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Audio-Visual Aids, Manipulative Materials, and Arithmetic Achievement in Grade Four

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AUDIO-VISUAL AIDS, MANIPULATIVE MATERIALS, AND ARITHMETIC ACHIEVEMENT IN GRADE FOUR

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CHAPTER I

INTRODUCTION

When in training to teach elementary school children, the college student is bombarded with the idea of using audio-visual aids and manipulative materials. His methods courses require him to collect or construct files of such materials. His psychology books stress the importance of appealing to as many of the learners' senses as possible.

But the construction and collection of these teaching aids takes a great deal of time and effort. Often their use takes extra teaching time. The writer has wondered if the time and effort involved in the use of these audio-visual aids and manipulative materials results in greater achievement on the part of the learner.

I. THE PROBLEM

Statement of the Problem. The central problem of this study was to determine whether children using an above-normal amount of audio-visual aids and manipulative materials would achieve significantly higher in arithmetic.

Importance of the Study. The planning, construction, and use of audio-visual aids and manipulative materials involves a great deal of time and effort on the part of the teacher. Their use also takes extra instructional time. If these aids do not result in better learning, valuable time and energies are being wasted. It is the intention of this study to add to the data available concerning the question of whether or not these aids are worthwhile.

Limitations of the Study. Since the writer was working with an actual classroom situation, it was very difficult to control every factor which might affect the outcome of the study. The writer attempted to keep all variables constant except the number of audio-visual aids and manipulative materials used.

Another difficulty encountered was the matching of groups. There were only fifty-four subjects to be paired.

There was a lack of standardized tests available to the writer which would test the learnings which the writer was trying to encourage.

II. HYPOTHESIS

The writer proposed the following hypotheses:

(1) There will be no significant difference between the means of the two groups as indicated by the Total Score of the Metropolitan Achievement Test.

(2) There will be no significant difference between the means of the two groups as indicated by the

Computation Score of the test.

(3) There will be no significant difference between the means of the two groups as indicated by the Problem Solving and Concepts Score of the test.

III. DEFINITION OF TERMS USED IN THE STUDY

Audio-Visual Aids. Any devices besides the textbook and chalkboard which the students could see or hear were called audio-visual aids.

Manipulative Materials. Any devices besides the textbook which the learners could touch and move about were classified as manipulative materials.

Aids. This term is intended to include all materials used except textbooks, duplicated exercises which the children do as assignments, and the chalkboard. It encompasses both audio-visual aids and manipulative materials.

Normal Amount of Aids. The quantity of aids which the writer would ordinarily use, and has previously used, in the mathematics program of her class, is termed "normal."

Above-Normal Amount of Aids. This term was used to describe the quantity of aids used with the experimental group. It denotes many more aids than the "normal" amount.

Experimental Group. The experimental group was the group with whom an above-normal amount of audio-visual aids and manipulative materials was used.

Control Group. The control group was the group with whom the "normal" amount of aids was used.

Achievement. Achievement was measured by comparing the pretest and post test scores of the Metropolitan Achievement Test, Arithmetic Section.

CHAPTER II

REVIEW OF THE LITERATURE

Recent advances in science have increased the body of mathematical knowledge and have led to a more extensive use of mathematics in technology and in daily life. More scientists are now needed than ever before. The citizen needs to be better founded in mathematics in order to understand his environment and to interpret it intelligently. These developments have placed upon the institutions of learning a greater responsibility to improve and accelerate the mathematics curriculum. The elementary school shares in this responsibility, for there the foundation of mathematics has to be laid, its underlying principles have to be taught, and a reasonable degree of skill in computation and problem solving has to be acquired (15:3).

These new and unprecedented demands on the individual for mathematical competence, both in his vocation and in his personal life, are now reflected in the comparatively dramatic modifications in the elementary curriculum (17:v).

In general, the instruction of mathematics in the past has been inadequate in preparing today's adults to meet these new demands. Most adults of today can solve arithmetical problems, but do not understand the "why" or "wherefore" of the processes (6:17). Kramer (15:28-40) states that children in the United States are no match in arithmetical computation and problem solving for their European counterparts, and that we should follow the example of European schools in spending more time for arithmetic instruction.

History of Mathematics Instruction. The name "modern" is probably a misnomer for the current elementary school mathematics program. Most mathematics in the "modern" program was created before 1900 (5:5). According to Westcott (25:19), modern mathematics has retained much of the proven procedures of the past. Teaching mathematics meaningfully has been a goal for a long time. Stressing understanding and the use of the discovery method have been used by good teachers all along (15:31). The use of concrete and semi-concrete materials is as old as the number system itself. The abacus was used over 2,500 years ago by the ancient Egyptians, Babylonians, and Greeks. Its exact origin is lost in antiquity (25:202).

In the past, there were two major ideas about the learning of mathematics. One was that the subject was to be learned as a tool, and was to be taught by "showing" the child how, and then having him practice to make perfect the skill to be learned (7:15). In the eighteenth, and the beginning of the nineteenth century, mathematics in America was presented by asking pupils to memorize a rule, and then to apply it to a concrete situation (15:17). The teacher was the authoritative source of information, and the child was the passive receiver (6:17).

