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A Comparison of Two Methods of Teaching Place Value

Bruce Alan Pennington
Central Washington University

A COMPARISON OF TWO METHODS OF TEACHING PLACE VALUE

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Bruce Alan Pennington

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APPROVED FOR THE GRADUATE FACULTY

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Daryl Basler, COMMITTEE CHAIRMAN

_____

Frank B. Nelson

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George C. Grossman

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CHAPTER I

THE PROBLEM AND DEFINITIONS OF TERMS USED

Number base systems other than base ten have been incorporated into many contemporary arithmetic texts. Teachers have been trained in the "new" mathematics and many schools of the state of Washington and the nation have adopted new mathematics programs. Many of the textbook companies have incorporated materials and recommendations of the School Mathematics Study Group.

In the past, many teachers and most texts began the arithmetic year with a brief presentation of place value of number. Some programs followed through with the idea of place value when involving the student in the four basic operations.

Several of the most recent arithmetic texts introduce base number systems to teach or reinforce the learning of place value. The authors of some texts, including the Holt, Rinehart, Winston series, state that place value may be taught by the use of number base systems.

Many comparative studies in arithmetic have been made in the recent past. A great number of these studies were made to compare the effectiveness of the more modern texts to the older and more traditional texts found in the classroom eight or ten years ago. Some of the research has been done on the teaching of the basic skills.

Do students learn more effectively through the newer mathematics? Just how much more effectively is place value taught through the use of other number base systems?

I. THE PROBLEM

It is the purpose of this study to determine how children who have been taught place value of number by number base systems on the fifth grade level achieve on two standardized achievement tests and a teacher-made test as compared to those children who have been taught using several ideas and techniques of the traditional approach.

The area of research will be toward the following questions:

1. Under which of the two methods do children show a higher level of understanding of place value?
2. Which of the two methods affords greater increase in level of understanding of place value for the student in the lower one-third of his class?
3. Which of the two methods affords greater increase in level of understanding of place value for the student in the middle one-third

of his class?

4. Which of the two methods affords greater increase in level of understanding of place value for the student in the upper one-third of his class?

Hypothesis. No statistical significant difference would be found in understanding of place value of number between children using number base systems approach and children using traditional arithmetic materials.

II. DEFINITIONS OF TERMS USED

Experimental Group. The arithmetic class that was taught place value through the use of number base systems was designated as the experimental group. This class used the text, Moving Ahead in Arithmetic, by J. Leo Brueckner, Elda L. Merton, and Foster E. Grossnickle which was published by Holt, Rinehart, and Winston, Inc. in 1963. This program presents some ideas that were formerly reserved for advanced mathematics.

Control Group. The arithmetic class that was taught place value of number through the use of the more traditional approach was designated the control group. This class used the text, The New Exploring Numbers, by J. Leo Brueckner, Elda L. Merton, and Foster

E. Grossnickle which was published by the John C. Winston Company in 1956.

Traditional Approach. This approach is represented by the text used by the control group.

Teacher-Made Test and Materials. Since the scope of this study was limited to the teaching of place value, it was the contention of the researcher that a standardized arithmetic test would possibly not measure adequately or fairly the topic under consideration. Since a more precise measuring device was needed, a fifty question test was devised by the researcher to measure the understanding by the students of place value. Also, during the experimental period, lessons were drawn from many different texts on the teaching of place value.

S. C. A. T. The School and College Ability Test is published by the Cooperative Test Division, Educational Testing Service, Princeton, New Jersey.

S. T. E. P. The Sequential Test of Educational Progress is published by the Cooperative Test Division, Educational Testing Service, Princeton, New Jersey.

California Achievement Test. This test was devised by Ernest W. Teigs and Willis W. Clark and is published by the California Test Bureau, Del Monte Research Park, Monterey, California.

CHAPTER II

REVIEW OF THE LITERATURE

Literature concerning the teaching of place value reveals methods with a similar approach among authorities. The most unique approach to the teaching of place value was that of using a metric ruler (1:55).

