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# A Model Mathematics Problem Solving Curriculum for Randle Elementary School in Randle, Washington in the White Pass School District

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# A MODEL MATHEMATICS PROBLEM SOLVING CURRICULUM FOR RANDLE ELEMENTARY SCHOOL IN RANDLE, WASHINGTON IN THE WHITE PASS SCHOOL DISTRICT

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A Project Report

Presented to

The Graduate Faculty

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In partial fulfillment

Of the Requirement for the Degree

Master of Education Administration

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by

Randy A. Torrey

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ABSTRACT

A MODEL, FIFTH GRADE, MATHEMATICS CURRICULUM

DEVELOPED IN ALIGNMENT WITH

WASHINGTON STATE ESSENTIAL ACADEMIC LEARNIG REQUIREMENTS,

WHITE PASS SCHOOL DISTRICT

by

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The purpose of this project was to design and develop a model fifth grade mathematics curriculum, in alignment with Washington State Essential Academic Learning Requirements, for the White Pass School District in Washington. To accomplish this purpose, a review of current research and literature was conducted and used to develop unit and lesson plans for a model problem solving mathematics curriculum. The results support the hypothesis that mathematic concepts are better understood when problem solving skills are used. Implications for using problem solving skills in a mathematics curriculum are discussed.

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This work is dedicated to my girlfriend Jenny, friends, and colleagues for their support and encouragement throughout my journey through this program.

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# Chapter 1

## BACKGROUND OF THE PROJECT

### Introduction

> Mathematics continues to grow at a rapid rate, spreading into new fields and creating new applications, in its open-ended search for patterns. Several factors...growth of technology, increased applications, impact of computers, and expansion of mathematics itself, have combined in the past century to extend greatly both the scope and the application of the mathematical sciences. The change must be reflected in the schools if our students are to be well prepared for tomorrow's world. (Office of Superintendent of Public Instruction, 2001,Redefined)

The statement above provides a clear reflection from the Office of Superintendent of Public Instruction about the importance of math competency in our rapidly changing society. Schools need to prepare and provide our youth with mathematical skills that will enable them to contribute to society.

According to RCW 28A.150.210, the goal of The Basic Education Act of 1998 is to provide students with the opportunity to become responsible citizens, to contribute to their own economic well being and to that of their families and communities, and to enjoy productive and satisfying lives. In addition, students must know and apply the core concepts and principles of mathematics. They also need to be able to think analytically, logically, creatively, and to integrate experience and knowledge to form reasoned judgments and solve problems.

Research conducted by the National Council of Teachers of Mathematics (1998) has argued that to be well informed as adults and to have the opportunity to obtain desirable jobs, students need to acquire more problem solving skills now than students in the past. Students must have a thorough understanding of mathematical concepts and processes. They need to have the skills to be able to reason, problem solve, and have the abilities to communicate when using mathematics to solve problems.

Finally, with the current emphasis on the Washington State Essential Academic Learning Requirements, all students need to have a fundamental understanding of mathematics. In order for students to obtain the knowledge and skills to be competent in mathematics and meet the tough requirements, mathematical content must be recognized, taught, and assessed in a problem-solving and constructive environment. For students to develop this deeper level of understanding, they must connect their knowledge to a variety of ideas and skills across topic areas and grade levels in mathematics and to situations outside the classroom. (Office of Superintendent of Public Instruction, 2001, Redefined)

## Purpose of the Project

The purpose of this project is to design and develop a model mathematics curriculum for 5th grade students in alignment with the Washington State Academic Learnings for White Pass School District #303, Randle, Washington. A review of research and literature regarding curriculum and instruction with the Washington State Academic Learnings will be performed. In addition, other relevant information from selected sources will be gathered and analyzed.

## Limitations of Project

For this project, the following limitations were set:

1. Research: the research and literature was limited to the past 20 years.
2. Scope: the model elementary level mathematics was designed for implementation in the White Pass School District, White Pass School District #303, Randle, Washington.
3. Target Population: the model program has been designed for students in grade 5.

## Definition of Terms:

The following terms used in this project have been defined as follows:

1. Ability- the capacity to do something and perform. It is influenced through educational experiences as well as by natural talents, aptitudes, or traits.
2. Assessment- the process of gathering information about students, instruction, and programs to learn more about them.
3. Benchmarks- different levels of anticipated performance based on each student grade level.
4. Concepts- general and fundamental ideas-for example, the ideas that are needed to guide reasoning, problem formulation, and problem solving situations.
5. Cooperative Learning- an environment which takes place with a collaborative effort of a group. The collaborative environment leads to greater student achievement and attitude.

6. Data Analysis- Using information collected to solve problems and come to conclusions.
7. Essential Academic Learning Requirement for Mathematics- a set of specific guidelines and skills in mathematics that a student should obtain to be competent in mathematics.
8. Evaluation- the process of determining the value of something on the basis of careful examination and judgement.
9. Geometry- the study of geometric shapes and properties.
10. Mathematics- the study of the measurement, properties, and relationships of quantities, using numbers and symbols.
11. Measurement- the understanding of measurable attributes of objects and applying appropriate techniques, tools, and formulas to determine these measurements.
12. Norm-referenced test- a test that compares quantitative scores to a normal distribution of scores for the same age or grade. The tests have been used to rank and sort students.
13. Open-ended questions- tasks that allow for various acceptable answers and multiple approaches to an effective solution. The tests allow many students of various abilities to work on the problems, make their own assumptions, develop creative responses, and communicate their solutions.
14. Performance criterion, or standard- a statement of expected performance quality that can be used to make judgements about performances. A set

performance criteria, or standards, include the evidence needed to demonstrate that a curriculum or standard has been achieved.