The second major idea regarding the learning of mathematics began with Piaget. He was one of the first to see that concept building is a long process requiring many experiences, before these experiences fit together in a pattern to form an "understanding" (5:35). Pestalozzi also began, from the beginning of the nineteenth century, to stress sense impressions as the foundations of all knowledge (15:18). When "Faculty Psychology" was in vogue, the "mental discipline" movement stressed the idea that abstract thinking and mental computation helped to strengthen and train the mind. While the advocates of this Formalist movement were influencing the curriculum, others were experimenting with it in a different way. In 1873, Francis Parker organized the Quincy, Massachusetts school system in a way that has come to be thought of as quite modern for that day. Arithmetic in this system was approached inductively, through objects rather than through rules (5:3).

After 1930 came the "Meaning Movement," which put forth the idea that meaning must be developed along with a skill (5:5). Associated with this theory were the Gestalt psychologists, who maintained that children learn mathematics better if they understand the number system, and then can perform the operations with meaning. They encouraged children to find rules inductively (15:25). Other authorities who advocated this method were George Katona, in the 1940's, and Jerome Bruner, in the 1960's (21:163).

Study Groups and Experiments in Mathematics Instruction. Widespread experimentation with curricular methods and materials has been a natural consequence of the recognition of changing demands on the mathematics programs in the schools (17:3). The period starting with the middle 1950's has been characterized by the organization of study groups and experimentation subsidized by the federal government (17:6). Professional organizations, colleges and universities, governmental agencies, foundations, individual educators, and commercial publishers have directed their energies to the production of new teaching materials at an unprecedented rate. Classroom experimentation is being directed at developing new materials and using effectively what is available (17:3).

One of the most widely publicized study groups which has been formed is the School Mathematics Study Group (SMSG), which operates on a subsidy from the National Science Foundation. It has prepared text materials in mathematics from fourth grade through junior high school. The National Science Foundation has also financed, on a small scale, summer institutes for elementary teachers. At the state and local levels, inservice training courses, workshops, and institutes on mathematics are commonplace (1:3).

Other exploratory mathematics programs for the elementary school which have been in the limelight for some time include the following: the University of Illinois Arithmetic Project, the Greater Cleveland Mathematics Program, the Syracuse University - Webster College Madison Project, the projects of Suppes and Hawley at Stanford University, the Minnesota Elementary Curriculum Project, and the University of Maryland Mathematics Project (15:30; 17:7).

Numerous experimenters, such as Glenadine Gibb, Esther Swenson, William Brownell, Henry Van Engen, and Harold Moser, have also helped to develop and publicize the "new" mathematics (5:7).

Many of these study and experimental groups are also developing new materials for the teaching of mathematics. These materials include many concrete and semi-concrete devices besides the textbooks included in their programs (21:161).

These materials are being tried out all across the country (15:3). Some early attempts at the creation of mathematics apparatus were made by Montessori, Cuisenaire, and Stern. The use of their materials by children is an individual type of activity, and numerous authorities claim that it leads to personal learning and self motivation. Comprehensive, formal, statistical evaluation of the effectiveness of this apparatus cannot yet be made (6:28).

<u>Trends Characteristic of the New Mathematics Program</u>. "A new arithmetic curriculum is emerging, and it will give more emphasis to the mathematical concepts underlying arithmetic, to generalization, and to mathematical structure (1:4)." Different emphases, different grade placement, new topics, changed methods for promoting learning, and new materials, books, and multisensory aids are characteristic at all levels (17:3). Experiences that provide an opportunity for pupils to investigate and discover rules and ideas are receiving increased attention. These experiences are often with objects, pictures, or charts (17:31).

Involvement of the student probably should rank at the top of a list describing the major distinctions between the traditional and the modern curricula. The child has ceased to be a spectator in the learning process and becomes instead an active participant (5:11). Many authorities in the field of mathematics endorse this change. According to J. Houston Banks (1:419), experimental evidence, as well as common sense, point to the fact that learning is facilitated by active participation. Dr. Banks also states that mental activity is essential to learning, and that physical activity is frequently involved. No amount of "teaching" that consists only of telling can guarantee learning. We learn by being active participants in the learning process (1:9).

Not only is activity important for itself in learning, but the type of activity is a determining factor in the effectiveness of the learning. The best learning takes place when the pupils acquire a concept and find a procedure leading to the correct answer, as a result of their being actively engaged in the learning process. This process of learning, and not just the correct product, is therefore emphasized (15:5).

Leading educators emphasize the teaching of mathematics for understanding rather than for the sole purpose of learning computational skills. The child understands the meanings best when he makes his own discoveries. Instead of showing children the "what" and "how," teachers should conduct the mathematics classes in such a way that the children will develop insight and will see the relationships that constitute the basic concepts in mathematics. Children will then be able to understand and explain the "why" of mathematics (19:7).

Development of problem-solving ability in mathematics requires a mastery of the subject as a coherent, complete body of knowledge. It is through concrete, practical situations that the child develops such mathematical abstractions as are necessary to this skill (1:419).

Mathematical understandings are an outgrowth of meaningful learning experiences. They grow slowly, and as a result of many and varied positive reinforcement experiences. The child's readiness to understand new mathematical concepts is also enhanced by a background of rich, varied, and meaningful experiences (25:8; 5:19). Many experiences are needed if the child is to build concepts or generalizations. It is practically impossible to abstract from only one set of experiences (6:54). If the child is exposed to only a few experiences, only association, and not generalization, will occur. Some of these experiences should be drawn from real situations, and others artificially induced by the use of varied materials (6:29).