Authorities agree on the importance of teachers having a knowledge of the history of the Hindu-Arabic number system (13:91).

This review of the literature will present a brief history of the Hindu-Arabic number system and the prevalent methods and procedures used to teach place value by traditional methods used ten years previously as compared to the more modern or recent methods used in texts of today.

I. HISTORY OF THE HINDU-ARABIC NUMBER SYSTEM

The Hindu-Arabic number system is one of many systems that has been invented by man. Our number system has evolved over a period of many hundreds of years which dates back into the time of unwritten history.

Since number is an organized system of thinking, many authorities believe that the need for numbers grew out of the desire and need for people to count things and keep records of their possessions.

Brueckner and Grossnickle (1:27-28) have identified several stages in the evolution of numbers: (1) devising ways to find the number of objects in a group, (2) development of simple systems of counting, using words to identify numbers, (3) invention of simple methods of computation to reduce the time required for counting when large numbers are involved.

According to Swenson, (18:45-46) the Chinese, Egyptians, Greeks, Romans, and Myans invented number systems that used varying techniques for writing numbers. None of these systems, however, used the written symbols in computation. Instead, they resorted to mechanical devices such as the abacus.

Marks (11:65) relates the history of the Hindu-Arabic number system in a very brief but concise manner which is very similar to other authors:

> Long before the Romans began manipulation of their cumbersome system, the Hindus in India were developing a number system that, after refinements and a lengthy struggle for acceptance, has become generally used throughout the world. This Hindu-Arabic number system finally incorporated the desirable properties that had occurred in earlier systems.
>
> Originating with the Hindus, it soon was widely used by Arabic peoples at the eastern end of the Mediterranean. Sometime prior to the eleventh century, the symbol for zero and the nine other symbols were developed somewhat as we write them today. During the period of the Crusades, these numbers were introduced into Europe through contact with the Moors in Spain and the Arabic nations in the Near East. At this time there began a long, slow struggle in Europe for acceptance of

> the Hindu-Arabic number system, continuing until the sixteenth or seventeenth century. Finally, with symbols somewhat refined, it replaced the Roman system throughout Europe.

Marks (11:65-66) states the simple properties of the Hindu-Arabic number system: (1) ten basic symbols, (2) place value, (3) a base-ten system, and (4) additive properties.

Another interesting point brought out by Swenson (18:50) is that the inventor of zero provided the "missing link" to complete the positional number system we use today.

TRADITIONAL ARITHMETIC

Many arithmetic textbooks of the past stressed computation rather than the understanding of the number processes. All teachers do not follow the procedures recommended in the teacher's guide and many teachers have resorted to the drill method of teaching computations with the four basic processes (13:1). In general, there was a lag in what authorities recommended and what was actually put into practice by the classroom teacher.

The New Exploring Numbers shows pictures of bundles of sticks, written instructions about place value, and compares the Hindu-Arabic number system with the Roman number system. The authors recommend that the class manipulate single sticks, bundles of ten, and

bundles of hundreds to make the meanings concrete. The abacus and place value chart with tickets is also recommended (3:2,7,380).

Brueckner and Grossnickle (1:195) state, "If a resourceful teacher accepts the philosophy that a pupil will understand a process better when he uses manipulative materials in initial learnings than when he uses symbolic materials, she usually will be able to provide the necessary materials for class demonstrations."

Overman (17:100) recommends three devices for teaching place value: (1) the abacus or number frame, (2) toy money, (3) sticks and bundles of sticks. He also states that other concrete materials such as the magnetic board may be used with counters. Overman considers the sticks to be indispensible.

The book, Learning to Use Arithmetic, has one page with a place value chart. Place value is "brought out" in the wording of the text. Problems refer to the 1st, 2nd, 3rd, and 4th places, or the writing of 3 in the one's place or 1 in the hundred's place. This system of teaching place value depends upon the ability of the student to read (8:7-8).

