teaching. After that the skills or understandings will be presented to those who need them until they, too, have discovered for themselves the understandings needed to use arithmetic as a tool. (2:14)

Limitations of the problem. This program of individualized arithmetic instruction has been designed, primarily, to meet the individual needs of those children

in grades four, five and six, attending Barnes Elementary School, District #403, Kelso, Washington. The program will encompass those concepts and skills now taught in grades one through eight according to the recently adopted Scott Foresman program of mathematics.

Included in the paper will be:

1. A review of literature concerning other individualized programs
2. A brief description of the local situation including,
  - a. The community
  - b. The school plant and facilities
  - c. The classroom
3. A brief review of the work done by the curriculum committee in selecting the Scott Foresman program
4. A summary of the sequences of instruction
5. A criteria for selecting instructional materials
6. Procedures to be used in the program, including evaluation
7. Sample schedules
  - a. One week's daily schedule of all curricular areas
  - b. Sample daily schedule of arithmetic instruction showing the use of the teacher's time and pupils' time
8. Samples of record sheets, assignment sheets, review work sheets, and tests

## CHAPTER II

### REVIEW OF LITERATURE

Very little has been written concerning programs of individualized arithmetic instruction. However, since 1963 at least three articles have been published that relate to such programs. These will be discussed briefly.

Searight gave a brief overview of an individualized arithmetic program used in the Lanphere Public Schools, Madison Heights, Michigan. He stated that those who have felt a need for a program can now begin to individualize instruction with a little more assurance that it can be done and that others are doing it, however, he believed that not all teachers should attempt to individualize their instruction.

Browning, in her article, "Individualized Instruction," said that to care for individual differences in the arithmetic area, grouping is inadequate and individualizing the arithmetic program will increase learning and help children to enjoy arithmetic.

The individualized instruction program in Montgomery County, Maryland, initiated by Caroline C. Potamkin began as a program in ability grouping. The children were permitted to progress at their own speed, with the expectation that they would establish their own groups. Instead, many

children were still working on the first assignment while others were working on a second and still others were working on a third. At this point the program began to evolve into the individualized program of instruction.

## CHAPTER III

### THE LOCAL SITUATION

The community of Kelso, Washington, is located on U. S. Highway 99, approximately forty miles north of the Oregon border. It is at the southernmost tip of the Puget Sound Basin and near the mouth of the Cowlitz River, which passes through its boundaries. It has a population of 8,379.

Many of the people in this small community are employed in occupations whose operations are situated in Longview, Washington, which is about three times larger than Kelso and borders directly on its west city limit. The economic status of Kelso is low, as there is little industry and the business district is quite small. The majority of the residential area is quite old and thus low in property value.

Kelso is a high equalization district. Although low in economic status and property valuation, it has a reputation for supporting its schools. Not once in over twenty years has a bond issue or special levy been defeated. This is mainly because the administration has made a continuous effort to provide adequate facilities and high quality instruction, and has made it clear to the public that the schools intend to provide the best education possible for the students in the system.

The school system is set up on a 6--3--3 plan with six years for the elementary school plus kindergarten, three years for the junior high school and three years for the senior high school. There are five elementary schools, two junior high schools and one senior high school plant. Money is now being set aside for the construction of a new senior high school which is to be built before 1970. There are 184 faculty members employed in the district and approximately 5,500 students in attendance.

The Barnes Elementary School, for which this program is being planned, is located on a seven acre site in the northern end of Kelso, the area of lowest socio-economic status in the community. It was constructed during the summer of 1960 and was put into operation for the school year 1960-1961. Three additional classrooms are now under construction. When completed, the physical plant will contain classrooms for one kindergarten, two first grades, two second grades, a combination room of second and third grades, two third grades, two fourth grades, a combination room of fourth and fifth grades, two fifth grades and two sixth grades. There is also an adequate library and a large multipurpose room.

Besides the regular staff members, the Barnes School utilizes the services of a librarian, an art consultant, a music teacher, a speech therapist and a psychologist.



Those instructional aids which are supplied by the district and are shared by the staff are kept in a central area for easy access when not in use. Other materials are available to the teachers from a large library of audio-visual aids maintained in the Cowlitz County Courthouse in downtown Kelso.

Each classroom at the Barnes School is furnished with portable desks and storage cupboards. This allows for much flexibility in the arrangement of facilities for instructional purposes. A diagram of a typical classroom at Barnes, showing one possible arrangement for instruction, can be seen on page 34 in the appendix.

Within the past few years the curriculum committee studying mathematics in Kelso made a thorough investigation of many programs being developed and put into practice throughout the nation. This was done for two purposes: (1) to find a program, or programs, which would provide a new and deeper understanding of arithmetical concepts and processes, and (2) to select one program to be used at the elementary and junior high school levels in the Kelso Schools.

Previously, the arithmetic program in the school system varied from building to building and in some instances within a building. The traditional methods of mathematics, such as the program presented by the Winston series, were in wide use.

Feeling that the program developed by Scott Foresman and Company used methods which would (a) make understanding of the base-ten system of numeration more meaningful and (b) provide a more systematic method of analyzing problem situations through the use of equations, the committee recommended the adoption of the Scott Foresman program. The recommendation was accepted and the program was put into use in September, 1963.