There is a definite sequence to this type of mathematics instruction, which Westcott calls the "Tri-Point Progression" (25:18). Fehr (7:425) suggests that teachers use sensory objects as a first means approach to concept formation. Other authorities label this stage of learning the stage of concrete materials (15:5). The first step toward abstraction consists of using pictures of concrete objects. These pictures may then be replaced by lines, circles, dots, etc. (1:137). Kramer (15:5) labels this the semi-concrete stage. The final abstraction is made to the number symbols and written words (1:137). This, of course, is the abstract stage (7:426).

One group of projects or programs seems to follow this approach quite closely. They follow the use of concrete materials as the media through which proficiency in conventional arithmetical operations may be achieved. Cuisenaire, Stern, and Dienes are associated with this movement, sometimes called the "manipulative materials movement" (21:20).

The method of teaching described above is commonly called the "discovery" approach, but it should be realized that the process is one of directed discovery (15:5). It can be deliberate and organized (25:122). Yet the children work in a relatively free atmosphere, and must feel free to experiment. Trial is an essential component of most learning. Trial may be for the purpose of eliminating wrong responses, or it may be to discover the relationship of the element to the whole configuration (1:9).

Hopefully, the result of the discovery method of teaching mathematics will be a greater understanding of mathematics itself. Understanding is crucial to the learning and use of numbers. Learning is not complete until the relationships of the situation become apparent and the generalized concept projected to other situations (1:9). One hundred per cent mastery of the facts and skills of mathematics is wasted effort if attendant meanings and understandings are missing. Without meaning there can be no understanding, and without understanding there cannot be intelligent application. The ultimate objective of instruction in mathematics is the development of problem-solving ability. Unless the child understands the significance of what he does, he cannot apply his knowledge to the solution of a problem (1:12). A study by Pace (15:364-65) indicated that greater problem-solving ability resulted from a better understanding of the fundamental processes and the number system itself.

Understanding also facilitates practice. If possible, understanding should be developed before intensive practice is undertaken (5:18-19). Processes that are understood require a minimum amount of practice to be mastered. Meaningful work also incites motivation for practice, since the pupil is more apt to see the need for drill (15:7).

Understanding lends permanence to acquired skills. A process that is understood is less easily forgotten, and the understanding serves as a framework for the reestablishment of a skill. The child can easily rediscover relationships which he once understood, but has forgotten through disuse (1:12).

Furthermore, it has been demonstrated by numerous experimenters, such as William Brownell, Harold Moser, Esther Swenson, Glenadine Gibb, and Henry Van Engen, that teaching the meaning behind algorithms leads to better transfer to new situations (5:4).

Another advantage which is claimed for the discovery method is that it fosters a good attitude toward learning mathematics (15:5), and it seems to promote intrinsic motivation (6:30). When a child is motivated, he applies greater effort to a problem, and consequently, he learns better (6:4). This motivation and increased interest on the part of the learner helps the child to study mathematics for its own sake, rather than only for the practical uses to which it can be put, or for external rewards (10:4). Attitude is always a crucial factor in learning. "The teacher of arithmetic has no greater responsibility than the development of satisfactory habits and attitudes (1:9)." Children are naturally curious, and want to know "why" before accepting information as being true. Introducing new concepts and pressuring a child for mastery before his conceptual background is sufficient to understand the "why" may be very harmful to his attitude (21:191).

On the other hand, children can be motivated by the sheer joy of discovery (17:32). It can be rewarding and satisfying to the learner. Westcott enumerates four stages of personal reactions during the discovery process. It begins when the child recognizes a problem and relates to it personally. He develops an interest in solving it. Then he collects pertinent data and materials that will help him to solve it. This is followed by spurious experimental behavior and flashes of insight. Finally, he finds a solution and tests it. The process begins with tension and anxiety, and is resolved in relief from tension and in satisfaction (25:123).

During the time allotted to mathematics instruction in most classrooms, it is difficult to use the discovery method (15:5). The building of an experiential background takes much time if the pupils are truly allowed to manipulate materials, and time must also be provided for practice. However, Kramer recommends that more time be spent in the instruction of arithmetic, and that teachers use time more wisely (15:40). It is easy to wonder if "play" sessions with materials are not a waste of valuable time (6:56), but one experience may have effects on the learning of concepts in several widely different branches of mathematics. Moreover, these effects may not be observable, or observed, until a long time — perhaps years — after the experience (6:29).

<u>The Use of Audio-Visual Aids and Manipulative Materials in the Discovery Approach</u>. Development of the discovery or meaning approach was accompanied by increased attention to the use of manipulative materials for aiding learning as it moves from the concrete to the abstract (17:18). Well known, supportive instructional materials of this character, such as the counting frame, the abacus, the place-value chart, the hundred-peg board, the flannel or magnetic display board, fact finders, domino cards, fraction kits, geometric forms, and measuring instruments have been reevaluated for use in a modern program (21:157). The need for more specially designed mathematics apparatus has begun to be appreciated (6:27). A large number of commercially prepared devices have been added to the catalogs of companies which market such materials (21:157).

People have used audio-visual aids and manipulative materials to help them to perform mathematical operations and to see relationships for many years. Herodotous, who lived in the fifth century, B.C., mentioned the use of the abacus in ancient Egypt. It was also used in Babylonia, and both the Greeks and the Romans also used it (11:4-6). In the eighteenth century, methods were suggested by which objects and pictures were used to make the fundamental processes more meaningful to children and to introduce arithmetical problems in a concrete way (14:vi). The greatest such influence exerted on the teaching of mathematics at this time was that of the Swiss educator, Johann Heinrich Pestalozzi (1746 - 1827). While most of the devices used in the modern arithmetic program are not startlingly new, they are being stressed now more than previously. The number line is an example of a known teaching tool that that has taken on an increased significance and much wider usage in connection with the new program (21:157).

