The Relative Effects of Immediate and Delayed Reinforcement on a Programed Learning Task

A. Lee Parks
Central Washington University

Follow this and additional works at: https://digitalcommons.cwu.edu/etd
Part of the Educational Assessment, Evaluation, and Research Commons, and the Educational Methods Commons

Recommended Citation
https://digitalcommons.cwu.edu/etd/981

This Thesis is brought to you for free and open access by the Master's Theses at ScholarWorks@CWU. It has been accepted for inclusion in All Master's Theses by an authorized administrator of ScholarWorks@CWU. For more information, please contact pingfu@cwu.edu.
THE RELATIVE EFFECTS OF IMMEDIATE AND DELAYED REINFORCEMENT ON A PROGRAMMED LEARNING TASK

A Thesis
Presented to
the Faculty of the Department of Psychology
Central Washington State College

In Partial Fulfillment
of the Requirements for the Degree
Master of Education

by
A. Lee Parks
April 1967
APPROVED FOR THE GRADUATE FACULTY

_____________________________________
Theodor F. Naumann, COMMITTEE CHAIRMAN

_____________________________________
Eldon E. Jacobsen

_____________________________________
H. Robinson
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. THE PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>The Problem</td>
<td>1</td>
</tr>
<tr>
<td>Statement of the problem</td>
<td>1</td>
</tr>
<tr>
<td>II. REVIEW OF THE LITERATURE</td>
<td>2</td>
</tr>
<tr>
<td>General Hypothesis</td>
<td>15</td>
</tr>
<tr>
<td>Specific Hypothesis</td>
<td>16</td>
</tr>
<tr>
<td>III. METHOD</td>
<td>17</td>
</tr>
<tr>
<td>Subjects</td>
<td>17</td>
</tr>
<tr>
<td>Experimental Design</td>
<td>17</td>
</tr>
<tr>
<td>Materials</td>
<td>18</td>
</tr>
<tr>
<td>Procedure</td>
<td>19</td>
</tr>
<tr>
<td>IV. RESULTS</td>
<td>21</td>
</tr>
<tr>
<td>V. DISCUSSION</td>
<td>29</td>
</tr>
<tr>
<td>VI. SUMMARY</td>
<td>38</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>42</td>
</tr>
<tr>
<td>APPENDIX</td>
<td>46</td>
</tr>
<tr>
<td>TABLE</td>
<td>PAGE</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>1. Summary of Data for Treatment Groups</td>
<td>21</td>
</tr>
<tr>
<td>2. Analysis of Variance for Groups</td>
<td>21</td>
</tr>
<tr>
<td>3. Summary of Data for 24 and 48 hour Post-test Treatment Groups</td>
<td>23</td>
</tr>
<tr>
<td>4. Analysis of Variance for 24 and 48 hour Groups</td>
<td>23</td>
</tr>
<tr>
<td>5. Summary of Data for Subjects Previously Achieving High and Low Reading Vocabulary Scores</td>
<td>25</td>
</tr>
<tr>
<td>6. Analysis of Variance for Data in Table 5</td>
<td>26</td>
</tr>
<tr>
<td>7. Summary of Data for Subjects Previously Achieving High and Low Reading Comprehension Scores</td>
<td>26</td>
</tr>
<tr>
<td>8. Analysis of Variance for Data in Table 7</td>
<td>27</td>
</tr>
<tr>
<td>9. Summary of Data for Errors on Program</td>
<td>28</td>
</tr>
<tr>
<td>10. Analysis of Variance for Errors on Program</td>
<td>28</td>
</tr>
</tbody>
</table>
CHAPTER I

THE PROBLEM

For many years there has been a commonly held assumption that the effect of immediate reinforcement is superior to that of delayed reinforcement. On the basis of these results programed instruction employs the principle of immediate knowledge of results. Recently however, there have been some convincing claims which show that for short delays there may be little difference between these two types of temporal treatments. Furthermore, one investigator has found that retention of learning may be superior under delayed reinforcement. It would appear from some of the current research that the effects of these two variables may need to be studied more closely.