## CHAPTER IV

### THE INDIVIDUALIZED PROGRAM OF INSTRUCTION

Procedures. Many attempts have been made to organize the curriculum to make provisions for individual differences. Among them are various kinds of administrative measures intended to make groups more homogeneous in ability. Homogeneity itself is a fallacy, for there are such wide variations among the specific abilities of pupils on quite similar levels of intelligence and such large differences among the traits of single individuals that no method of grouping will insure homogeneity.

Grouping is a very important part of classroom organization and teaching. Each teacher must consider that grouping may consist of a large part of the class or one or two individuals, depending on the objectives of the lesson to be presented, and the needs of the children to be taught. Grouping can be used for many reasons ranging from remedial instruction to enrichment for the high achiever. Grouping's main purpose is to gain the most effective use of pupil's and teacher's time.

Teachers should have a very flexible classroom program at every level because no matter what the year in school, the age of the children, or the abilities of the class,

there is always a range of individual differences among the pupils with respect to every attribute. Once a teacher realizes the range of differences within the classroom, he will inevitably find a need to work with groups and individuals.

At the beginning of each school year the teacher should determine exactly where a child is in his knowledge and understanding of arithmetic concepts and skills. This should be done after the children have had at least a week or two for review. The review work is necessary simply because after their summer vacations, children often fail to remember items learned during the previous year's work. This also allows each child sufficient time to become used to the new classroom situation and, what is more, to establish rapport with the teacher. To evaluate the children's work and determine their knowledge and understanding of arithmetic concepts and skills, the teacher may use observations of the children working, interviews with them about their work and the difficulties they are having, analysis of their written work and oral responses and/or evaluation through the use of standardized tests. (Evaluation will be discussed further in the last section of this chapter.)

After the review work is completed and the children's work has been evaluated, the teacher then should have determined a point at which each child may be started in the sequence of instruction in the individualized program.

When instruction is started on an individualized basis, each child should be given learning experiences which pertain to the mastery and understanding of those skills and concepts which have not been fully learned or understood in previous grades. This is necessary in order to assure continuous progress through the sequence of instruction, with maximum learning.

After a child has completed all areas which had not been learned satisfactorily in previous grades, he should then continue on an individual basis in the sequence of instruction, studying new material as soon as he is ready for it and progressing through the sequence at his own speed.

Once the child has started in the sequence, the teacher should check periodically on the child's progress. This should be done as he completes the work for each concept or skill, to determine whether or not it has been learned. If it has been learned, the child should then be allowed to progress to the next topic. If not, more instruction must be given in the area presenting the difficulty and additional assignments may be made to reinforce the instruction given on the topic. After new assignments have been given and the child has completed the work, the teacher should retest him to be sure that the skill or concept has been learned. If the child shows a satisfactory

level of accomplishment, he should be allowed to go on. If not, the process must be repeated until the child can demonstrate that he has learned the material at hand. It is suggested that each time the child must repeat instruction on the same skill or process, the teacher should try different approaches in the instruction, using as many different techniques as necessary to teach the child. Because children differ in the speed at which they learn as well as in the way in which they learn, the differentiation of instruction by the teacher is necessary to insure optimum learning for all pupils.

Each new concept or skill should be presented in some meaningful social situation so that the pupils will see the importance of the new work and understand more fully the processes involved. As soon as the instruction is completed, work should be done to develop skill and mastery of the topic. While the child is working, the teacher should make observations so that mistakes and incorrect procedures may be corrected immediately. This will help keep bad work habits to a minimum and will save much corrective work on the parts of the teacher and the pupil.

In the graded elementary school where all children in a grade have had a program in which the same predetermined area of arithmetic has been presented, the individualized arithmetic program should be introduced gradually. Starting

slowly and gradually working into the program will give the children more confidence in the program and make them more willing to accept the responsibility for working independently.

Although a somewhat rigid schedule may be needed at first, the teacher will change to a more flexible one as quickly as possible. In any case, four things should take place in any given period:

First, there must be a time to give instruction in any new work. At the beginning of the period, the teacher should call those students ready for new work to a designated area of the room for instruction. Those students who need review work in the same areas should also be included at this time. The teacher should then proceed to introduce the new work, dismissing each child as he is ready to work independently, and giving more assistance or instruction to the remaining children until they too can work independently.

The second thing for which the teacher must allow some time is reteaching. In evaluating the work of the children, the teacher may find that a child does not understand a concept or has not learned a skill well enough to perform satisfactorily; this means reteaching. This may vary from the correction of a simple mistake or misunderstanding to the reteaching of an entire process. Reteaching

may not be necessary every day, nor will the same amount of time be used for instruction every day, thus the need for flexibility in the schedule.

Third, the teacher must allow time to give instruction to any child having minor difficulties with his individual work. Many times a child's problem can be solved simply by pointing out some simple misunderstanding or misconception he may have about the process on which he is working.

Fourth, a short period of time should be spent at the end of each arithmetic period for an individual pupil check. At this time the teacher should check the amount of work accomplished by each child and any problems encountered. (See appendix, page 39.)