These aids have been classified into two types. Manipulative materials, frequently referred to as "concrete" or "exploratory" materials, include objects of all kinds that the children and teacher can group, count, combine, separate, and manipulate in various ways. Visual aids, or audio-visual aids, which are not manipulated or moved about, are viewed or heard by the children in order to help them to visualize processes, number systems, and other concepts (14:vi).

The type of aid used depends upon the learning stage of the pupil. Manipulative materials are used for the concrete stage of mathematics learning. Physical experiences such as working with the counting frame, the abacus, toy money, rods, and pegs will be helpful for most pupils in the acquisition of concepts. For the semi-concrete stage, representations such as pictures, diagrams, dots, and marks are employed. During the abstract stage, the pupils work with mathematical symbols. The manipulation of materials by the child, the representation of the situation on paper, and the gradual increase in the difficulty level of the stages offer the opportunity to even the below-average pupil to learn to understand the processes and to acquire the concepts involved (15:5).

There are many aids to be found in the natural environment of the child. Strings, rubber bands, watches, shadows, alignment of objects, grouping of objects, are all available in the ordinary home or classroom (7:16). Pictorial aids, models, mathematics films and filmstrips, and television can also be used effectively (6:18).

It is important for each student to have an opportunity to prove and discover mathematical principles for himself by using concrete devices (25:196). In order to accomplish this goal, there must be sufficient manipulative materials and devices so that each child can use them on an individual basis. Examples of such aids which can be used by each child are: number blocks, abaci, fraction kits, counting frames, placeholder charts, and number lines.

The National Council of Teachers of Mathematics publication, "A Guide to the Use and Procurement of Teaching Aids for Mathematics," includes a good list of available commercial materials and sources (21:157).

The use of audio-visual aids and manipulative materials helps the mathematics teacher provide for individual differences in her instruction (15:394). Different children learn mathematics in different ways. The use of many approaches, many varied experiences, and many aids will have a chance of reaching more children than the use of just a few, or only one (6:29). Because of the wide range in the mathematical ability of children, some will not need to spend much time in the first stage of learning — the concrete stage. Others may have to spend a great deal of time in the first and second stages before they will be able to abstract (15:5). If sufficient concrete and semi-concrete materials are available, each child will be able to work, individually, at an appropriate level of learning.

Aids can also be a vital part of an enrichment program for the gifted child. It is desirable to allow gifted children to explore widely, and in depth, rather than to merely master material at a more rapid rate. The use of manipulative materials and aids can enable the gifted child to work more in accordance with his potential than if he were forced to use the same material and work at the same level as the rest of the group (1:13).

Still another possibility for using aids is in the process of practice or drill. Even though the emphasis in mathematics is on learning for understanding, practice and drill are still necessary. The pupil should understand before drilling, so that he may know and compare what he is to learn, and so that he will know why he should learn it (17:54). Moreover, drill or practice on material which is understood is more efficient. Ideally, understanding should precede drill, but no amount of drill can ensure understanding (1:12). When the child does have a proper background of understanding, committing the addition, subtraction, multiplication, and division facts to memory will consist largely of establishing immediate recall and identifying the more difficult combinations for intensive study. To this end, flash cards can be most helpful (1:152). Drill cards displaying families of related facts can be used effectively for this same purpose (1:250). Practice on a skill should include different kinds of exercises and activities, so that monotony is avoided (15:7). Many manipulative materials and aids are best for this task, not just the use of textbooks, duplicated materials, and workbooks (17:54).

<u>Evaluating the Use of Audio-Visual Aids and Manipulative Materials</u>. Aids can be valuable, but they must be used correctly. They are not a guarantee of a superior, or even a good, mathematics program. Textbooks and other teaching materials are of value only to the extent that they will help to produce the desired behavioral changes in pupils (1:14). The classroom equipped with many manipulative devices does not automatically enable children to "discover" mathematical concepts and to learn with meaning. In even the most well equipped classroom it is possible to find children who can provide correct answers to exercises from the textbook, but who have little understanding of the ideas behind the solution processes. This type of learning is probably little or no better than that found in the classrooms where there are no learning aids. Only when aids are used correctly do they lead to greater understanding on the part of children (14:vi).

Physical objects and other multisensory aids can be overemphasized and used in improper and incorrect ways (7:16). Overdependence on them can hinder the child's progress to the abstract level of learning (19:8; 14:vi).

Aids are used correctly when they help children to visualize the abstract ideas of mathematics. Once a child has grasped these ideas, he should no longer use the various devices (14:vi). A teacher who gradually develops for himself a theory on the use of materials in teaching mathematics, modified from time to time, and sees how the use of such aids leads to mathematical understanding, cannot help but develop a broader and more structured viewpoint on the whole of mathematical learning (7:429).

Aids are a means to an end, and never an end in themselves (7:425-26). "Filling a classroom with junk, simply because funds are available, is no assurance of improved instruction (1:14)." The variety and kinds of aids commercially available today are literally unlimited, but the teacher should carefully weigh their essential value for learning before purchasing them. Before using any materials, the teacher should always attempt to think through the kind of concept that will be the outcome, and the probable use of the concept (7:425-26). The important question about aids is, "Is it worth the cost; in money, in terms of teacher time and energy, and in terms of pupil time and energy (1:14)?" The instructional program and the aids used by a teacher will depend upon the needs and the abilities of the children (14:vi-vii).