Statement of the problem. It was the purpose of this study (1) to compare the effects of immediate knowledge of results and delayed knowledge of results on a programed task to determine which, if either, would result in superior learning; and (2) to explore the effect of these two variables on the amount of retention of learning over time.
CHAPTER II

REVIEW OF THE LITERATURE

Since the first experimental study on the effects of delayed reinforcement by Hunter in 1913, subsequent studies in this area have been largely confined to the study of animals—rats being the primary subjects. In general the results of these studies tend to support the positions of several of the major learning theorists. The superiority of immediate reinforcement is consistent with the learning theories proposed by Thorndike (1931), E. R. Guthrie (1952), Clark Hull (1943), Tolman (1951), and B. F. Skinner (1953).

The bulk of the studies done with animals on the issue of delayed reinforcement on a variety of learning tasks support the contentions of the above-cited theorists. Using a skinner box situation, Harker (1956), Perin (1943), and Roberts (1930) found that delayed reinforcement, as compared with immediate reinforcement, decreased responding speed. Cogan, et al. (1961), Crum, et al. (1951), Fehrer (1956), and Logan, et al. (1956), using a straight alley with a goal box as the end, also found delay of reinforcement to decrease learning.

There are, however, animal studies concerned with
the effectiveness of a delay of reinforcement on learning which are contradictory to the above-cited findings. Watson (1917), one of the first to do studies in this area, found that a 30 second delay of reward had no effect on the acquisition of a digging response. Warden and Hass (1927), using a maze situation, found that a one or five minute delay of food reward did not increase errors or trials to mastery of the task. In a historical review of the literature on delay of reinforcement Renner (1964) summarized the results of several animal studies by saying that a constant delay of reinforcement will retard acquisition. Of course, there are many variables in all of the above studies which make each study subtly different from the others. However, it is not the purpose of this review to discuss these variations but rather it is simply to show a trend. Briefly stated, animal studies in general support the notion that immediate reinforcement is more beneficial to learning in lower animals. Hilgard and Marquis (1961) sum up the results of several animal studies as follows:

Evidence of many kinds indicate that responses which are followed by reward immediately are learned more rapidly than responses for which reward is delayed. . . . At the present time it seems unlikely that learning can take place at all with delays of more than a few seconds. . . .

Briggs (1964) states, "Of course, delay in reinforcement
retards learning in animals; often no learning occurs with delays exceeding a very few seconds."

Recently, however, a few experimenters have been studying the effects of delayed reinforcement on human learning. The same trend in results has been appearing in humans as with the lower animals. It has been shown by Greenspoon and Foreman (1956) that delay of knowledge of results (KR) exerts an appreciable effect upon performance of a motor task. This particular task was a line-drawing task. In a replication of the Greenspoon (1956) study, Bilodeau and Ryan (1960), using 0 seconds and 20 seconds delay of KR, gave support to the results of Greenspoon. Dyal (1966) found that delaying of KR results in an increased frequency of errors of the same type as the original response of a line-drawing task. The immediate KR results in a tendency to make errors in the direction opposite the original response bias. In an earlier study by Bilodeau and Bilodeau (1958) the results of five studies on the effect of delayed KR showed delay not to be a significant variable on performance. They suggest though that this was due to the simplicity of the task, no special interfering tasks were interpolated, and control of temporal variables which may have had confounding effects. They suggest that the intertrial interval rather than KR is the critical variable and that
better learning occurs with a shorter interval. Denney, et al. (1960) supports Bilodeau's (1958) position on the relatively greater importance of intertrial interval (post KR delay) over KR delay. Leavitt (1944) found retention for rotary-pursuit tasks better than nonsense syllable retention under immediate KR. This tends to suggest that verbal performance may not necessarily be facilitated by immediacy of KR. Angell and Troyer (1948) found that learning is significantly enhanced by immediate KR through the punch-card technique. This study did not control for novelty effect however. Sax (1960) had subjects pair nonsense syllables with complex Chinese characters under different reinforcement schedules. He used a green light as a reinforcer. The green light was thus roughly equal to KR. His results showed that as latency in the presentation of the reinforcement increased, there was a significant increase in the number of trials needed to reach learning criterion.