While the teacher is giving instruction to small groups or to individuals, as the case may be, the remainder of the class should be at their seats working independently. Many times a child will have a question to be answered while the teacher is giving instruction to another child. The one having the difficulty should be allowed to ask the assistance of any other student in the classroom who would have the knowledge necessary to solve the problem.

To minimize the need for teacher-instruction, self-instruction should be encouraged as much as possible. The Scott Foresman Seeing Through Arithmetic books, which are to be used in this program, are arranged in such a way



that self-instruction is possible even on the introduction of many concepts and skills.

The context in the STA series is organized under nine headings--"Learning How," "Exploring Problems," "Moving Forward," "Keeping Skillful," "Checking Up," "Thinking Straight," "Looking Back," and "Side Trip." The lessons headed "Learning How" employ the four-step teaching method.

In Step 1, called SEE, the pupil is shown what happens to the objects involved. The process with the symbols and numerals is shown and is closely connected with the visual representation. Verbal explanations are included, but they are simple and brief. In Step 2, called THINK, another example is worked out and the pupil is asked some questions about important details. He is encouraged and helped to think about the example. In Step 3, called TRY, other completely worked out examples are given, but the pupil is expected to work these out for himself and then compare his work with the completed examples in his book. In Step 4, called DO, examples for practice are given. In this step the child is expected to work independently. (5:10)

The remaining eight headings pertain to lessons as follows: (1) "Exploring Problems"--lessons developing specific problem-solving techniques; (2) "Moving Forward"--lessons developing other new material, including basic facts, numeration system, money, fractions, and measurement; (3) "Using Arithmetic"--maintenance and practice in solving verbal problems; (4) "Keeping Skillful"--maintenance and practice in computation, measurement, numeration system, and money; (5) "Checking Up"--tests of various types; inventory, achievement, diagnostic (end-of-block), and

problem solving; (6) "Thinking Straight"--lessons involving special aspects of quantitative thinking; (7) "Looking Back"--lessons involving reteaching or review of previously taught content; and (8) "Side Trip"--lessons on various topics related to the arithmetic program: Roman numerals, geometric figures, maps, etc. (5:12)

As the children progress through the STA series on an individualized basis, they may move through the text according to the following plan:

- I. All children, should do the work in sections headed "Learning How," "Exploring Problems," "Moving Forward," "Using Arithmetic," "Keeping Skillful," and those parts in sections headed "Checking Up" that may be used for diagnostic reasons.
- II. The children who demonstrate that they have mastered the particular processes being evaluated, should then be allowed to go on to new work in the sections headed "Thinking Straight" and "Side Trip."
- III. The children who do not perform satisfactorily in the evaluation of the concept or skill should then do the next section headed "Looking Back," which is designed for this purpose. The teacher may also assign additional work over the same area from different sources if there is a felt need.
- IV. After the child has completed the "Looking Back" section, the teacher should retest to insure that the child has learned and understands the concept or skill involved.
- V. If the child has completed all of the work in the text plus additional assignments, the teacher should retest the child on those concepts and skills necessary for the new work, as it is possible that the child may have forgotten one or more of them. If this is the problem, then the child should be taken back and instructed in the particular concept

or skill, given assignments pertaining to the problem and then returned to work on the process where the difficulty was first encountered. At this point, the teacher may wish to use special teacher-constructed assignments or other suitable materials that are available.

An important phase of the program of individualized instruction is record keeping. When every child is at a different place in the sequence of instruction, the task of keeping a daily record for the entire class would require an undue amount of a teacher's time. Therefore, each child should take the responsibility of keeping track of his own progress. The necessary records to be used by the children should be simple, easily read and clearly understood. Instruction should be given on record keeping at the start of the program. (A sample record sheet may be found on page 37 of the appendix.)

Also, the children should take the responsibility for correcting their own written work. They should check their answers in a teacher's edition of the text or use specially prepared answer sheets. (See the appendix, page 41.)

The child should check all exercises or problems done incorrectly, mark the number of wrong answers at the top of the page and turn in the paper. The child should proceed, immediately, to the next work. When it is apparent that the child is having difficulty with a certain process,

the teacher should allow time to review the work individually or call him up with a group that is to receive instruction in the same process.

\*

natural numbers (0-99): ideas of order, betweenness, "greater than," "less than," "equal to" • recognition and symbolization of additive and subtractive situations • relationship between addition and subtraction • idea of commutative property of addition • ordinal use of number

ideas of "many" and "few" • sets: determining equivalence by one-to-one correspondence: grouping and regrouping (through 10) • statements of "equal to," "greater than," "less than" • recognizing combining and separating actions

use of an ordered pair of numbers to indicate position of an object within a set of objects

combining and separating actions associated with pictured situations

natural numbers (0-999): ideas of order, betweenness, "greater than," "less than," "equal to" (extended) • additive and subtractive situations (extended) • commutative property of addition (extended): associative property of addition • multiplicative and divisive situations • relationship between processes • ordinal use of number (extended)

sets: determining equivalence (extended): grouping and regrouping (through 18) • statements of "equal to," "greater than," "less than" • statements of equality that involve conditions for problem situations

use of an ordered pair of numbers to indicate position (extended) • introduction to rate pairs: use of expressions like "2 out of 3," "2 to 3," etc.