It is difficult to test behavioral outcomes of the mathematics program which uses many aids. In evaluating by means of written standardized tests, it is difficult to measure such outcomes as understanding and ability to compute mentally. Also, tests may lack curricular validity. They may not test outcomes commensurate with the goals of instruction in a particular situation (15:381). Both standardized and teacher-made tests tend not to allow for divergent thinking, which is highly correlated with problem-solving ability and creativity. Most of these tests allow for only one correct answer, while divergent thinking may lead to many answers (25:188).

There has been insufficient time since the beginning of the "new" mathematics programs for teachers to construct, use, and test modern and successful devices through which number ideas may most effectively be communicated to children (21:140). In order to accomplish the goal of better teaching and learning through the use of improved, properly balanced programs, promising new materials should be tried out in the classroom under carefully controlled conditions before they are promoted for countrywide use or rejected as inferior (15:31). More research is needed concerning the effectiveness of the use of these aids and models (6:18).

CHAPTER III

PROCEDURES

The study was conducted with two matched groups of fourth graders. It extended from November 7, 1966, to June 2, 1967, and involved a sample of fifty-four subjects from Arrowhead Elementary School, Bothell, Washington. The textbook used was Seeing Through Arithmetic, which was published by the Scott Foresman Company. Each group was composed of twenty-seven children. Pairs of children were matched on the following criteria: scores on the Arithmetic Section of the Metropolitan Achievement Test, Form B, administered in October; scores on the Seeing Through Arithmetic Test, administered the previous May by the third-grade teachers; sex; age; and whether or not the child had repeated a grade. The criteria were considered in the order listed above. The data used and the resulting pairings may be seen in Appendix B. The table of random numbers was then used to place one member from each pair in the experimental group. The other member was placed in the control group.

Each group received thirty-five minutes of instruction, per day, including work time to complete assignments.

During the period when one group was having a mathematics lesson, the other group went to another room and another teacher for a social studies lesson. This prevented the control group from watching the activities of the experimental group. The arithmetic lessons were conducted at 9:40 a.m. and 12:55 p.m. The groups' mathematics times were periodically exchanged so that each group had the same number of morning lessons and the same number of afternoon lessons. This was done to counteract any affect that the time of day which instruction was received would have on the results of the study.

An effort was made to keep all other factors as equal as possible, and vary only the factor being tested — the quantity of aids used. Although the progress of the two groups was not always exactly even throughout the study, both groups had covered the same number of pages and topics, and had done the same written assignments at the close of the experiment. The writer taught both groups.

Before the experiment was initiated, the writer obtained permission from the school principal to conduct the study. Explanatory letters were also sent to the parents of the children who were to be involved with the study. It was made clear that the control group would not be deprived of any aids which they would ordinarily have, and would receive the same quality of instruction they ordinarily would have received if the study were not conducted. When no objections were voiced by the parents, the experiment was begun.

CHAPTER IV

RESULTS

The Metropolitan Achievement Test, Arithmetic Section, Form A, was administered as a post test to both groups in three sittings each, from May 31 through June 2, 1967. Three scores were obtained: Total Score, Computation, and Problem Solving and Concepts. The raw data from this post test is to be found in Appendix B. The net gain in points was computed for each child, and the mean gain was found for each group. This was done for all three scores from the test. The test statistic chosen to test the hypotheses of no difference between the means of the two groups was the t. The level of significance chosen was .05.

For the Total Score, the mean of the experimental group was 17.31, and the mean of the control was 17.44. The result of the t-test was .08, which was not significant at the .05 level. For the Computation section, the mean of the experimental group was 23.14, and the control group scored a mean of 23.59. The t-score was .26, which was not significant. However, for the Problem-Solving and Concepts subtest, the experimental group had a mean of 12.59, and the control group's mean was 11.33. The t-score of this section was 2.43, which was significant at the .05 level, and shows more gain for the experimental group. A summary of the data from these test scores is shown in Table I, page 30.

TABLE I

COMPARISON OF TEST RESULTS

	Experimental Group			Control Group				
Scores	N	$\overline{X}$	s^2	N	$\overline{X}$	s^2	t	Hypothesis
Computation	27	23.14	35.03	27	23.59	43.69	.26	Retain
Problem Solving and Concepts	27	12.59	46.16	27	11.33	42.30	2.43	Reject
Total Score	27	17.31	27.16	27	17.44	29.83	.08	Retain

CHAPTER V

SUMMARY, CONCLUSIONS, AND IMPLICATIONS

Although the analysis of the data from the study shows a trend, general conclusions concerning the use of aids cannot be drawn from a single experiment. More research and more time are needed before firm judgments can be made. The study also raised other questions which are themselves material for further research.

I. SUMMARY

A survey of the literature relative to this problem would indicate that the use of audio-visual aids and manipulative materials tends to increase the understanding of mathematics, and leads to the formation of more positive attitudes on the part of the learner. This study was carried out to see if achievement would be affected by the use of an above-normal amount of such materials. The results of the study showed no significant difference between the means of the groups as indicated by the Total Score and the Computation section. However, the group with whom an above-normal amount of aids was used did make significantly higher gains on the Problem Solving and Concepts section of the test.

II. CONCLUSIONS

From the statistical results of the study, it was concluded that the use of aids did not increase achievement, as indicated by the Total Score of the Metropolitan Achievement Test, on the part of the children in the experimental group. The first hypothesis was retained.

The t-test results also showed no significant difference between the means of the groups as indicated by the Computation section of the Metropolitan Achievement Test. Hypothesis number two was therefore retained.