III MODERN ARITHMETIC

Many of the latest versions of arithmetic text-

books may be identified by the word "mathematics" which is found in many of their titles. The more recent books usually contain more written instructions and directions for the pupil. Some of the pages of the texts contain only questions to stimulate the understanding of the mathematical concepts.

The authors do not discard all of the techniques of traditional arithmetic, but do attempt to reach a balance between computations and understanding of the mathematical concepts. Place value charts, the abacus, concrete materials, and bundles of sticks or other objects are still employed as teaching devices and are recommended in teacher's editions of classroom texts.

Several of the newer classroom texts use number base systems other than base ten. One text (21:19) uses groupings called "sets" to teach place value.

The authors of another book (21:19) use the idea of grouping with sets other than ten. Grouping by four's is used and is developed into a base four number system. Writing base four numerals for a given set of objects is used. Interest is aroused by presenting material on the Egyptian, Greek, East Arabic, and Roman number systems. The authors state,(21:20) "Material on base four is not vital background for future work. This is included to increase understanding of base ten and to

serve as a stimulation for the children. Mastery is not expected."

The authors of Discovering Mathematics 5 stress the understanding of numeration and place value. They use expanded notation, reading and writing of numbers, and a base five number system with a number line in teaching place value (7:246-251).

SUMMARY OF THE REVIEW OF THE LITERATURE

Since many authorities on the teaching of arithmetic agree that teachers should know the history of the Hindu-Arabic number system, a brief history of this number system was given in the review of the literature. The history reveals that the number system evolved in a logical sequence over a long period of time and that its acceptance was a slow process.

In the past, the traditional approach to arithmetic concentrated on computation with the four basic operations and little attention was given to the understanding of concepts. This usually developed into a drill program.

The modern mathematics for the elementary school was developed in response to the findings of the School Mathematics Study Group (15:vi). The leading principles of this group were the discovery method and learning for meaning.

CHAPTER III

PROCEDURES

During the 1967-68 school year, the writer, with the permission of the Superintendent of Royal School District, conducted an experimental study of a comparison of teaching place value of number using number base systems with that of the traditional method. The cooperation of Mr. Jule Crabtree, the other fifth grade teacher, was solicited. The experimental study was designed to test the hypothesis that there is no significant difference in the understanding of place value as developed from the use of number base systems contrasted to the use of the traditional method.

The subjects were matched as closely as possible, using the scores from the arithmetic section of the School and College Ability Test given the previous year. The score and name of each pupil was then written on a slip of paper. The slips of paper were then grouped into stacks of equal scores. The subjects were then paired by chance drawing from each stack.

When the pairing was completed, two equivalent groups of subjects were formed. The group to be taught by the traditional method was designated as the control group, while the subjects to be instructed using number base systems formed the experimental group.

After the two equivalent groups were formed, each group's scores were further subdivided into three groups consisting of the upper third, the middle third, and the lower third according to their scores on the arithmetic section of the S. C. A. T. These sub-groupings were used only for the purpose of statistical analysis.

While the experimental group of twenty-four students was being instructed for one hour in arithmetic by the experimenter, the control group of twenty-five students had reading in the other fifth-grade classroom.

At the end of the period, the subjects returned to their regular classrooms. After recess, the experimenter taught the control group for one hour while the experimental group had reading in the other fifth-grade room.

The New Exploring Numbers was used in instructing the control group while the experimental group used the Holt, Rinehart, and Winston, Moving Ahead in Arithmetic. These books are by the same authors and most of the printed pages are the same. Moving Ahead in Arithmetic contains some of the ideas presented in Chapter II, but neither text contains any material on number base systems other than base ten. These texts were used for review of word problems and the four basic fundamentals prior to presentation of the experimental materials. The texts were used for the remainder of the school year after the

experimental materials had been presented.