It has been these results on rats, human motor skills, and other simple forms of learning acquisition in men which have been used to support the claim that human learning is most effective when feedback of KR is immediate. It seems, however, in light of more current studies that there may be instances or types of learning which are not necessarily facilitated by immediate
feedback of results. There seems to be little doubt that rats and men respond similarly on simple learning tasks and conditioning situations. Humans (older humans at least) have the ability to regulate their present behavior with respect to past events and future expectations. In the lower animals the motivational effects of reward seem to be limited to the immediate present. As Brackbill, Wagner, and Wilson (1964) point out, when dealing with verbal learning of people beyond infancy, language is used to bridge gaps of time, enabling reinforcement to be effective several hours or days after the behavior as long as the giver of the reinforcement lets the person know for what it is he is being reinforced. Goldbeck and Campbell (1962) found that overt responding to moderately difficult material resulted in higher criterion scores on an immediate test than did any of the three modes (overt, covert, and reading responses) with easier material. It was supposed by these experimenters that the relatively longer period of time on the more difficult program was partially spent in "self-administered" delay of reinforcement, as well as in response latency and item-item delay, since the subjects could expose the feedback item whenever they wanted.

Landsman and Turkewitz (1962) reviewed the results of Greenspoon and Foreman (1956) and suggested that the
principle of immediate KR may not be as applicable to cognitive tasks. In this study subjects were to choose one of a pair of four digit numbers which was designated by the examiner as right. Subjects were required to go through all of the pairs (7) of digits until two successful trials were completed. One group received immediate feedback and the other received a six second delay after each response. It was found that delay had a significant decremental effect on learning. It was suggested that something other than the correct response was being reinforced for the delay group during the time that they waited to find out if their answer was correct. They stated that the effect of delay should be explored for tasks ranging from purely motor to purely cognitive.

In a discussion with B. F. Skinner (1965) the present experimenter asked if there might be types of learning, such as more cognitive tasks, which might proceed best with slight delays in reinforcement. Dr. Skinner stated that there is only one type of learning and immediate reinforcement is always best. The matter was not explored further.

Moore (1961) conducted three experiments, utilizing auto-instructional materials. The first two dealt with the assumption that information on the correctness of each response must be provided with a minimal delay. It
was found that KR groups did not differ significantly from non-KR groups. However, Moore did not check the effect of KR delay. In a testing situation (not a teaching one) Bierbaum (1965) found, using punch cards, that the immediate KR group did significantly worse than the no KR group. Here again, the delay factor was not investigated.

Crawford and Sturges (1963) did a series of four experiments involving factual material, nonsense material, and inductive generalization (one group receiving the correct answer; the other receiving a cue to the correct answer). The first two groups received the correct answer as the reward. Three groups were used for all four types of material—experimental with 24 hour delay, experimental with immediate reward, and a control group with no reinforcement. In none of the three groups, using four types of material, were any of the immediate reinforcement groups superior to the 24 hour delay. They found that for factual material the delay group was superior to the immediate reward group; for nonsense material both immediate and delayed reward were significantly greater than the control group but there was no difference between them; for the inductive generalization material (no cue) the delayed reward groups showed significantly greater learning than the immediate reward
group; for the inductive generalization material (cue provided) there was no significant difference between the amount of learning of both groups.