According to the t-test applied to the data from the Problem Solving and Concepts section of the Metropolitan Achievement Test, there was a significant difference between the means of the two groups, in favor of the experimental group. The third hypothesis was therefore rejected.

III. IMPLICATIONS

Constructing and procuring the materials used with the experimental group in this study involved a great deal of time and work. Their use also took increased class time. The experimenter recognizes the need for much more research to be done in this area. Studies which involved a greater number of students in the sample would be helpful in identifying trends. Also, it would be advantageous to carry out such a study for a longer period of time.

The amount of time during the instructional period which was spent for the use of aids was so great that it left very little actual work time when the students could do seatwork assignments at their desks on an individual basis. In order to provide sufficient time for such work, it would seem appropriate to lengthen the time allotted to arithmetic instruction. Whether or not this would result in better learning could be reason for yet another study. Although not statistically shown, it might be that this lack of practice time was the reason for the lower score of the experimental group on the Computation section of the Metropolitan Achievement Test. The difference between the means on this section was small enough not to be significant at the .05 level, however.

It also seemed to the investigator that the subjects in the experimental group were able to attend to lessons for a longer period of time than those in the control group. Again, this was not statistically shown, but if true, it would concur with learning theory which holds that children's attention span is increased with interest and personal involvement and activity.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. Banks, J. Houston. *Learning and Teaching Arithmetic*. Boston: Allyn and Bacon, Inc., 1964.

2. Blommers, Paul and E. F. Lindquist. *Elementary Statistical Methods*. Boston: Houghton Mifflin Company, 1960.

3. Brueckner, Leo J., Foster E. Grossnickle, and John Reckzeh. *Developing Mathematical Understandings in the Upper Grades*. Philadelphia: The John C. Winston Company, 1957.

4. Cohen, Louis S. and David C. Johnson. "Some Thoughts About Problem Solving," *The Arithmetic Teacher*, 14:261-62, April, 1967.

5. D'Augustine, Charles H. *Multiple Methods of Teaching Mathematics in the Elementary School*. New York: Harper and Row, Publishers, 1968.

6. Dienes, Z. P. *Building Up Mathematics*. London: Hutchinson Educational Ltd., 1961.

7. Fehr, Howard F. and Jo McKeeby Phillips. *Teaching Modern Mathematics in the Elementary School*. Palo Alto: Addison-Wesley Publishing Company, 1967.

8. Gattegno, Caleb. *From Actions to Operations*. New York: Cuisenaire Company of America, 1959.

9. Glennon, Vincent J. "Research Needs in Elementary School Mathematics Education," *The Arithmetic Teacher*, 13:363-368, May, 1966.

10. Howard, Charles F. and Enoch Dumas. *Teaching Contemporary Mathematics in the Elementary School*. New York: Harper and Row, Publishers, 1966.

11. Howe, Hartley, "Abacus," *Collier's Encyclopedia* (17th ed.), I, 4-6.

12. Hungerman, Ann D. "Achievement and Attitude of Sixth-Grade Pupils in Conventional and Contemporary Mathematics Programs," *The Arithmetic Teacher*, 14:30-39, January, 1967.

13. Johnson, Donovan A. and Gerald R. Rising. Guidelines For Teaching Mathematics. Belmont: Wadsworth Publishing Company, 1967.

14. Kennedy, Leonard M. Models for Mathematics in the Elementary School. Belmont: Wadsworth Publishing Company, 1967.

15. Kramer, Klaas. The Teaching of Elementary School Mathematics. Boston: Allyn and Bacon, Inc., 1966.

16. Lerch, Harold H. and Charles T. Mangrum II. "Instructional Aids Suggested by Textbook Series," The Arithmetic Teacher, 12:543-46, November, 1965.

17. Marks, John L., C. Richard Purdy, and Lucien B. Kinney. Teaching Elementary School Mathematics for Understanding. New York: McGraw-Hill Book Company, 1965.

18. The National Council of Teachers of Mathematics. The Growth of Mathematical Ideas. Washington, D.C.: Twenty-Fourth Yearbook of the National Council of Teachers of Mathematics, 1959.

19. Rappaport, David. Understanding and Teaching Elementary School Mathematics. New York: John Wiley and Sons, Inc., 1966.

20. Schaaf, William L. Basic Concepts of Elementary Mathematics. New York: John Wiley and Sons, Inc., 1965.

21. Scott, Lloyd. Trends in Elementary School Mathematics. Chicago: Rand McNally and Company, 1966.

22. Spitzer, Herbert Frederick. Teaching Elementary School Mathematics. Boston: Houghton Mifflin Company, 1967.

23. Van Engen, Henry, Maurice L. Hartung, and James E. Stochl. Foundations of Elementary School Arithmetic. Chicago: Scott-Foresman Company, 1965.

24. Weber, Evelyn and Sylvia Sunderlin (ed.). New Directions in Mathematics. Washington: Association for Childhood Education International, 1965.

25. Westcott, Alvin M. and James A. Smith. Creative Teaching of Mathematics in the Elementary School. Boston: Allyn and Bacon, Inc., 1967.