The control group and the experimental group were taught place value for a period of three weeks. The lessons for the experimental group were prepared by the writer using ideas from several texts and the knowledge gained from courses in modern elementary arithmetic. The writer had taught number base systems to sixth-grade pupils the previous year.

At the beginning of the three week experimental period, both groups reviewed place value by the use of place value charts and writing the names for numbers. Many of the same techniques were used from this point on.

Each group was introduced to tallying by suggesting that this was probably the way early man grouped objects in counting. Tallying in a one to one ratio was done by the students. This developed into grouping tally marks into groups of fives and tens and a comparison was made with the Roman number system.

The control group was led into the development of a base ten system by using tally marks and the idea of progressively larger groupings of ten. This same process was used with the experimental group to develop a base five system.

The experimental group used the five symbols of the base five system and converted base ten numbers to the base five system. Practice in counting in base

five was done by writing the numbers from 1 to 100. Instruction and practice was also given in converting base five numbers to base ten numbers.

A base five place value chart and an abacus were used by teacher to further clarify the idea of place value in a base five system. Students used small rectangular strips of construction paper to form groups of five. The idea of place value was stressed when each larger group was formed.

A base ten place value chart and an abacus were used with the control group. They were given practice in grouping by ten by using strips of constrcution paper.

At the end of the three week experimental period, the Sequential Test of Educational Progress and a teacher-made test were administered to the subjects of both groups.

The subjects were tested again after an elapse of two weeks in which no stress was placed on the teaching or learning of place value.

Two months after the second test, the subjects were re-tested using the S. T. E. P., the teacher-made test and the arithmetic section of the California Achievement Test.

Only the results of the final three tests were used in the statistical analysis. Since the groups

were evenly matched, both groups had the opportunity of learning from the tests given previously. The California Achievement Test was given in order to have at least one test to which the subjects had not been previously exposed.

The experimenter taught two matched groups in an experimental study of the teaching of place value of number by number base systems as compared to a traditional approach. The subjects were tested after an interval of two and one-half months and statistical analysis were applied to the data.

CHAPTER IV

ANALYSIS OF DATA

In order to answer the questions listed in Chapter I, the collected data were analyzed through the application of the t-test to determine statistically significant differences which might have existed between the experimental and control groups. All statistical findings are reported at the .05 level of confidence.

Two and one-half months following the teaching of place value by the two methods, as described in Chapter III, achievement tests were administered to the subjects to determine educational progress. These tests were given after a relatively long interval to check retention of understanding.

TABLE I

MEAN DIFFERENCES FOR
SEQUENTIAL TEST OF EDUCATIONAL PROGRESS, FORM 4A

Group	N	Obtained Means	S^2	S.E.	Obtained t	Required t
Experimental Group	24	32.3	54	3.22	1.056	2.020
Control Group	25	28.9	95			

Table I presents the difference between mean scores

on the arithmetic section of the Sequential Test of Educational Progress, Form 4A, administered on January 8, 1968.

It may be seen on Table I, page 16, that the obtained t of 1.056 was not found to be statistically significant. However, the experimental group had the higher mean score.

Table II, page 18, shows the mean scores for the upper one-third, the middle one-third, and the lower one-third of the subjects tested on the S. T. E. P. It may be seen from this table that the lower one-third of the experimental group scored statistically higher than the lower one-third of the control group.

Table III, page 19, shows the mean scores for the subjects tested on the teacher-made test. It may be seen from this table that the experimental group had the highest mean score.

Table IV, page 20, shows the mean scores for the upper one-third, the middle one-third, and the lower one-third of the subjects tested on the teacher-made test. It may be seen from this table that the lower one-third of the experimental group had a statistically significant mean score. The other sub-groups of the experimental group had higher mean scores than the control group.