To this writer's knowledge there are virtually no studies done comparing immediate and delayed KR on a programed learning task. Leslie Briggs (1964) reports that at present he is checking the effects of delayed KR with a programed task. The most current and applicable sources which are closely related to this problem are five studies done by Brackbill, et al. Brackbill and Kappy (1962) used an apparatus and procedure which is essentially the same in all their following studies. The subject sits in front of a 14" x 42" upright piece of plywood which is divided into two columns. Each column contains the following parts from top to bottom: A lamp, a stimulus aperture, a marble aperture, a marble receptacle, and a lever. If the subject pressed the lever under the correct stimulus, then following the appropriate delay, the lamp flashed on, a loud buzz sounded, and a marble dropped into the receptacle. If the lever for the incorrect stimulus was pressed, then a click was heard and above the correct stimulus flashed on. The subject was then able (after the experiment) to get a pre-chosen toy with his marbles. These experimenters found that learning a series of discriminations under 0, 5, 10
second delays that mean number of trials to criterion and mean number of errors increased as the delay was increased. But these temporal effects were not significant. It was found, however, that retention was facilitated by delay during acquisition for a one day interval. The facilitation effect faded when retention was tested after eight days.

Brackbill, Bravos, and Stern (1962), using third grade boys and using a series of discriminations under 0, 5, and 10 second delay intervals, were tested for recognition and relearning one day or eight days after learning. Again delay facilitated retention for the one day interval but not for the eight day period. The difference between the groups for mean number of trials to criterion was not significant but there were significantly more acquisition errors for the 10 second delay group.

Brackbill, Isaacs, and Smelkinson (1962) say that Brackbill's et al. previous experiments used material of high familiarity. To check the delay-retention effect to other types of material, they used nonsense bigrams rather than names and pictures of common objects. Again the design, apparatus, and procedure were essentially the same. Neither the mean number of trials to criterion or the mean number of acquisition errors for the immediate
and 10 second delay group differed significantly. The results agreed with their previous studies, despite the change in learning material.

In a later study by Brackbill, Boblitt, Douglas, and Wagner (1963) it was found that retention was facilitated by delay. Studying the effects of an amplitude of responses, it was found that retention was facilitated by high amplitude of motor response, but not by amplitude of verbal response. There was no interaction between reinforcement delay and response amplitude.

In all of Brackbill's studies cited thus far, the learning material has been only roughly similar to that used in the classroom. The purpose of Brackbill's, et al. (1964) last experiment was to extend the generalizability and usefulness of the previous findings to education by using learning material that is more representative of elementary school material. The subjects in this experiment were asked to learn French words on the same apparatus which was used in all the previous experiments—a simulated teaching machine. One group received immediate feedback and the other received a 10 second delay in feedback. It was found, as in previous studies, that delayed feedback is as conducive to learning efficiency as is immediate feedback. Furthermore, delayed feedback was more effective for difficult material. The immediate
feedback group made 10.5 times as many errors in relearning the difficult items as they did relearning the easiest items. By contrast, the subjects in the delayed-feedback group retained the difficult items as well as the items which were easy to learn. Other things being equal, Brackbill, et al. (1964) feel that the more difficult the learning material, the more important it is for its retention that the material be learned under delayed rather than immediate feedback.

Despite the many studies which find evidence to the contrary, it appears that for human learning, delayed feedback may not reduce learning efficiency—especially on a programmed learning task. The relation between reinforcement, as reinforcement is defined in these studies, and knowledge of results will not be discussed since those who have generalized the results of animal studies to human learning situations do not in general consider the relation between the two. Knowledge of results shall be used synonymously with reinforcement. Since knowledge of results provides a motivation which enables the learner to learn then this knowledge of results is presumed reinforcing. McGuigan (1960) states this point thusly: Knowledge of results can be reinforcing if the learner's motivation is intrinsic to the task to be learned. Brackbill (1964) suggests that humans, other than small
children, are self-rewarding or self-motivating. This comes about once the child has developed sufficient language ability to span time cognitively.

Finally, the present study is not concerned with the issue of whether primary or secondary reinforcement is operating during the delay period. The issue here is only the effect of immediacy of knowledge of results on a programmed learning task in the usual classroom situation, letting whatever mediating responses which occur in this setting occur.