APPENDIX A

LIST OF AIDS USED ONLY WITH THE EXPERIMENTAL GROUP

Abacus
Abacus sheets (individual)
Charts, thirty-eight (see samples in Appendix C)
Clocks and watches (individually constructed)
Coin and money kits (individually constructed)
Coin kit for flannel board
Coins (real)
Counting and grouping objects (many kinds)
Cuisenaire rods
Facts-family cards
Felt board
Fraction kit for flannel board
Fraction kits (individually constructed)
Fraction paper-folding kits (individual)
Fraction rods (individual)
Geoboards
Hundred-peg board
Hundreds pocket charts (individual)
Magazine pictures (brought by children to illustrate story
    problems)
Markers (individual)
Measuring instruments
    Cups (graduated measuring)
    Pints
    Quarts
    Half-gallons
    Gallons
    Measuring spoons
    Thermometers (Centigrade, Fahrenheit, and paper-and-ribbon)
    Various shapes and sizes of jars (for estimating volume)
Money (paper bills)
Multiplication wheels
Number lines for desks
Overhead transparencies (with matching worksheets for children)
Place-value boxes and paper strips (individual)
Place-value frames (individual
Plastic boxes and BB shot to show place value on overhead
Regrouping frames (individual)
Related-facts cards
Tally boxes (individual)
Tally sheets (individual)

APPENDIX B

TABLE II

RAW DATA USED FOR PAIRING SUBJECTS

Pair	Metropolitan Score	STAT Score	Sex	Age	Previous Retention
1	67.5	85	M	9-7	No
	61.5	84	M	9-2	No
2	62.0	78	F	9-7	No
	61.0	77	M	9-7	No
3	56.0	84	F	9-5	No
	55.5	86	F	9-3	No
4	55.5	78	F	9-11	No
	55.0	77	M	10-0	No
5	54.5	83	F	9-5	No
	54.0	85	F	9-1	No
6	54.5	77	M	9-4	No
	53.5	71	M	9-5	No
7	53.5	65	F	9-5	No
	53.0	75	F	9-8	No
8	52.5	78	M	9-4	No
	52.0	76	F	9-2	No

TABLE II (continued)

Pair	Metropolitan Score	STAT Score	Sex	Age	Previous Retention
9	51.5	78	F	9-9	No
	51.5	72	M	9-9	No
10	52.0	54	M	10-9	Yes
	51.0	-	F	9-1	No
11	51.0	73	M	9-3	No
	51.0	77	F	9-9	No
12	50.5	34	M	10-0	Yes
	50.0	68	M	10-8	Yes
13	50.5	72	F	9-7	No
	50.0	74	F	9-4	No
14	49.5	69	M	9-7	No
	49.0	77	M	9-5	No
15	49.5	82	F	9-7	No
	49.0	78	F	8-11	No
16	49.0	68	F	9-8	No
	49.0	68	F	9-2	No

TABLE II (continued)

Pair	Metropolitan Score	STAT Score	Sex	Age	Previous Retention
17	48.5	70	M	10-8	No
	47.5	72	M	9-4	No
18	47.5	-	F	9-8	No
	47.5	68	F	9-9	No
19	47.0	54	F	9-6	No
	45.5	57	F	9-5	No
20	46.0	-	M	9-10	No
	46.0	77	M	9-4	No
21	45.5	64	M	9-6	No
	45.0	61	M	9-8	No
22	45.0	57	M	9-7	No
	45.0	60	F	9-10	No
23	45.0	20	M	10-9	Yes
	44.0	-	F	9-0	No
24	43.5	64	F	9-3	No
	43.5	67	F	9-2	No

TABLE II (continued)

Pair	Metropolitan Score	STAT Score	Sex	Age	Previous Retention
25	43.5	72	M	9-10	No
	43.0	73	M	10-2	No
26	43.5	77	F	10-0	No
	43.0	69	F	9-9	No
27	42.0	52	F	9-10	No
	38.0	58	F	9-10	No

TABLE III

NET GAIN IN POINTS FOR THE TOTAL SCORE OF THE TEST

Pupil	Experimental Group	Control Group
1	12.5	8.0
2	14.0	14.5
3	15.0	21.0
4	16.0	14.5
5	17.5	15.0
6	20.0	4.0
7	13.0	14.0
8	22.0	19.0
9	20.5	28.5
10	14.5	22.0
11	19.0	17.0
12	7.5	15.0
13	17.5	24.0
14	13.0	26.5
15	24.5	22.5
16	19.0	14.0
17	14.0	15.0
18	19.0	13.5
19	11.0	20.5
20	25.0	19.5
21	23.5	18.0
22	14.0	16.5
23	25.0	9.5
24	12.5	20.0

TABLE III (continued)

Pupil	Experimental Group	Control Group
25	30.5	22.0
26	15.5	23.0
27	12.0	14.0
	$N = 27$	$N = 27$
	$\Sigma X = 467.5$	$\Sigma X = 471.0$
	$\Sigma X^2 = 8{,}823.75$	$\Sigma X^2 = 9{,}017.50$
	$\bar{X} = 17.31$	$\bar{X} = 17.44$
	$s^2 = 27.16$	$s^2 = 29.83$

t-score = .0878

TABLE IV

NET GAIN IN POINTS FOR THE COMPUTATION SECTION OF THE TEST

Pupil	Experimental Group	Control Group
1	25	23
2	28	18
3	26	24
4	23	21
5	23	22
6	23	3
7	16	25
8	27	31
9	27	33
10	13	28
11	29	26
12	9	14
13	22	29
14	25	30
15	26	33
16	22	16
17	21	19
18	27	18
19	17	25
20	26	27
21	23	21
22	19	23
23	31	16

TABLE IV (continued)