15. Probability- the measurement of the likelihood of events.
16. Problem Solving- one of the four mathematical processes identified in the Essential Academic Learning Requirements for Mathematics that involves using strategies to investigate, question, define, and solve problems.
17. Standardized test- a test that is administered, scored, and interpreted in a consistent matter regardless of who, when and where the test is given.

# Chapter 2

# REVIEW OF RELATED LITERATURE

## Introduction

The review of research and literature summarized in Chapter 2 has been organized to address the following topics:

1. The Importance of Mathematics in Today's World
2. Mathematics as an Essential Learning Requirement
3. Current Trends in mathematical instruction, two selected approaches...
    - Problem-solving
    - Cooperative Learning
4. Assessment in Mathematics
5. Summary

Data and research current within the last 15 years were identified through Educational Resources Information Center (ERIC) and Proquest computer searches. Other various sources were collected and used.

## The Importance of Mathematics in Today's World

According to the Washington State Commission of Student Learning (1998), society is placing demands on citizens to interpret and use mathematics to make sense of information and complex situations. Computers and other technologies have increased our capacities for dealing with numbers, for collecting, organizing, representing, and

analyzing data. Tables, lists of numbers, graphs of data, and statistics summarizing information occur in every form of the media. In order to be well informed as adults and to have access to desirable jobs, students today require an education in mathematics that goes far beyond what was needed by students in the past. All students must develop and sharpen their skills, deepen their understanding of mathematical concepts and processes, and hone their problem solving, reasoning, and communication abilities while using mathematics to make sense of, and to solve, compelling problems. All students need a deep understanding of mathematics; for this to occur, rigorous mathematical content must be recognized, taught, and assessed in a problem-solving environment. For students to develop this deeper level of understanding, their knowledge must be connected to a variety of ideas and skills across topic areas and grade levels in mathematics, to other subjects taught in school, as well as to situations outside the classroom.

The Mathematical Sciences Education Board (1997) explains mathematics is the key to opportunity. No longer just the language of science, mathematics now contributes in direct and fundamental ways to business, finance, health, and defense. For students, it opens doors to careers. For citizens, it enables informed decisions. For nations, it provides knowledge to compete in a technological community.

Mathematics provides a set of tools for describing, analyzing, and predicting the behavior of systems in the real world. This practical utility has always provided a major justification for the important role of mathematics in the elementary school curriculum. The inclusion of application problems particularly traditionally in the form of arithmetic words problems was intended to develop pupils' skill in knowing when and how to apply

their mathematics effectively to various kinds of problems in everyday life (De Corte, & Verschaffel, 1996).

## Mathematics as an Essential Learning Requirement

In 1993, the Washington State Legislature led a major reform movement. It created legislation in which its primary goal was the improvement of teaching and learning (RCW 28.A.630.885). As part of the effort, the Legislature created the Commission on Student Learning and charged it with three important tasks:

- To establish Essential Academic Learning Requirements that define what all students need to know and be able to do in eight subject areas,
- To develop a performance-based assessment system to measure student progress toward achieving the Essential Academic Learning Requirements,
- To recommend an accountability system that will recognize and reward schools in which students are achieving the performance standards and provide support and assistance to those schools in which students are not reaching the standards.

The educational reform movement in Washington State resulted from the adoption of the new Basic Education Act (RCW 28A.150.210) in 1993. The goals of the act were to involve parents and community members in providing opportunities for all students to develop the knowledge and skills essential to the following:

1. Read with comprehension, write with skill, and communicate effectively and responsibly in a variety of ways and settings;

2. Know and apply the core concepts and principles of mathematics, social, physical, and life sciences; civics and history; geography, arts and health; and fitness;
3. Think analytically, logically, and creatively, and to integrate experiences and knowledge to form reasoned judgments and solve problems; and
4. Understand the importance of work and how performance, effort, and decisions directly affect career and educational opportunities.

The Washington State Commission of Student Learning identified five Essential Learning Requirements in Mathematics. The publication is entitled, *The Essential Academic Learning Requirements in Mathematics* (2001) and include the following:

1. The student understands and applies the concepts and procedures of mathematics, how to use them, and why they work;
2. The student uses mathematics to define and solve problems;
3. The student uses mathematical reasoning
4. The student communicates mathematical ideas in mathematical and everyday language;
5. The student understands how mathematical ideas connect to other subject areas, real-life situations, and career goals.

The math learning goals and accompanying benchmarks, or Essential Academic Learnings for Mathematics, designed by the Commission of Student Learning and modeled after the recommendations from the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) are available to school districts to identify what students in the state of Washington should know, understand, and be able to do in order

to become mathematically literate, responsible, and productive citizens. Scoring guides are provided to describe the performance criteria for each of the Essential Academic Learning Requirements for Mathematics. They are available for teachers and students to help interpret each of the specific benchmarks.

The benchmarks are used to measure student progress in relation to the Essential Academic Learning Requirements for Mathematics. The assessments that used to monitor benchmark success are given at grades 4, 7, and 10. Benchmark One relates to K to 4$^{th}$ grades, Benchmark Two relates to the 5$^{th}$ to 7$^{th}$ grades, and Benchmark Three relates to 8$^{th}$ to 10$^{th}$ grades. By grade 10, students must show a mastery of the performance requirements in order to obtain a certificate of mastery.

## Instructional Methodology in Mathematics
## Two Selected Approaches

Current research obtained from ERIC and Proquest focused on two specific instructional methods, problem solving and cooperative learning. Both of these methods are proven successful in attaining student achievement in mathematics.