This study will also attempt to measure the effect of delay of results on retention. Other than the above-cited experiments by Brackbill, et al., there have been virtually no studies that have explored this relationship. Retention has been considered an important issue in the field of education. Learning without retention or using methods which reduce retention would be a very unfortunate practice indeed. Brackbill, et al., (1964) comments on this by saying:

The goal of education is not simply to teach students but to teach them so they stay taught—so that they may apply educationally the solution of more acquired skills and knowledge directly to life's problems or may transfer them to advanced educational problems. Educators and laymen alike would consider ridiculous any proposal to use an instrumental method known to maximize learning efficiency and to minimize retention.

While Brackbill used relearning as the measure of
retention, the present study will simply employ two post-test situations—one 24 hours after learning and the other 48 hours after learning. Thus, rather than measuring the amount of savings for relearning, this study will consider retention as the amount of retention on a post test from the original learning situation 48 hours earlier.

The Brackbill studies used a toy as the ultimate or final reward for doing well on the learning task. This was apparently done because of the relative young age of all the subjects involved. The present writer does not feel that this is a valid procedure to be used in determining the effects of delayed knowledge of results on a programed learning task since this is not the type of motivation provided in the typical classroom situation.

Another way in which this study will differ from those of Brackbill, et al., will be in actual learning material. All their studies used a method of stimulus pairing, ranging from relatively unfamiliar stimuli to familiar stimuli. This again is not typical to the programed-instruction approach. Brackbill's (1964) last study attempted to remedy this by using material which is more typical of that used in the usual classroom situation. But again, in this writer's opinion, Brackbill diverges from the methods used in programed learning. The material presented was, on any particular trial, two stimulus cards
with the same English word on each card plus a French word on each card—one of which was the correct equivalent of the English word. This procedure is simply an extension of Brackbill's previous studies—pairing of stimuli.

The usual programmed learning approach is to use the Socratic method of teaching by asking questions and the Cartesian method of analyzing a problem into its smallest parts and proceeding from the simple to the complex. Thus the student goes through a sequence of questions which lead him step by step to general principles of the material being studied or to specific skills. Brackbill's studies involved material which could merely be memorized and the parts of which were not logically related with complex principles.

It would seem that in order to test adequately the effects of delayed knowledge of results on a programed task, programed material should be used. This is what the present study proposes to do. It will also study the effects of a delay between frames on a self-instructional programed lesson.

**General hypothesis.** Delay of knowledge of results will have no significant effect on a programed learning task; however, retention will significantly be facilitated.
Specific hypotheses. (1) If 6th grade pupils are presented a programed learning task in mathematics and if knowledge of results is delayed for 10 seconds for one group and immediate for the other, the delay group will be found to retain significantly more than the immediate group. Delay of knowledge of results will facilitate retention. (2) If 6th grade pupils are presented a programed learning task in mathematics and if knowledge of results is delayed for 10 seconds, there will be no significant difference in the amount of learning between these subjects and those who receive immediate knowledge of results throughout the program.
CHAPTER III

METHOD

Subjects. The Ss used for this study were two 6th grade classes from the Ellensburg Schools. The total number of Ss was 56.

Experimental design. The Ss from each grade were randomly assigned to one of four learning condition groups. These were (1) immediate knowledge of results (KR), (2) delayed KR, (3) immediate KR who turn the timing mechanism before KR, and (4) immediate KR with a delay between frames.

The immediate KR group simply did the program in the traditional manner, following the instructions that were given in the original published form of the program. All Ss worked independently on their own program, going to the correct answer as soon as they had written down the answer of their choice.

The delayed KR group differed from the immediate in that they were instructed to wait 10 seconds after writing down the answer of their choice for each particular frame before going on to see what the correct answer was. This delay was accomplished by giving each S in the delayed KR group an "hour glass" with an amount of sand in it which would last for only 10 seconds. The original program
instructions were changed to accommodate this temporal modification.