problem situations as described by the following mathematical sentences:

$$2 + 3 = 5$$

$$5 - 3 = 2$$

concept of a unit of measure • intuitive judgment of precision of measurement • idea of linear measure • idea of capacity measure

closed and open curves • interior and exterior of curves • betweenness for curves

base-ten system (through 99) • place value • grouping by ones and tens

basic facts: addends of 1, 2, 3, 4; subtrahends and differences of 1, 2, 3, 4 • extension of basic facts: sums and minuends of 6, 7, 8

use of money (coins through quarter): counting, purchasing items, making change

concept of unit of measure (extended) • concept of standard units: inch, foot, pint, quart • relationship between standard units: inch and foot, pint and quart • intuitive judgment of precision of measurement (extended)

curves (extended) • points • lines, segments, intersecting lines, and parallel lines • betweenness, simple closed curves, interior and exterior (extended) • polygons

base-ten system (through 999) • place value • grouping by ones, tens, hundreds

basic facts: sums, minuends, products, and dividends through 12 • extension of basic facts: sums, minuends, products, and dividends through 18

use of money (through dollar): counting, purchasing, making change • use of standardized measuring instruments: inch and foot rule, pint and quart containers

natural numbers (0-9999) • fractions: unit fractions, nonunit fractions • commutative property of multiplication; commutative and associative property of addition (extended) • relationship between processes (maintained) • identity property of addition • ordinal use of number (maintained)

recognition of numerosness of sets by grouping and regrouping (through 36) • statements of equality that involve conditions for problem situations (extended)

idea of equivalent rate pairs • preparation for reduction of ordered pairs of numbers

introduction to imagined action • comparison by subtraction • mathematical sentences for problem situations extended to include the following:

$$\begin{array}{rcl} 2 & + & 5 \\ 4 & - & 3 \end{array} \quad \begin{array}{rcl} 12 & - & 3 \\ 12 & - & 4 \end{array}$$

concept of standard units: extended to include yard, gallon, dozen ounce, pound, units of time • reduction of measures (within scope of processes taught)

recognition of simple closed curves (extended) • finding distinctions and similarities among circles, squares, triangles, and other polygons • recognizing halves and fourths in squares, circles, and triangles

base-ten system (through 9999) • place value • regrouping as preparation for "carrying" and "borrowing" • reading and writing numerals for fractions through eighths

basic facts: sums and minuends through 18, products and dividends through 36 • addition of three or more numbers expressed by 1-figure numerals • addition and subtraction of numbers expressed by 2- and 3-figure numerals • addition and subtraction involving money

use of money (coins and bills): purchasing, making change, computing • use of standardized measuring instruments: extended to include yard rules, gallon containers, weight scales, clock, calendar

natural numbers (0-999,999,999) • rounding numbers • fractions (extended) • commutative property of addition and of multiplication, associative property of addition (extended) • relationship between processes (maintained) • identity property of multiplication

recognition of numerosness of sets by grouping and regrouping (through 81) • recognition of sets of equivalent fractions • statements of equality that involve conditions for problem situations (extended to more complex problem types)

meaning of equivalent fractions • reduction of ordered pairs of numbers • common divisors and common multipliers

comparison by division • averages • mathematical sentences for problem situations extended to include the following:

$$12 \div 3 = 9$$

concept of standard unit: units of temperature (Fahrenheit), dry measure, liquid capacity, weight, length, time (extended) • scale measures • abbreviations of measurement words • reduction of measures

parallel lines and right angles • recognition of rectangles, squares, triangles (extended): determining which have parallel sides or right angles; distinguishing rectangles from trapezoids • linear scale to measure distances on map

base-ten system (through 999,999,999) • place value • regrouping for "carrying" and "borrowing" (extended) • Roman numerals (to XXXIX) • numerals for fractions: proper and improper fraction numerals, mixed numerals

basic facts: products and dividends through 81 • addition and subtraction of natural numbers (extended) • multiplication involving multipliers expressed by 1- and 2-figure numerals • division involving divisors expressed by 1- and 2-figure numerals; remainders

use of money (maintained) • use of standardized measuring instruments: extended to include thermometer, writing of abbreviations for common units of measure • use of a scale with a map

\* \* \* \* \*

natural numbers (maintained) • fractions (extended) • number line: corresponding natural numbers and fractions to points in a line • rounding numbers (extended) • commutative property of addition and of multiplication, associative property of addition (maintained)

recognition of numerosness of sets by grouping and regrouping (extended) • recognition of sets of equivalent fractions (extended) • statements of equality that involve conditions for problem situations (extended to more complex problem types)

use of ratios to express ordered pairs involving rates and comparisons • distinguishing among fractions, rates, and comparisons • introduction to per cents

introduction of  $n$  as a placeholder • multiple-step problems • finding areas of rectangles • use of ratios in rate and comparison problem situations, as follows:

$$\frac{3}{1} = \frac{n}{4}$$

$$\frac{3}{1} = \frac{12}{n}$$

$$\frac{n}{1} = \frac{12}{4}$$

measurement equivalents and reduction of measures (extended) • measures of area: square inch, square foot, square yard, acre, square mile • finding areas: rectangle