## Problem Solving

As stated in the *Principles and Standards for School Mathematics* (NCTM, 2001), problem solving is the cornerstone of school mathematics. Without the ability to solve problems, the usefulness and power of mathematical ideas, knowledge, and skills are severely limited. Students who can efficiently and accurately multiply, but who cannot identify situations that call for multiplication are not well prepared. Students who can both develop and carry out a plan to solve a mathematical problem are demonstrating

knowledge that is much deeper and more useful than simply carrying out a computation. Unless students can solve problems, the facts, concepts, and procedures they know are of little use. The goal of school mathematics should be for all students to become increasingly able and willing to engage with and solve problems.

Problem solving should permeate the mathematics curriculum so that students can apply, with increasing confidence, mathematical problem-solving strategies to solve problems within and outside mathematics (NCTM, 1989).

A problem can be defined as any task or activity for which students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific "correct" solution method (Hiebert et al., 1997).

Solving problems is not only a goal of learning mathematics but also a major means of doing so. Problems solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas described in these standards...Good problems will integrate multiple topics and will involve significant mathematics (NCTM's Principles and Standards, 2000).

There is no other decision that teachers make that has a greater impact on students' opportunity to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages the students studying mathematics (Lappan and Briars, 1995).

In the 1960s, a new math movement floundered because it failed to reach one million elementary school teachers of mathematics (Fiske, 1990). Problem solving is destined for the same fate at all levels of the curriculum unless the mathematics

community quickly realizes the need to educate its members about how to use and teach problem-solving skills.

Teaching math through problem solving does not mean that the teacher can sit back and provide a problem and expect magic to happen. The teacher is responsible for creating a positive atmosphere and making the lesson work. Each problem-solving lesson should consist of three parts: before, during, and after. Each part has specific agendas and requires teacher actions that are necessary to make the lesson effective. (Van De Walle, 2001)

The agenda of the "before" part of the lesson is to get the kids mentally ready and have them start thinking of ideas that will help them the most. It is essential that they understand the task at hand and their responsibilities. At the completion of this part, the student should fully understand what the task is.

The "during" part of the lesson allows the students the opportunity to work without instructor guidance. It is important that the students are given the opportunity to use their ideas and not have to follow any specific directions. Also, the teacher should listen to all of the groups and find out how different individuals and groups are thinking. The teacher should observe what ideas they are using and how they are approaching the problem.

The last portion of the lesson should provide the students a chance for discussion. Students must learn to both contribute to and participate in these discussions. Students need to communicate with each other and decide which approaches and solutions make the most sense and why. This is the time in which the class reflects on solutions, methods, and extensions.

According to Steen (1989), industry expects school graduates to be able to use a wide variety of mathematical methods to solve problems. Students must, therefore, experience a variety of problems—variety in context, in length, in difficulty, and in methods. They must learn to select appropriate strategies for solving problems, recognize and formulate several solutions when that is appropriate, and work with others in reaching an agreement on solutions that are effective.

Students learn mathematics as a result of solving problems. Mathematical ideas are the outcomes of the problem-solving experience, rather than elements that must be taught before problem solving (Hiebert et al., 1996,1997). Furthermore, the process of solving problems is now completely connected and children learn mathematics by doing mathematics.

Simon (1986) states that students who are exploring and discovering new concepts for themselves have the opportunity to do mathematics rather than passively learn about mathematics. The discovery approach allows students to think more deeply about the concept and to create representations and explanations of it that connect with their experience in a personally meaningful way. This will in turn create a greater understanding of the mathematical concept being taught.

Learning is the making of connections between new and unfamiliar information and the learner's existing network of knowledge, the construction of knowledge by the learner, and instruction should make these connections. Teachers are being encouraged to teach according to these different cognitive strategies.

Metacognition is a word that refers to conscious monitoring and regulation of your own thought process. Good problem solvers monitor their thinking regularly and

automatically. They recognize when they are stuck or do not fully understand. They make conscious decisions to switch strategies, rethink the problem, search for related content knowledge that may help, or simply start afresh (Schoenfeld, 1992).

There is evidence that metacognitive behavior can be learned (Campione, Brown, & Connel, 1989). Therefore, students who learn to monitor and regulate their own problem-solving behaviors do show improvement in problem solving.

In the early 1990's it was popular to select problems for the purpose of teaching a particular strategy ( Van De Walle, 2001). The following strategies are likely to appear where mathematical content is the main objective.

- Draw a picture, act it out, use a model.
- Look for a pattern.
- Make a table or chart.
- Try a simpler form of the problem.
- Guess and check.
- Make an organized list.
- Working backwards.
- Use logical reasoning.

When it comes to teaching strategy, it takes considerable creativity to use time and space effectively, to assure that every minute in your classroom is used productively to engage students in your subject (House, 1988). The challenge is finding the right problem that is both challenging and engaging for the students, and is not too time consuming.

## Cooperative Learning.

Student understanding of mathematical concepts can also be effectively learned by pair and group problem solving. Students working together on an unfamiliar problem are faced with the tasks of generating possible solution strategies, selecting strategies, deciding how long to stay with the strategy, and determining what strategy to use next.

Through cooperative learning, students are led into knowledge about their own cognitive processes with their ability to use them. Having an arsenal of knowledge and strategies is useless without the ability to select appropriate information to use to solve a particular problem (Simon, 1986).

Many studies note the social nature of learning and the power of group dialogue in cognitive operations (Noddings, 1988), (Gredler, 1992), (Whimbey and Lochhead, 1991), (Silver, 1996). Peer collaboration is widely known to be effective in contributing to student learning. Before a student is ready to solve problems independently, he or she make meaning out of problems and develop understanding through peer collaboration.