In the immediate KR group that only turned the hour glass each S was given an "hour glass" with an undetermined amount of sand in each of these glasses. This group was given the same instructions except that they were asked to turn the hour glass after recording their answer for any particular frame. They were further instructed not to wait for any of the sand to drain down, but to look at the correct answer immediately.

The immediate KR group with a 10 second delay between frames was given instructions exactly the same as those of the immediate KR group except that they were asked to turn their hour glasses after looking at the correct answer and then wait for the sand to drain down. Each S in this group was given an hour glass with an amount of sand which would take 10 seconds to drain.

**Materials.** A programmed lesson on Ratios and Proportions, grade level 4-6, (Encyclopedia Britanica) was selected for use, in part, on the assumption that all Ss would be unfamiliar with it since it was approached via the set concept. The teachers of both grades affirmed this assumption. The program was of a linear type. Only the first 90 frames were used so that the entire experiment could be conducted in one session, and consequently
reducing variables. Within these 90 frames three main mathematical concepts were presented in relation to ratios and proportions. These were addition, subtraction and division. The types of responses required of the subject varied throughout the program. Some required the S to figure the problem and then write down the numerical answer. Another method was to have the S fill in the missing word. Others involved the multiple choice procedure. The correct answers were covered by a sliding margin.

The timing apparatus consisted of small "hour glasses." The hour glasses used by the delay group and the immediate KR group (with a delay between frames) contained 10 seconds worth of sand. Those hour glasses which the immediate KR group (who turned the glass before KR but then went immediately on) had an undetermined amount of sand in them since there was no delay factor involved. A pre-test and a post-test were also used.

Procedure. Each class and each group was run separate from any other class and experimental group. The two 6th grade classes used were run on different days. Within each class each student was randomly assigned to one of the following experimental groups:

Group I - Immediate KR
Group II – Immediate KR with 10 second delay between frames

Group III – Immediate KR, turning glass only before KR

Group IV – Delay of KR for 10 seconds

The procedure for all groups was as follows: Each S was seated in the respective testing room and given a pre-test and asked to fill in the answers. After all Ss had completed the pre-test, dittoed samples of different types of frames to be found in the program were handed to each S. The E then read the instructions and went through the sample frames at the same time. (See Appendix for instructions) Instructions for all groups were made equal in both length and quality. Each group was told that their particular treatment would lead to superior learning. This was done in an attempt to provide equal motivation for all Ss to follow the particular instructions given them. After reading the instructions the Ss were asked to begin work on the program. The E remained in the room to answer any questions about the procedure or to clarify any illegible words in the program, but no help was given which would help the child solve the problem. As each S finished with the program he was dismissed from the room. One 6th grade was post-tested 24 hours later and the other was post-tested 48 hours later.
CHAPTER IV

RESULTS

Analysis of variance tests were used to analyze the data. Analysis of the post-test scores for all of the four groups showed no significant difference in learning. Tables 1 and 2 summarize these results.

Table 1

Summary of Data for Treatment Groups

<table>
<thead>
<tr>
<th></th>
<th>Immed.</th>
<th>Delay</th>
<th>Immed.</th>
<th>Immed.</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Ss</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Sum of scores</td>
<td>288.5</td>
<td>299.5</td>
<td>307.5</td>
<td>310.5</td>
</tr>
<tr>
<td>Mean</td>
<td>20.61</td>
<td>21.40</td>
<td>21.97</td>
<td>22.18</td>
</tr>
</tbody>
</table>

Table 2

Analysis of Variance for Groups

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean sq.</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>20.71</td>
<td>3</td>
<td>6.90</td>
<td>1.13</td>
</tr>
<tr>
<td>Within groups</td>
<td>316.72</td>
<td>52</td>
<td>6.09</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>337.43</td>
<td>55</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

F (3,52) (.05) = 2.80
Null supported
The difference between the groups was not significant at the .05 level of confidence. In fact the results were not significant at the .250 level. This suggests that none of the differential treatments employed in this study, including immediate reinforcement, had any superiority of effect on the learner. Thus the null hypothesis was not rejected. On a programed learning task in mathematics with the knowledge of results delayed for 10 seconds, there was no significant overall difference between these subjects and those who received immediate knowledge of results. As the results show a delay between frames appears to have an insignificant effect on the learner.