parallel lines and right angles (extended); perpendicular lines • recognition of polygons and circle (extended) • metric geometry: using dimensions to describe polygons and circles • concept of area: rectangle • concept of perimeter: polygons • scale drawings

base-ten system, place value (maintained) • Roman numerals (extended) • common fraction numerals (maintained) • decimal fraction numerals: tenths, hundredths • regrouping for "carrying" and "borrowing" (extended)

natural numbers: addition, subtraction, multiplication, division (extended) • fractions: addition and subtraction of fractions expressed by proper fraction numerals and mixed numerals

idea of divisibility: tests, even numbers, odd numbers

introduction to graphs: pictographs and bar graphs

use of money (maintained) • use of standardized measuring instruments (maintained); interpreting time zones, standard time • finding averages • use of scale drawings and graphs

natural numbers (maintained) • fractions (maintained) • number line: extended to include negative numbers and numbers expressed by decimal numerals • idea of distributive property of multiplication over addition

recognition of numerosness of sets (extended) • use of statements of equality (extended) • use of variety of symbols as placeholders • statements of inequality that involve conditions for problem situations: ideas of "greater than," "less than," and "is not equal to" symbolism • concept of solution set

use of ratio test • fraction numerals as terms of ratios • per cents expressed as ratios (extended)

generalizing the use of symbolism • rate and comparison problems extended to include: "three cases" of per cent; problems involving the use of fractions in ratios • finding volumes of rectangular prisms

measurement equivalents and reduction of measure (maintained) • finding areas: extended to include parallelogram • precision in measurement • measures of volume: cubic inch, cubic foot, cubic yard • metric measures: meter, gram, liter, kilogram, kiloliter

parallel lines and right angles (extended) • recognition of polygons and circle (extended) • identification of geometric solids (cylinder, various prisms, cone, pyramid, sphere, half sphere) • metric geometry (extended) • area: extended to include any parallelogram • perimeter extended to any polygon • volume of rectangular prism

base-ten system, place value (maintained) • common fraction numerals (maintained) • decimal fraction numerals: extended to include ten-thousandths • regrouping for "carrying" and "borrowing" involving decimal fraction numerals

natural numbers: four processes (maintained) • fractions: addition and subtraction (maintained); multiplication and division of fractions expressed by proper fraction numerals and mixed numerals • fractions expressed by decimal fraction numerals (all processes)

idea of divisibility (maintained)

graphs (extended) • introduction to statistical tables

use of money (maintained) • use of standardized measuring instruments (maintained) • use of metric measures • finding averages (extended) • computing per cents • use of frequency charts and graphs of statistical data



\* \* \* \* \*

definitions: natural number, fraction, rational number of arithmetic, power • properties of a number system (closure, commutative, associative, distributive) • systems: of natural numbers, finite, of rational numbers of arithmetic • operations: definition of addition and multiplication • identity properties: addition, multiplication

sets: of numbers, of points, tabulation of, finite, infinite, intersection of, union of, standard description of, Cartesian product of • conditions: simple and compound, in one and two variables; for equivalence, for inequality; solution set of • connective "and"

concept of a set of equivalent ordered pairs • concept of rate pairs as members of a proportional relation • definition of equivalent fractions • natural numbers and rational numbers of arithmetic as components of rate pairs

development of systematic problem-solving approach involving universe, condition, and solution set • problems involving simple and compound conditions in one and two variables • use of rate pairs to solve per cent problems (retought and extended) • use of logical connective "and" (A)

standard unit and reduction of measures (extended) • precise development of segment measure, angle measure • concept of area of a closed region: rectangle, triangle, parallelogram • metric system (extended)

nonmetric geometry: concept of point; sets of points (line, plane, circle, etc.); congruence • intersections and unions of point sets used to define geometric objects (ray, angle, triangle, etc.) • metric geometry: angles formed by coplanar lines; measures of segments, angles; measures associated with polygons, congruent triangles

systems of numeration and their properties: tally, code, grouping, place value; Egyptian, Roman, Babylonian; base ten; other bases (binary, duodecimal, etc.) • scientific notation, involving positive exponents

natural numbers: multiplication and division (extended) • rational numbers of arithmetic: all processes (extended) • computation involving 0 and 1 • computation in bases other than ten

composite and prime numbers: complete factorization, unique factorization property, greatest common factor

preparation for probabilities in a sample space (set of all possible outcomes of an experiment)

complete factorization of natural numbers • using methods of replacement and inverse operations to find solution sets of conditions • use of charts and graphs to obtain solution sets

use of logical connective "and" (A) • use of logic in the development of mathematical definitions and properties, including the transitive property of equality • deriving statements from accepted statements

business uses of mathematics: involving interest, discount, per cent of increase, commission, and other applications of per cent • use of standardized measuring instruments: extended to include protractor • use of metric measures (extended)

**NOTE:** The sequence for grade eight will be the sequence used in Seeing Through Mathematics 2 (Scott-Foresman)

Instructional materials. Instructional materials that correlate with the vertical program of individualized instruction are necessary to make learning more meaningful. For use in this program, they may be defined as anything used for instruction, including textbooks, supplementary reading materials, workbooks, and visual aids. Below are criteria that may be used in the selection of instructional materials (including teacher-made items).