According to Slavin (1995), cooperative learning has typically referred to a variety of teaching methods in which students work in small groups to help one another learn about academic content. In cooperative classrooms, students are expected to assist each other, to discuss and disagree with each other, to assess each other's current knowledge and fill in gaps in each other's comprehension of a particular concept.

Employers repeatedly stress the importance of being able to work with a team on common objectives. The hardest problems demand the talents of many different people. Students of mathematics must learn how to work with others to achieve a common goal.

Students must plan, discuss, compromise, question each other, and organize their discoveries. Teamwork in the classroom not only teaches these skills, it is a very effective way to learn mathematics (Steen, 1989).

Students in lecture-oriented classrooms may be thinking, but student interactions that occur in a cooperative learning environment are seen to be more productive. In a cooperative learning environment students discuss, observe, compare, imagine, describe, create, listen, evaluate, and solve problems (Slavin, 1995).

Davidson (1992), states that frequent use of cooperative learning methods in mathematics can offer numerous benefits for a student. These benefits include:

- Opportunities to discuss and clarify concepts, to exchange ideas freely, ask questions, give and receive help, explore situations, look for patterns and relationships in sets of data, and formulate and test conjectures
- Learning varied approaches for solving the same problem
- Support for problem-solving, logical reasoning, and making mathematics connections
- Learning to communicate the language of mathematics
- The chance to learn from "mistakes" in a nonthreatening environment
- Decreasing math anxiety and increasing math confidence
- Accommodating diverse learning styles
- Making friends with group members across boundaries of race, class, and gender
- Improving the ability to cooperate with others and to develop social skills

## Assessment in Mathematics

Assessment has been a key component of educational reform. Though, Americans have been content to using standardized, norm-referenced tests to serve as a proxy for looking more directly at the ends of education (Diez, 1997). The current trend of educational reform has created a tension between the desire to use more complex and performance-oriented standards and the practice of using easy-to-administer and score testing methods. To be able to achieve the goals of reform, it will require a change in the relationship between assessment and learning by making assessment integral to learning.

The *Assessment Standards for School Mathematics* (NCTM, 1995) presented six standards about exemplary mathematics assessment. They addressed how assessment should:

- Reflect the mathematics that students should know and be able to do.
- Enhance mathematics learning.
- Promote equality.
- Be an open process.
- Promote valid inference.
- Be a coherent process.

Stiggins (1997) suggests that there are four basic assessment methods. When selecting proper assessment methods, it should be kept in mind that the assessment method choice in any classroom context is a direct function of the purpose and the target.

Stiggins (1997) outlines four basic assessment methods:

1. The Selected Response Assessment: This includes all of the objectively scored paper and pencil test formats including: multiple-choice items, true and false items, matching exercises, and short answer fill-in items. Standardized achievement tests and chapter tests that accompany many textbooks often rely on selected response assessments.

2. Essay Assessment: A respondent is provided with an exercise that calls for the preparation of an extended written answer. They might be asked to answer a question or to provide an explanation of the solution in a complex problem, compare, interpret, or solve open-ended problems.

3. Performance Assessment: The respondent actually carries out a specified activity under the watchful eye of an evaluator, who observes performance and makes judgments as to the quality of achievement demonstrated. They can be based either on the process while skills are being demonstrated, or on the evaluation of products created.

4. Personal Communication as Assessment: One of the most common way teachers gather information about day-to-day student achievement in the classroom is to talk with the students. These forms of personal communication include questions posed and answered during instruction, interviews, conferences, conversations, listening during class discussions, conversations with others about student achievement, and oral examinations. The examiner listens to responses and tallies them right or wrong if correctness is the criterion, or makes subjective judgments according to some quality criterion.

Diez (1997) mentions that most teachers agree that assessments focusing on "right answers," such as multiple choice and true/false forms of objective testing provide very little data for student improvement. Objective test scores give little information about how the learner arrived to an answer. Therefore, there is no immediate feedback on the next needed steps for improvement.

The Washington Assessment of Student Learning (WASL) is an example of a criterion-referenced test. These tests measure students' performance against set standards. They show whether students have learned the specific skills, concepts, facts and ideas found in the state standards, not how they performed against fellow Washington students or their peers nationally. By using a combination of multiple choice, short answer and essay questions, the WASL allows for a deeper assessment of important skills and knowledge found in the state standards.

External assessment expectations and instruments have a "profound influence" on what teachers, administrators, and parents' value in the classroom. Because teachers know the format and characteristics of these assessments, they tend to adjust their teaching and curricula to reflect this knowledge. However, the changes teachers make are not always consistent with professional recommendations such as the *Curriculum and Evaluation Standards in School Mathematics* (NCTM 1989).

Research supports the current trends toward alternative assessments. Mathematics instruction and mathematics assessments must be "interdependent" to achieve the maximum benefits of performance-based assessments (Frederikson and Collins, 1989).

Students learn usable knowledge and skills more effectively and efficiently through experiences with open-ended mathematics problems than with traditional goal-specific problems or exercises. When solving goal-specific mathematics problems, students use strategies that successfully solve the specific problem at hand but are less effective for making connections among concepts and procedures for organizing knowledge. When solving open-ended mathematics problems, students create, adapt and use solution strategies that make important relationships more meaningful. Thereby, helping students to develop knowledge that is better organized and skills that are more usable (Sweller and Levine, 1982).