Since one-half of each treatment group was tested 24 hours after completion of the math program and the other half was tested 48 hours later, there was an analysis of variance done between these groups. Tables 3 and 4 summarize these results. It was found that of the eight groups there was no significant difference between treatments. Specific to the hypothesis, the performance of the delay group, post-tested 48 hours later, was not significantly different from the delay of immediate groups post-tested 24 hours later. Therefore, the hypothesis that retention of learning under delay of reinforcement is enhanced was not supported. In fact the
means for all groups tended to decrease from the 24 to 48 hour post-test for all groups except the immediate KR group. In the case of the immediate group post-tested 48 hours after program completion the mean post-test score was higher than that of the 24 hour group, though as was pointed out earlier the difference was not significant.

Table 3

Summary of Data for 24 and 48 hour Post-test Treatment Groups

<table>
<thead>
<tr>
<th></th>
<th>Immed.</th>
<th>Delay</th>
<th>Immed. (delay btwn.)</th>
<th>Immed. (turn glass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours after P-T</td>
<td>24</td>
<td>48</td>
<td>24</td>
<td>48</td>
</tr>
<tr>
<td>No. of Ss</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Sum of scores</td>
<td>141.5</td>
<td>147.0</td>
<td>156.5</td>
<td>143.5</td>
</tr>
<tr>
<td>Mean</td>
<td>20.21</td>
<td>21.00</td>
<td>22.36</td>
<td>20.43</td>
</tr>
</tbody>
</table>

Table 4

Analysis of Variance for 24 and 48 hour Groups

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean sq.</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>56.87</td>
<td>7</td>
<td>8.12</td>
<td>1.31</td>
</tr>
<tr>
<td>Within groups</td>
<td>296.63</td>
<td>48</td>
<td>6.18</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>353.50</td>
<td>55</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ F (7,48) (0.05) = 2.21 \]
Since achievement test scores were available for all subjects, the experimenter made an attempt to analyze the relationship between achievement scores on a standard test and the experimental treatment, e.g. do good readers tend to do better or worse than poorer readers in relation to treatment received. Since half of the subjects (half of each treatment group) took their achievement test six months after the other half, the experimenter undertook to "normalize" the scores by adding three months to the latter groups' grade equivalent achievement scores and by subtracting three months from the formers' grade equivalent achievement scores. The assumption of equivalence will be discussed in the next chapter. The highest and lowest achievement scores for any particular subtest were determined for each of the four treatment groups. This resulted in an eight group design (immediate knowledge of results, low reading achievement; immediate knowledge of results, high reading achievement; delayed knowledge of results, low reading achievement; and delayed knowledge of results, high reading achievement; etc.).

The results of this analysis for the subjects' "normalized" achievement scores on reading vocabulary showed no significant difference between any of the groups. More specifically, there was no difference in
learning for high reading comprehension and low reading comprehension test scores on the new learning task either within or between treatment groups.

Proficiency on reading vocabulary test did not appear to be a variable positively effecting performance on the programmed math task, either within or between treatment groups. See Tables 5 and 6. The overall mean post-test score of those subjects with the high reading vocabulary scores was higher than that of the group with the low reading vocabulary score but it was not significantly so.