1. Should be accurate, authentic, and up-to-date.
2. Should present meaningful content in a clear, well organized manner.
3. Should be in satisfactory physical condition.
4. Should meet the needs of the student and fit the concept or process on which he is working.
5. Should be appropriate to the age, interests and intelligence of the child for which it is intended.
6. Should broaden the intellectual and possibly the emotional experience of the user(s).
7. Should justify its cost in time, money and effort.
8. Should be the best of its type for the purpose for which it is to be used.

Evaluation. Evaluating each child periodically as he moves forward in the individualized program is very important, as this can do much to help improve the effectiveness of instruction and guidance.

Tests are the means of providing information upon which to base diagnosis. The tests make no analyses or suggest what should be done. However, they do indicate things which will aid the teacher in making judgments based on experience, training and understanding.

Four means of evaluation are to be used in this vertical program:

1. Teacher-made tests are to be used at the conclusion of work on a specific concept or skill. These tests will indicate whether or not a child will be allowed to go on to the next concept or skill or whether a review is necessary. (See appendix, page 40.)
2. Observation of pupil performance will enable the teacher to give immediate assistance in an area of difficulty which a child may encounter while working independently or in a small group.
3. Pupil-teacher conferences will be used to discuss any teacher-felt or pupil-felt difficulty which cannot be satisfactorily cared for during independent work at the child's desk, at the chalk board, or during a time of instruction. These conferences should be held at the teacher's desk.
4. Commercial tests, edited by Scott Foresman are to be administered at the beginning of the program and at the beginning of each following year. These tests are designed to evaluate all areas of arithmetic competence. (See appendix, page .)