Though, several challenges can occur for both students and teachers when open-ended performance assessments and rubric scoring are incorporated into a mathematics classroom (Peressini and Webb, 1999):

- Students need many opportunities to become familiar with this new form of assessment and to become "comfortable performing on demand."
- Students need to realize the importance of communicating their mathematical reasoning in a variety of formats.
- Students need to accept and respond appropriately when requested to "leave clear trails" of their computations and other mathematical work.
- Teachers need to gain confidence in their ability to correctly analyze student responses in this more complex assessment environment.
- Teachers need to be open to unconventional student responses and try to follow these responses "through the eyes of their students."

- This new assessment process can be quite time-consuming for both students and teachers.

From research work with performance-based assessment over a decade, Aschbacher (1991) concludes that they must have these key features:

- Students need to be asked to produce, do, or create something that requires higher-level thinking or problem solving skills.
- Students need to respond to assessment tasks that are meaningful, challenging, engaging, and instructional.
- Students should face assessment tasks set in real-world contexts or close models.
- Students' processed behavior must be assessed equally along with a product.
- Criteria and standards for performance need to be public knowledge and made known to students in advance.

In order for assessment to truly be meaningful and valid, teachers must understand their mathematical goals deeply. They must understand how their students may be thinking about mathematics. Teachers must have a good grasp of possible means of assessing students' knowledge. They should also be skilled in interpreting assessment information from multiple sources. Therefore, assessment needs to be a major focus in teacher preparation and professional development ( Diez, 1997).

If we want students to be able to use the mathematical knowledge they master in productive ways to reason and solve problems, teachers must use a variety of ways to define what it means to be a proficient problem solver. No single method can meet all of

our assessment needs, therefore teachers must learn to use all methods that are applicable to the achievement target (Stiggins).

## Summary

The research and literature summarized in Chapter 2 support the following themes:

1. Problem solving and cooperative learning can be incorporated in the classroom to help students learn essential mathematical skills and concepts.
2. The need for a problem-solving curriculum continues to grow in order to prepare students for today's constantly changing society.
3. Understanding the concepts and procedures of mathematics, defining and solving problems, using mathematical reasoning, and communicating mathematically are essential skills that have been identified by the Washington State Legislature.
4. Assessment should be used as a tool to assist teachers and students in gathering information and data to help students excel and meet higher standards in mathematics.

## Chapter 3

## PROCEDURES OF THE PROJECT

The purpose of this project is to design and develop a model mathematics curriculum for 5th grade students in alignment with the Washington State Academic Learnings for White Pass School District in Washington. A review of research and literature regarding curriculum and instruction with the Washington State Academic Learnings will be performed. In addition, other relevant information from selected sources will be gathered and analyzed. Chapter 3 contains background information describing the following:

1. Need for Project
2. Procedures
3. Planned Implementation and Assessment of the Project

### Need for the Project

The need for this project was influenced by the following considerations

1. The author (Randy Torrey), a 5th grade elementary teacher in the White Pass District since 1998 has been engaged in a search for up to date instructional methods, techniques, and strategies that provide an optimal learning environment.
2. The need to prepare students at Randle Elementary to meet the new standards mandated by the Washington State Legislature in 1993 (RCW 28A,150,210).

3. The need to align the 5th grade mathematics curriculum with the Washington State Academic Learning Requirements.
4. The development of this project relates with the writer's graduate studies in Educational Administration at Central Washington University.

## Development of Support for the Project

The development of support for the project was received by the following:

1. The writer participated in various district and ESD workshops focusing on mathematics.
2. The writer contacted other fifth grade teachers in the White Pass School District as well as other districts for information regarding mathematics and problem solving.
3. Information was obtained from math specialists in ESD 113 and the Northwest Regional Laboratory.

## Procedures

Computer search programs were used to obtain literature and research that was relevant to the topic. The Educational Resource Information Center (ERIC) and Proquest were the primary sources used to review and obtain current and background knowledge. Also, other resources such as the OSPI, ESD 113, fellow 5th grade teachers, various district websites, and the Northwest Regional Educational Laboratory were used in the development of this project.

## Planned Implementation and Assessment of the Project

The model, 5th grade, mathematics curriculum aligned with Washington State Essential Learnings and instructional strategies developed for this project will be incorporated into lesson plans that will be presented in chapter 4 of this project. The model will be field tested and implemented in the author's 5th grade classroom during the 2001-2002 school year. Performance-based assessment procedures will be used to determine program success. Examples of assessment will include classroom observations, student interviews, evaluation of student mathematical skill development, and student ability to use mathematics in real-world situations.

# CHAPTER 4

## THE PROJECT

The Model Mathematics Problem Solving Curriculum for Randle Elementary School in Randle, Washington, which is the subject of this project, is presented in five instructional units in Chapter 4, including:

| | |
|---|---|
| Unit 1: | Introduction |
| Unit 2: | Numbers and Operations |
| Unit 3: | Data and Probability |
| Unit 4: | Measurement |
| Unit 5: | Geometry |

# A Model Mathematics Problem Solving Curriculum

# For Randle Elementary School

# In Randle, Washington in the

# White Pass School District


**By**

**Randy A. Torrey**



**Central Washington University**

**July, 2002**

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## Introduction

The project of a mathematics problem-solving curriculum is based on the increasing demand for individuals in our society to be able to take everyday problems and have a foundation of skills to solve them. The goals and objectives of the project are designed to teach and administer problem-solving strategies related to real world problems.

Students are demanded to have problem solving abilities to be successful in our society. There is a need to improve these abilities to stay competitive with international competition and improve test scores. In order for students to be able to compete within a rapidly changing world, they must learn and have a basic foundation of problem solving skills. These essential skills provide students a framework to solve a routine problem and be successful.