Pupil	Experimental Group	Control Group
24	15	28
25	39	30
26	22	30
27	21	24
	$N = 27$	$N = 27$
	$\Sigma X = 625$	$\Sigma X = 637$
	$\Sigma X^2 = 15{,}403$	$\Sigma X^2 = 16{,}205$
	$\bar{X} = 23.14$	$\bar{X} = 23.59$
	$s^2 = 35.02$	$s^2 = 43.69$

t-score = .26

TABLE V

NET GAIN IN POINTS FOR THE PROBLEM SOLVING SCORE OF THE TEST

Pupil	Experimental Group	Control Group
1	0	- 7
2	0	11
3	14	18
4	9	8
5	12	8
6	17	5
7	10	3
8	17	7
9	14	24
10	16	16
11	9	8
12	6	16
13	13	19
14	21	23
15	23	12
16	16	12
17	7	11
18	11	9
19	5	16
20	24	12
21	24	15
22	9	10

TABLE V (continued)

Pupil	Experimental Group	Control Group
23	19	3
24	10	13
25	22	14
26	9	16
27	3	4
	$N = 27$ $\Sigma X = 340$ $\Sigma X^2 = 5,526$ $\bar{X} = 12.59$ $s^2 = 46.16$	$N = 27$ $\Sigma X = 306$ $\Sigma X^2 = 4,608$ $\bar{X} = 11.33$ $s^2 = 42.30$

t-score = 2.43

APPENDIX C

Learn to Use the Right Word

$56 \div 7 = \boxed{8}$

dividend

divisor

quotient

$\boxed{8}$ → quotient

divisor ← $7\overline{)56}$

dividend

"Monkey Bar" Division

8 | 509	
−160	20
349	
−320	40
29	
24	3
r 5	63

Steps:

1. Estimate
2. Multiply
3. Subtract

Repeat steps 1-3 until the remainder is smaller than the divisor.

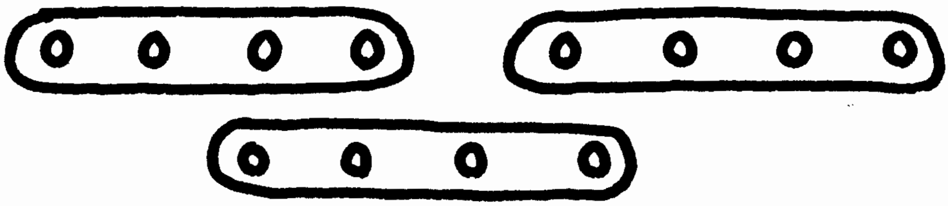
Division

Finding How Many Groups

- The Question:

  "How many 4¢ stamps can you buy for 12¢?"

- You can draw the division.



- You can write it this way: $12 \div 4 = 3$ or $4\overline{)12} = 3$

MAKING CHANGE

\$10.00	money given
−7.50	cost of item
\$ $\boxed{2.50}$	change

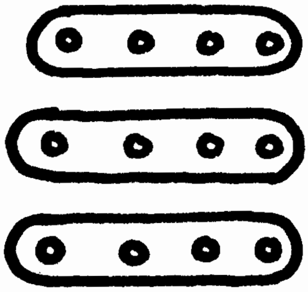
Equation:

$$\$10.00 - \$7.50 = \square$$

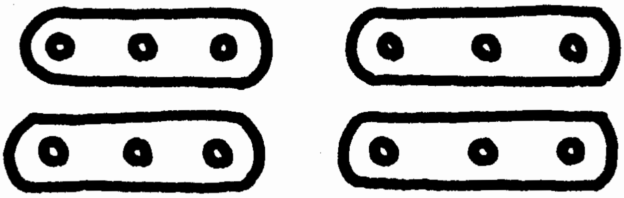
Reversing the Facts

$3 \times 4 = 4 \times 3$

$3 \times 4 =$



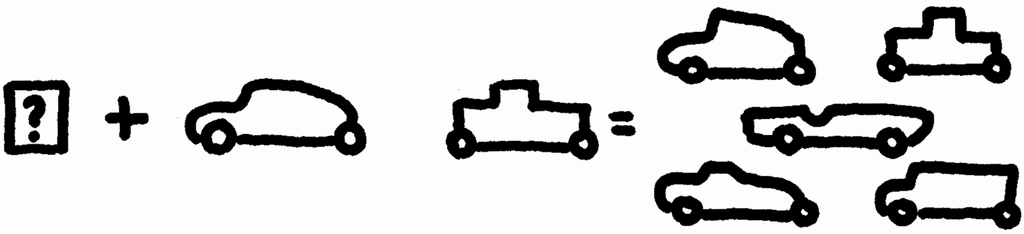
$4 \times 3 =$



3×4 means – three groups with four in each group.

4×3 means – four groups with three in each group.

Daryl got 2 new cars. Then he had 5. How many did he have at first ?



Equation:

$\square + 2 = 5$ Example: $\begin{array}{r} 5 \\ -2 \\ \hline \boxed{3} \end{array}$ cars

Equations & Examples

Addition:

1. EQ: $a+b=\square$ EQ: $\begin{array}{r} a \\ +b \\ \hline \boxed{c} \end{array}$

2. EQ: $a+\square=c$ EQ: $\begin{array}{r} c \\ -a \\ \hline \boxed{b} \end{array}$

3. EQ: $\square+b=c$ EQ: $\begin{array}{r} c \\ -b \\ \hline a \end{array}$

Use the Right Word

$$\begin{array}{rl} 9 & \text{multiplicand} \\ \times\, 3 & \text{multiplier} \\ \hline 27 & \text{product} \end{array}$$

$3 \times 9 = 27$

multiplier

multiplicand

product