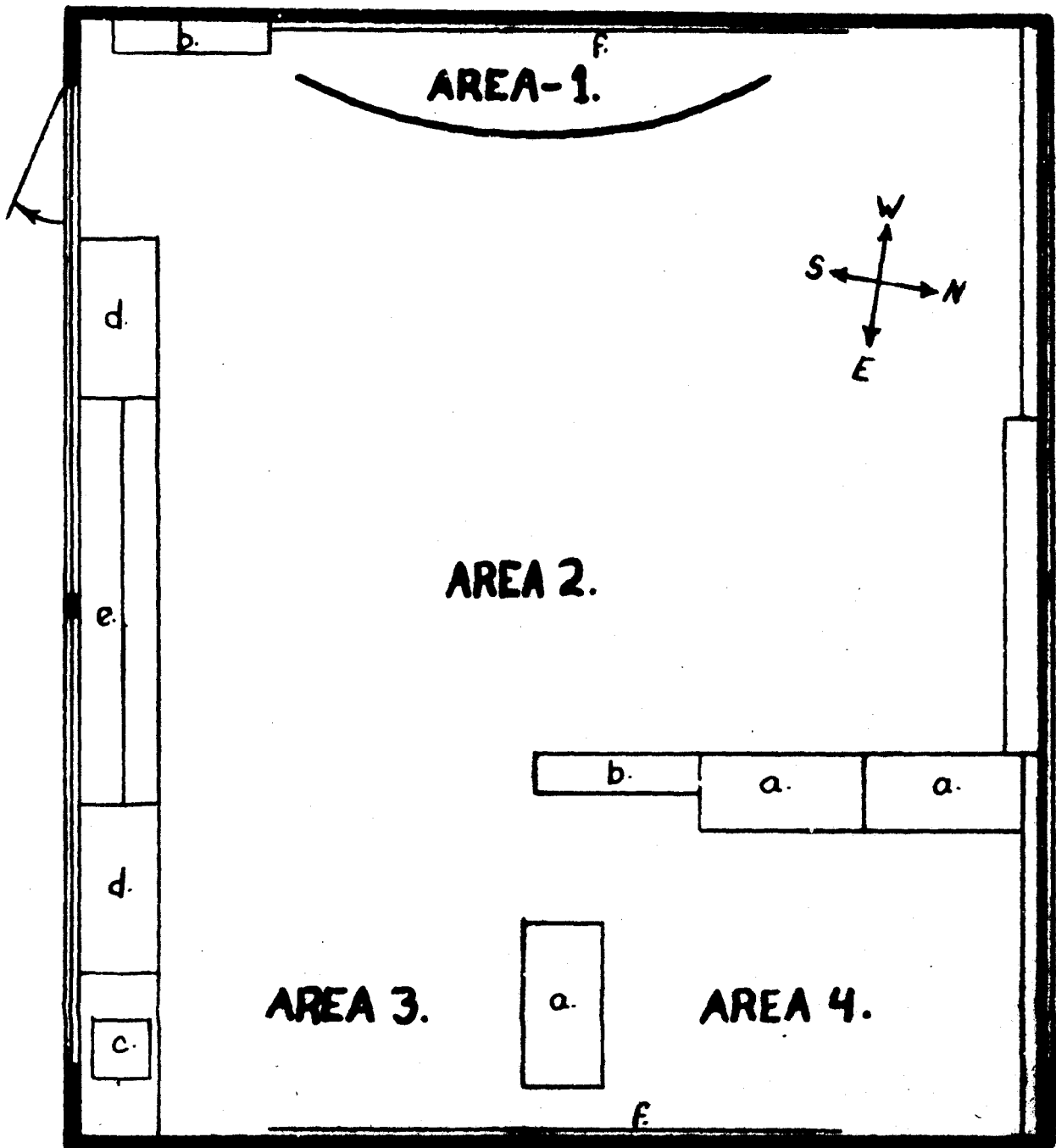
## B I B L I O G R A P H Y

## BIBLIOGRAPHY

1. Biber, Barbara. Child Life in School. New York: E. P. Dutton and Company, Inc., 1942.
2. Browning, Virginia W. "Individualized Instruction," School and Community, October, 1963, p. 14.
3. Bruecknew, Leo J. Adapting Instruction in Arithmetic to Individual Differences. Minneapolis: University of Minnesota Press, 1941.
4. Goodlad, John I., and Paul Anderson. The Nongraded Elementary School. New York: Harcourt, Brace and Company, 1959.
5. Hartung, Maurice L., Henry Van Engen and Lois Knowles. Seeing Through Arithmetic 4. Chicago: Scott Foresman and Company, 1963.
6. Potamkin, Caroline C. "An Experiment in Individualized Instruction," The Elementary School Journal, December, 1963, pp. 155-162.
7. Searight, Franklyn. "You Can Individualize Arithmetic Instruction," The Arithmetic Teacher, Vol. II, No. 3, March, 1964, pp. 199-200.
8. Salvin, Joseph P. The Philosophical Basis for Individual Differences, According to Saint Thomas Aquinas. Washington, D. C.: The Catholic University of America, 1936.

## A P P E N D I X

# TYPICAL BARNES CLASSROOM



AREA 1--Small group instruction

AREA 2--Seating area used for individual work and large group instruction

AREA 3--Construction area used for individual art projects

AREA 4--Individual, silent study area

- a. Portable storage cupboard
- b. Portable bookcase
- c. Sink and storage drawers
- d. Storage area for art supplies and instructional materials
- e. Coat rack
- f. Chalk board

# SAMPLE WEEKLY SCHEDULE FOR ALL CURRICULAR AREAS

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
9:50	Room business Reading Music	Room business Reading Language	Room business Reading Music	Room business Reading Language	Room business Reading Language
10:15					
10:15	Recess	Recess	Recess	Recess	Recess
10:30					
10:30	Arithmetic Spelling	Arithmetic Spelling (Dictionary & reference instruction)	Arithmetic Spelling	Arithmetic Spelling (Hand- writing)	Arithmetic Spelling
11:50					
11:50	Lunch	Lunch	Lunch	Lunch	Lunch
12:50					
12:50	Library Science	Social studies	Science	Social studies	Art
2:00					
2:00	Phys. Ed.	Phys. Ed.	Phys. Ed.	Phys. Ed.	Phys. Ed.
2:20					
2:20	Science (cont'd)	Individual study & teacher assistance	Same as Tues.	Same as Tues	Same as Tues.
3:00					



SAMPLE SCHEDULE FOR ONE PERIOD OF  
ARITHMETIC INSTRUCTION

- I. A. TEACHER--Call those students ready for new work to small group instruction area. Work with the group, dismissing the children as they are ready to work independently, and give more assistance or instruction to those in need.
- B. PUPILS--Those children not included in the instruction should be working independently at this time.
- II. A. TEACHER--Call those students who need reteaching in a particular concept or skill. Work with the group as in I., A.
- B. PUPILS--Same as I., B.
- III. A. TEACHER--Give assistance to those children having difficulty with their written work. This time may also be used for individual teacher-pupil conferences if there is a need for them.
- B. PUPILS--The children may ask for individual assistance from the teacher on assigned work, independent work, special projects, or extended activities.
- IV. A. TEACHER--At the end of each period, record all information necessary on the Individual Pupil Progress Chart. (See page        in the appendix.)
- B. PUPILS--Have ready all information needed for the chart discussed under IV., A.

WORK SHEET<sup>1</sup>

NAME : \_\_\_\_\_

[illegible]

1The child is responsible for completing all of the information on this sheet. He should be instructed that the column headed CONCEPT OR SKILL BEING STUDIED need not be filled in every day, only when new work is started. This record should be kept up to date at all times so the students can readily see their own progress.

## REVIEW ASSIGNMENT SHEET<sup>1</sup>

NAME: \_\_\_\_\_

DATE: \_\_\_\_\_

CONCEPT OR SKILL BEING REVIEWED: \_\_\_\_\_

ASSIGNMENTS

	DATE STARTED	DATE COMPLETED
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		
11.		
12.		

<sup>1</sup>The column headed ASSIGNMENTS is to be filled in by the teacher and the two columns marked DATE STARTED and DATE COMPLETED are to be filled in by the child. The designated work is to be completed by the child and the sheet returned with the assignments completed and corrected.