The project is designed for students in the fifth grade at Randle Elementary. The goals and objectives, stated as guiding questions are based on the White Pass School District's mathematics curriculum and align with the Washington State Essential Learning Requirements. The objectives and activities are motivating for fifth grade students, which connect to practical applications. The objectives and activities also integrate with other subject areas across the curriculum, such as physical education, geography, reading, writing, and technology. The project consists of five units: introduction, data and probability, number and operations, measurement, and geometry.

## Unit Two

## Data and Probability

## Data and Probability

## Unit Overview

In this unit, students gathered game scores of various sport teams as well as statistical information about teams and players. With the use of this numerical data, students determined the results of the different teams by comparing and contrasting the different numerical outcomes. Students met unit objectives by using problem solving strategies and skills to solve various unit problems and scenarios.

## Objectives of Lesson Plans

The student will be able to…

-find the mean, median, mode, and range of a given set of numbers.

-understand percentages and problem solve on how to compute them.

-learn how to collect, read, and record data to solve various problems.

-use data and present it in various forms such as graphs and charts.

-formulate and solve problems that involve collecting and analyzing data

-explore different concepts of chance.

## Learning Activities

Learning activities focusing on using problem solving skills were designed to correlate with the unit student learning objectives. The activities include...

-students will work in pairs. Each pair will use the Internet to find and record all of the selected sport game scores of a season.

-pairs will use data to calculate the mean, median, mode, and range of a specific team.

-the use of journals to record data and to note any significant findings with the data.

-using data to predict future outcomes.

-class discussions on seasonal patterns.

-creating a class data chart at different times during the basketball season.

-class discussions comparing and contrasting the data from the different teams.

-recording and predicting outcomes of individual player performances.

-predicting the teams that will qualify for end of the season playoffs and their success.

-predicting teams' final standings using ratios and proportions.

## Teaching Strategies

Teaching Strategies included...

-using the Internet and effective data recording.

-direct discussion.

-pairs sharing information and data among themselves and with other pairs.

-whole class discussions and sharing of data and findings.

-individual seatwork and journal reflection.

-problem solving using manipulatives (computers, calculators, assessment tools, etc.).

## Instructional Materials

-Textbook, Saxon Math 65 An Incremental Development, 2nd Edition.

-Computer and computer software.

-Calculator, overhead projector.

## Student Assessment

For this unit, students were assessed by the following methods…

-self-assessment using a rubric to check for mastery.

-peer assessment using a rubric to check for mastery.

-teacher observation using checklists to see if students have met the objective.

-journals to see student reflection and understanding.

-classroom assignments to check for mastery of the objectives.

-daily quizzes that covered the daily or weekly lessons.

-unit assessment to assess the student for the entire unit.

## Grade 5

## Unit Two: Data and Probability

## Lesson Sample

Anticipatory Set: Teacher will explain and discuss how different sports use and calculate averages.

Objective: Students will learn how to find the mean of a set of numbers.

EALR's Addressed: Benchmarks 1-1, 1-4, 2-1, 2-2, 2-3, 4-1, 4-3, 5-1, 5-2, 5-3.

Instructional Input: The teacher will tell the student that he/she shot 4 sets of 5 free throws each. The teacher will then show his/her average for the 4 sets.

Modeling: The teacher will demonstrate on the white board how to calculate an average.

Check for Understanding: The teacher will ask and answer questions that relate to the process of finding an average.

Guided Practice: The teacher will then give each individual a problem with a set of numbers that represent free throws made. Then the students will calculate the average for the problem.

Independent Practice. Once the teacher feels that everyone understands how to calculate an average, he/she will have the students go into the gym and shoot 4 sets of 5 free throws and record the data. Once they are finished shooting, they will then figure out their average of made shots.

## Assessment Sample

Name:_____
Date: _____

Sheena went to a free throw shooting contest. She shot 25 free throws in five different rounds. In the first round she made 15. In the second round she made 18. In the third round she made 12. In the fourth round she made 22. Finally, in the last round she made 23 free throws. What is mean of Sheena's shots made in the five rounds?

Use pictures, words, and numbers to help you solve the problem.

## Unit Three

## Numbers and Operations

## Numbers and Operations

### Unit Overview

In this unit students worked with the data that they had gathered and recorded. This unit provided various problem-solving activities that made the students use their skills to solve problems that focused on multiplication, division, whole numbers, estimation, ratios, fractions, percentages, and decimals. The activities allowed the students to gain a deeper understanding of the meanings of these different concepts. Students were able to recognize the relationships that these concepts had with each other. The activities provided the students an opportunity to use problem-solving skills to solve real world problems.

### Objectives of Lesson Plans

The student will be able to...

-understand various meanings of multiplication and division.

-identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems.

-use visual models to help solve problems.

-know when to add, subtract, multiply, and divide whole numbers.

-be able to convert and solve problems dealing with decimals, percents, and fractions.

-develop and use problem-solving strategies to estimate computations involving fractions and decimals in situations relevant to students' experience.

-organize and analyze relevant information.

-understand the place-value structure and the base-ten number system and be able to represent and compare whole numbers and decimals.

## Learning Activities

Learning activities focusing on using problem solving skills were designed in accordance with the unit student learning objectives. The activities included...

-recording basketball scores and finding averages.

-measuring different dimensions of a basketball court and converting the measurements from whole numbers to decimals and fractions.

-using players' statistics and calculating percentages and decimals from fractions.

-comparing and contrasting the statistical information of different sets of data.

-predicting outcomes of teams and players according to numerical data collected.