MEAN DIFFERENCES FOR THE SUB-GROUPS
ON THE S. T. E. P., FORM 4A

Level	Group	N	$\bar{X}$	S	S.E.	Obtained t	Required t
Upper	Experimental	7	39.28	13.4	2.374	.013	2.160
	Control	8	39.25	22.5			
Middle	Experimental	8	31.75	17.1	3.558	1.040	2.145
	Control	8	28.12	71.5			
Lower	Experimental	8	26.5	4.7	2.007	3.538	2.145
	Control	8	19.4	23.5			

TABLE III

MEAN DIFFERENCES FOR THE TEACHER-MADE TEST

Group	N	$\bar{X}$	S	S.E.	Obtained t	Required t
Experimental	24	36.8	39	2.49	1.690	2.021
Control	25	32.6	105			

TABLE IV

MEAN DIFFERENCES FOR THE SUB-GROUPS
ON THE TEACHER-MADE TEST

Level	Group	N	$\bar{X}$	S	S.E.	Obtained t	Required t
Upper	Experimental	7	42.3	17	3.330	.510	2.160
	Control	8	40.6	63.2			
Middle	Experimental	8	38.4	5.8	4.357	1.331	2.145
	Control	8	32.5	24.7			
Lower	Experimental	8	31.4	20.6	3.500	2.429	2.145
	Control	8	22.9	65.5			

Table V, page 22, shows the mean scores for the subjects tested on the California Achievement Test, Form W. It may be seen that there was no statistically significant difference between the means. The experimental group had the higher mean score.

Table VI, page 23 shows the mean scores for the upper one-third, the middle one-third, and the lower one-third of the subjects tested on the California Achievement Test, Form W. The lower one-third of the experimental group excelled the lower one-third of the control group.

After examination of the six tables of data, it may be concluded that the lower one-third of the experimental group showed statistically significant higher scores on all tests given. The experimental group had slightly higher mean scores on all tests with the exception of the California Achievement Test, Form W.

TABLE V

MEAN DIFFERENCES FOR THE CALIFORNIA ACHIEVEMENT TEST, FORM W

Group	N	$\bar{X}$	S	S.E.	Obtained t	Required t
Experimental	23	65.4	77.2	3.72	.743	2.021
Control	24	62.6	231.3			

TABLE VI

MEAN DIFFERENCES FOR THE SUB-GROUPS ON THE CALIFORNIA ACHIEVEMENT TEST, FORM W

Level	Group	N	$\bar{X}$	S	S.E.	Obtained t	Required t
Upper	Experimental	7	71.1	36.8	5.166	-.553	2.160
	Control	8	74.0	129.8			
Middle	Experimental	8	66.3	72.4	4.740	-.079	2.145
	Control	8	66.6	85.0			
Lower	Experimental	7	60.3	50.8	5.920	2.229	2.160
	Control	8	47.1	168.0			

CHAPTER V

SUMMARY AND CONCLUSIONS

The purposes of this chapter are to summarize the findings of this study, to draw some conclusions, and to make some recommendations based on the results of the research. The initial chapter presented the problem and introduced the study. Chapter II provided a digest of related literature. The two chapters immediately preceding Chapter V reported the procedures followed and the findings.

I. SUMMARY

The purpose of this study was to determine how children who had been taught place value of number by number base systems on the fifth grade level achieved on two standardized tests and a teacher-made test as compared to those children who had been taught place value of number by the traditional approach, represented by The New Exploring Numbers.

Two fifth grade classes of Red Rock Elementary School, Royal City, Washington were involved in this research. These two classes were divided into two equivalent groups by using the scores from the School

and College Ability Test given the previous year. All of the students participating in the study were given Form W of the California Achievement Test, Form A of the Sequential Test of Educational Progress, and a teacher-made test at the end of two and one-half months. The comparisons of the results of these three tests make up the findings of this study.

The null hypothesis was retained with the following exception: the lower one-third of the subjects who had been taught place value by number base systems showed significantly higher achievement scores on all tests administered.

II. CONCLUSIONS

Although the statistical analysis as applied to the two groups as a whole do not conclusively indicate that either method of teaching place value of number used in this study was significantly more effective than the other, the findings do support several noteworthy conclusions:

1. In every instance the lower one-third of the students of the experimental group scored significantly higher than the lower one-third of the control group.