# INDIVIDUAL PUPIL CHECK SHEET

		DATE	DATE	DATE
		9/23	9/24	9/27
PUPILS' NAMES	LEVEL	PAGE	PAGE	PAGE
1. <i>John Jones</i>	<i>4</i>	<i>46 add/min no.</i>	<i>49</i>	<i>stuck 50 Mult. fractions</i>
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				
11.				
12.				

<sup>1</sup>At the end of each arithmetic period, the teacher should fill in this chart. Each child gives the following information: (1) the page number on which he is now working, (2) if he needs instruction in a new area, indicate by circling the page number, and (3) whether or not he is having difficulty in his work, indicate by writing the word "stuck" by the page number.

# TEACHER-MADE TEST

NAME: \_\_\_\_\_

No. Wrong: \_\_\_\_\_

DATE: \_\_\_\_\_

1. Tom bought a gallon of paint for \$4.29 and a paint brush for \$2.89. He gave the clerk at the paint store a ten-dollar bill. How much money should the clerk have given him in change?
2. Sue went to the post office to buy some stamps. She bought twenty-five 3¢ stamps, eighteen 2¢ stamps, and fifteen 6¢ stamps. How much money did she spend for all the stamps?
3. The steam engine that Don wants to buy costs \$26.50. He had \$19.32. How much more money does he need before he can buy the steam engine?
4. Jim's bicycle cost \$52, and Don's bicycle cost \$67.95. Jim's bicycle cost how much less than Don's?
5. The fifth-grade boys and girls hoped to sell 200 tickets for a play. Eight of the girls said they would make the tickets. If each of the girls made the same number, how many tickets did each girl make?
6.  $917 + 2542 = n$
7. Find the sum:  
365, 812, 581, 297
8.  $5634 + n = 1218$
9.  $287 \times 509 = n$
10.  $7809 + 63 = n$

# ANSWER SHEET

PAGE 203

Green Squares White Numbers	Plain Black Numbers	Plain Green Numbers
A 513 and 1 rm.	A 170	A 13/18
B 7 and 22 rm.	B 970	B 2/9
C 187 and 350 rm.	C 782	C 3 4/5 or 3 8/10
D 410	D 91	D 9 1/2 or 9 8/16
E 615 and 2 rm.	E 36	E 1 3/16
F 104	F 85	F 8 7/12
G 34 and 141 rm.	G 15	G 2 7/8
H 82	H 82	H 23
I 49 and 5 rm.	I 73 and 72 rm.	I 1 4/15
J 16 and 205 rm.	J 83	J 3 11/24
K 205	K 272	K 2 1/16
L 97	L 682	L 7 5/9
M 90	M 57 and 153 rm.	M 4/9 or 8/19
N 802 and 42 rm.	N 750 and 24 rm.	N 3 1/18
	O 408	O 1 1/3 or 1 4/12
	P 816	P 9 1/12
	Q 370 and 135 rm.	
	R 629	

ntroducing

# Seeing through arithmetic tests

for Grades 3, 4, 5, and 6

*by Maurice L. Hartung, Henry Van Engen, E. Glenadine Gibb, Lois Knowles*

**SCOTT, FORESMAN AND COMPANY**

Chicago 11    Atlanta 5  
Dallas 2    Palo Alto    Fair Lawn, N.J.

Please note: This and the following pages were redacted due to copyright concerns,

Maurice L. Hartung  
Henry Van Engen  
E. Glenadine Gibb  
Lois Knowles

STAT

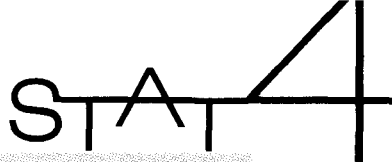
Grades 3, 4, 5, 6 | **Teacher's guide**

# Seeing through arithmetic tests

Scott, Foresman and Company *Chicago, Atlanta, Dallas, Palo Alto, Fair Lawn, N.J.*



Maurice L. Hartung  
Henry Van Engen  
E. Glenadine Gibb  
Lois Knowles



# Seeing through arithmetic test

Scott, Foresman and Company Chicago, Atlanta, Dallas, Palo Alto, Fair Lawn, N.J.

Pupil

Teacher

Grade

Date

Copyright © 1960 by Scott, Foresman and Company  
Printed in the United States of America  
International Rights Reserved  
To reproduce this test, or any part of it,  
is in violation of the copyright laws unless  
permission to do so is secured from the publishers.

Part

Number  
correct

Problem solving: Selecting answers 1

Computation 2

Problem solving: Selecting equations 3

Problem solving: Solving equations 4

Information 5

Concepts 6

Total



ndicated by sample questions from STA Tests 3, 4, 5, and 6

Directions

Here are teacher-to-pupil instructions for the first half of the tests as outlined in the **Teacher's Guide**. Directions for Parts 4, 5, and 6 are similarly presented in the Guide.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.



Maurice L. Hartung  
Henry Van Engen  
E. Glenadine Gibb  
Lois Knowles

STAT

Grades 3, 4, 5, 6 | Cumulative individual record

Seeing through arithmetic tests

Scott, Foresman and Company Chicago, Atlanta, Dallas, Palo Alto, Fair Lawn, N.J.