## Teaching Strategies

Teaching strategies included:

-individual and group class work.

-modeling of the conversions and connections between whole numbers, fractions, percentages, and decimals.

-small group discussions.

-whole class discussions.

-journal reflection and sharing.

## Instructional Materials

-textbook, Saxon Math 65 An Incremental Development, 2nd Edition.

-calculator, paper, overhead projector.

-computer and computer software.

-Actions with Fractions, AIMS Education Foundation.

## Student Assessment

For this unit, students were assessed using the following methods…

-self-assessment using a rubric to check for mastery.

-journals to see student reflection and understanding.

-peer assessments using a rubric to check for mastery.

-teacher observations using checklists to see if students have met the objectives.

-performance assessments to see if students mastered the concept being taught.

-daily quizzes that covered the daily or weekly lessons.

-classroom assignments to check to see if students have mastered the objectives.

-unit assessment to see if students understand the unit concepts that were taught.

## Unit Three: Numbers and Operations

### Lesson Sample

Anticipatory Set: The teacher will discuss how decimals and fractions are often used in different sports and how it is necessary to be able to convert the two.

Objective: Students will be able to convert fractions into decimals.

EALR's Addressed: 1-1, 1-3, 2-1 - 2-3, 4-1 - 4-3, 5-1, 5-3.

Instructional Input: The teacher will discuss and demonstrate how to convert each.

Modeling: The teacher will demonstrate in front of the class how to change a fraction into a decimal.

Check for Understanding: The teacher will randomly call on different students to demonstrate the procedures.

Guided Practice: The teacher will provide 5 fractions and have each student convert the fractions into decimals.

Independent Practice: Each student will draw some type of playing court or field and will break the field into 10 different fractional pieces and then convert each fraction into a decimal.

## Assessment Sample

1. 50% is equivalent to which fraction? ( a) 3/4 (b) 2/3 (c) 1/2 (d) 1/4

2. 60% is equivalent to which fraction (a) 4/5 (b) 1/2 (c) 3/5 (d) 2/5

3. 5% is equivalent to which fraction? (a) 5/10 (b) 1/20 (c) 1/10 (d) 5/50

4. 3/4 is equivalent to which percentage? (a) 60% (b) 75% (c) 50% (d) 35%

5. 1/10 is equivalent to which percentage? (a) 5% (b) 15% (c) 10% (d) 50%

## Unit Four

## Measurement

## Measurement

## Unit Overview

In this unit, students measured the dimensions of a basketball court in the school gym. The students used both the standards units of measurement and metric systems in determining different measurements. They used problem solving strategies to solve and calculate different measurements and problems.

## Objectives of Lesson Plans

The students will be able to…

-understand such attributes as length, width, area, and perimeter.

-understand the need for measuring in standard units and become familiar with standard units in the customary and metric systems.

-carry out simple unit conversions, such as converting centimeters to meters.

-understand that measurements are approximations and how differences in units affect precision.

-develop strategies for estimating the perimeters and areas of irregular shapes.

-develop, understand, and use formulas to find the area of rectangles.

-organize and analyze relevant information.

-use measurement to help solve every day problems.

-predict and estimate measurements.

## Learning Activities

Learning activities focusing on using problem solving skills were designed in accordance with the unit student learning objectives. The activities included...

-choosing the correct type of measurement tools to measure court dimensions.

-measuring various dimensions of the basketball court.

-create a detailed plan of the basketball court's measurements and dimensions.

-work in pairs to use prior knowledge to estimate and predict the width and length of the court's dimensions.

-class discussions of comparisons of the measurement data recorded.

-use journals to record measurements and note any similarities and differences discovered.

-work in pairs to convert yards to feet and meters to centimeters.

## Teaching Strategies

Teaching strategies included...

-small group discussion.

-small group activities.

-individual sketching of court dimensions.

-whole class discussions on measurement discoveries.

-teacher modeling of converting different units of measurement.

-journal use and reflection.

-problem solving.

## Instructional Materials

-textbook Saxon Math 65 An Incremental Development, 2nd Edition.

-calculator, paper, ruler, yard and meter sticks, overhead projector.

-computer and computer software.

## Student Assessment

For this unit, students were assessed using the following methods...

-small group assessment using a rubric to check for mastery.

-individual assignment using a checklist or rubric to check for mastery.

-teacher observation using checklists to see if students have met the objective.

-performance assessment to see if students could identify different shapes.

-classroom assignments to check to see if students have mastered the objectives.

-daily quizzes that covered the daily or weekly lessons.

-unit assessment using an end of the unit test.

## Grade 5

## Unit Four: Measurement

## Lesson Sample

Anticipatory Set: The teacher will explain to the class how most countries in the world use the metric measuring system.

Objective: Students will convert meters to centimeters.

EALR's Addressed: 1-2, 2-1 – 2-3, 4-1 – 4-3, 5-1 – 5-3.

Instructional Input: The teacher will explain how the U.S. uses a different type of measuring system compared to a majority of the world. The teacher will explain how it is actually easier to convert different measurements using the metric system than our system of measurement by using place value.

Modeling: The teacher will measure an object or distance in the classroom using meters and then convert the measurement to centimeters.

Check for Understanding: The teacher will ask and answer various questions.

Guided Practice: The teacher will give 5 meter measurements to the students and have them convert the measurements to centimeters.

Independent Practice: Once the teacher feels that everyone understands how to convert the meters to centimeters, he/she will have the students get into pairs and measure selected lines in a gym using a meter stick. Once the pairs have the measurements in meters, they will convert them to centimeters.