2. For the arithmetic groups as a whole, there was no significant difference between the methods used to teach place value.
3. The experimental group showed a trend of obtaining higher mean scores.

III. RECOMMENDATIONS

1. Number base systems should be used in teaching place value to low achievers in arithmetic.
2. Number base systems should be used in teaching all arithmetic students since the experimental group set a trend of obtaining higher scores.
3. The experimenter feels that experimental studies should be carried on in an atmosphere as closely related to the regular classroom atmosphere as possible. Too many controls would seem to invalidate the conclusions as they may possibly be applied to the general classroom.

BIBLIOGRAPHY

1. Brueckner, Leo J., and Foster E. Grossnickle. Making Arithmetic Meaningful. Philadelphia: The John C. Winston Company, 1953.

2. _____, Elda L. Merton, and Foster E. Grossnickle. Moving Ahead in Arithmetic. Teacher's Edition, Book 5. New York: Holt, Rinehart and Winston, Inc., 1963.

3. _____, Elda L. Merton, and Foster E. Grossnickle. The New Exploring Numbers. Teacher's Edition. Philadelphia: The John C. Winston Company, 1956.

4. Buswell, Guy T., William A. Brownell, and Irene Sauble. Teaching Arithmetic We Need. Boston: Ginn and Company, 1961.

5. Daverin, Austa L. "A Comparative Study of the Traditional and the Scott, Foresman Approaches to the Teaching of Addition and Subtraction of Fractions." Unpublished Master's thesis, Central Washington State College, Ellensburg, 1962.

6. Deans, Edwina. Learning Mathematics. Teacher's Edition. New York: American Book Company, 1966.

7. De Vault, M. Vere. Discovering Mathematics. Teacher's Edition, Book 5. Columbus: E. Merrill Books, Inc., 1966.

8. Fehr, Howard F. and others. Learning to Use Arithmetic. Teacher's Edition. Boston: D. C. Heath and Company, 1962.

9. Grossnickle, Foster E., and Leo J. Brueckner. Discovering Meanings in Arithmetic. Philadelphia: The John C. Winston Company, 1959.

10. Hartung, Maurice L. Seeing Through Arithmetic. Chicago: Scott, Foresman and Company, 1961.

11. Marks, John L., C. Richard Purdy, and Lucien B. Kinney. Teaching Arithmetic for Understanding. New York: Mc Graw-Hill Book Company, Inc., 1958.

12. Mc Swain, E. T. Arithmetic 5. River Forest, Illinois: Laidlaw Brothers, 1965.

13. Morton, Robert Lee. Teaching Arithmetic in the Elementary School, Intermediate. New York: Silver Burdett Company, 1938.

14. _____, and others. Modern Arithmetic Through Discovery. Book 5. Morristown: Silver Burdett Company, 1963.

15. _____, and others. Modern Arithmetic Through Discovery. Teacher's Edition, Book 6. Morristown: Silver Burdett Company, 1963.

16. Pflueger, Evalyn B. "A Comparative Study of Two Methods of Teaching Arithmetic." Unpublished Master's thesis, Central Washington State College, Ellensburg, 1962.

17. Overman, James Robert. The Teaching of Arithmetic. Chicago: Lyons and Carnahan, 1961.

18. Swenson, Esther J. Teaching Arithmetic to Children. New York: The Macmillan Company, 1964.

19. Upton, Clifford B. and Kenneth G. Fuller. Achieving Goals, American Arithmetic. Second Edition. New York: American Book Company, 1963.

20. Van Dalen, Deobold B. Understanding Educational Research. New York: Mc Graw-Hill, Inc., 1966.

21. Addison-Wesley Publishing Company, Inc. Elementary School Mathematics. Book 5. Reading: Addison-Wesley Publishing Company, Inc., 1964.