## Assessment Sample

Student Performance Checklist

_____ 1. Student can measure an object using millimeters.

_____ 2. Student can measure an object using centimeters.

_____ 3. Student can measure an object using meters.

_____ 4. Student can convert centimeters to meters.

_____ 5. Student can covert millimeters to centimeters.

_____ 6. Student can convert millimeters to meters.

## Unit Five

## Geometry

## Geometry

### Unit Overview

In this unit, students used the dimensions of a basketball court to recognize geometric ideas and relationships and applied them to other disciplines and to problems that arise in everyday life. Students used the dimensions of the basketball court to identify, compare, and analyze characteristics and properties of two-dimensional geometric shapes.

### Objectives of Lesson Plans

The student will be able to...

-investigate, describe, and reason about the results of subdividing, combing, and transforming shapes.

-explore congruence and similarity.

-draw geometric shapes.

-create and describe mental images of objects, patterns, and paths.

-use geometric models to solve problems in other areas of mathematics, such as number and measurement.

-recognize geometric ideas and apply them to real world problems.

-make and use coordinate systems to specify locations and to describe paths.

-use visualization, spatial reasoning, and geometric modeling to solve problems.

## Learning Activities

Learning activities focusing on using problem solving skills were designed in accordance to unit students learning objectives. The activities included...

-measuring and recording the perimeter, area, width, and length of the court dimensions.

-measuring and recording the different angles on the court.

-finding the circumference, radius, and diameter of the different circles located on the court.

-investigating and recording the relationships among different shapes on the court.

-create a grid of the court and locate various points on the grid.

-identify similar, congruent, and symmetrical shapes on the court.

-reason about the area of a triangle by visualizing its relationship to a corresponding rectangle.

-compare and contrast the perimeter and area of different areas of the court.

## Teaching Strategies

Teaching strategies included...

-individual work.

-small group work.

-direct instruction and modeling of calculating the different measurements.

-whole class practice and discussions of results.

-journal reflection and use.

-problem solving.

## Instructional Materials

-textbook Saxon Math 65 Incremental Development, 2nd Edition.

-computer and computer software.

-calculator, overhead projector.

## Student Assessment

For this unit, students were assessed by the following methods…

-performance assessment to see if students have met the unit objectives.

-teacher observation using checklists to see if students have met the objectives.

-peer assessment using a rubric to check for mastery.

-self-assessment using a rubric to check for mastery.

-classroom assignments to check to see if students have mastered the objectives.

-daily quizzes that covered the daily or weekly lessons.

-unit assessment to see if students understand the unit objectives that were taught.

## Grade 5

### Unit Five: Geometry

### Lesson Sample

Anticipatory Set: Explain to students how different sports use different lines and boundaries to play on.

Objective: Students will measure and record the perimeter and area of a basketball court.

EALR's Addressed: 1-3, 2-1 – 2-3, 3-1 – 3-3, 4-1 – 4-3, 5-1 – 5-3.

Instructional Input: The teacher will teach and demonstrate how to calculate perimeter and area.

Modeling: The teacher will find the area and perimeter of a white or chalk board in the classroom.

Check for Understanding: The teacher will ask various questions about finding perimeter and area.

Guided Practice: The teacher will give each student a couple of sample problems to figure out the area and perimeter.

Independent Practice: The students will get into pairs and calculate and record various shapes on a basketball court.

## Assessment Sample

Personal Communication Assessment

1. What is the width of the gym floor?

2. What is the length of the gym floor?

3. Explain the steps in finding the perimeter of a shape.

4. Explain the steps in finding the area of a shape.

5. How do you find the circumference of a circle?

6. How do you find the perimeter of a circle?

# Chapter 5

## Summary, Conclusion, and Recommendations

### Summary

The purpose of this project was to develop a model Problem Solving Curriculum for Randle Elementary in Randle, Washington the White Pass School District, aimed at fifth grade mathematics classrooms. To complete this project, a review of related literature and research was conducted using ERIC, OSPI, and NCTM sources. These sources were used to develop unit and lesson plans for a model problem solving curriculum.

### Conclusions

Conclusion reached as a result of this project were:

1. The need for a mathematics problem solving curriculum is essential for success in math as well as other subject areas. Problem solving is a life long skill that will empower an individual with the ability to solve different problems as they arise.

2. Students develop a greater understanding and appreciation for mathematics when it is used in various subjects and when it is taught using concepts that

coincides with their personal interests. As a result, this opens their eyes to the importance of problem solving skills in their everyday lives.

3. Assessment is a critical component of a mathematics curriculum. A variety of methods should be used to assess students in mathematics. Students must be able to demonstrate, write, and explain the processes involved in answering a problem.

## Recommendations

As a result of this project, the following recommendations have been suggested:

1. It is recommended that schools use a mathematics curriculum that is based on problem solving skills. Problem solving should not be a supplemental piece of a mathematics curriculum. It should be part of the foundation of a mathematics curriculum in which all other components of a curriculum come together.
2. It is recommended that problem solving skills should not be limited to mathematics. Teachers should encourage problem solving in other subjects to the point that students naturally problem solve without consciously thinking about it.
3. It is recommended that teachers use a variety of assessments to ensure that students meet the Essential Academic Learning Requirements.

4. It is recommended that anyone who is interested in creating a project similar to this to start by using the NCTM and OSPI web sites.
5. It is recommended for future considerations that it would be beneficial to make sure that students are taught how to successfully work in cooperative groups before having them work on math problem solving lessons.
6. Schools wanting to develop a fifth grade mathematics problem solving curriculum may want to adapt and use materials from this project. Any one who is interested is also invited to contact the author of this project for further information.

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