Table 5
Summary of Data for Subjects Previously Achieving High and Low Reading Vocabulary Scores

<table>
<thead>
<tr>
<th></th>
<th>Immed.</th>
<th>Delay</th>
<th>Immed.</th>
<th>Immed.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(turn glass)</td>
<td>(delay btwn.)</td>
</tr>
<tr>
<td>Ss in each group</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lowest reading vocab. score</td>
<td>N = 6</td>
<td>7</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>ΣX = 118.00</td>
<td>160.00</td>
<td>129.00</td>
<td>126.50</td>
</tr>
<tr>
<td></td>
<td>x = 19.67</td>
<td>22.86</td>
<td>21.50</td>
<td>21.08</td>
</tr>
<tr>
<td>Ss in each group</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>highest reading vocab. score</td>
<td>N = 7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>ΣX = 149.00</td>
<td>149.50</td>
<td>162.00</td>
<td>158.50</td>
</tr>
<tr>
<td></td>
<td>x = 21.29</td>
<td>21.36</td>
<td>23.17</td>
<td>22.64</td>
</tr>
</tbody>
</table>
Table 6

Analysis of Variance for Data in Table 5

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean sq.</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>81.15</td>
<td>7</td>
<td>11.59</td>
<td>-2.74</td>
</tr>
<tr>
<td>Within groups</td>
<td>-190.35</td>
<td>45</td>
<td>-4.23</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>-109.20</td>
<td>52</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ F(7, 45)(.05) = 3.07 \]

The same eight group analyses were done with high and low scorers on reading comprehension achievement tests. Tables 7 and 8 present these results.

Table 7

Summary of Data for Subjects Previously Achieving High and Low Reading Comprehension Scores

<table>
<thead>
<tr>
<th></th>
<th>Immed.</th>
<th>Delay</th>
<th>Immed. (turn glass)</th>
<th>Immed. (delay btwn.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ss in each group with lowest reading</td>
<td>( \bar{X} = ) 17.50</td>
<td>21.00</td>
<td>21.50</td>
<td>22.00</td>
</tr>
<tr>
<td>Ss in each group with highest reading</td>
<td>( \bar{X} = ) 88.00</td>
<td>91.50</td>
<td>22.00</td>
<td>20.38</td>
</tr>
</tbody>
</table>

\( N = 4 \)

\( \bar{X} = \) mean reading comp. score
Table 8

Analysis of Variance for Data in Table 7

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean sq.</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>74.34</td>
<td>7</td>
<td>10.62</td>
<td>1.92</td>
</tr>
<tr>
<td>Within groups</td>
<td>132.88</td>
<td>24</td>
<td>5.53</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>207.22</td>
<td>31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

F (7,24) (.05) = 2.43

Here again the difference between the high and low achievement scorers on reading comprehension appeared to be insignificant, even at the .250 level of confidence. There appears to be no positive relationship between a high reading comprehension score and achievement on a programed math lesson under any of the various treatments used in this study. As in the previous analysis, the overall mean post-test score for the high reading comprehension group was higher than that of the low reading comprehension group, but not significantly so.

The mean number of errors made on the programed task was 7.55. An analysis of variance was run to find any possible differences between the four experimental groups' number of errors made in doing the program. The immediate knowledge of results group had the highest number of errors; the mean being 8.07. The delayed knowledge of results group had the lowest number of errors.
with a mean of 6.64. This difference was found to be non-significant. See Tables 9 and 10.

Table 9
Summary of Data for Errors on Program

<table>
<thead>
<tr>
<th></th>
<th>Immed.</th>
<th>Delay</th>
<th>Immed.</th>
<th>Immed. turn glass</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>10 sec. between frames</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>$\Sigma x$</td>
<td>113</td>
<td>93</td>
<td>106</td>
<td>111</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>8.07</td>
<td>6.64</td>
<td>7.57</td>
<td>7.93</td>
</tr>
</tbody>
</table>

Table 10
Analysis of Variance for Errors on Program

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean sq.</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>74.53</td>
<td>3</td>
<td>24.84</td>
<td>1.21</td>
</tr>
<tr>
<td>Within groups</td>
<td>1,062.31</td>
<td>52</td>
<td>20.43</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1,136.84</td>
<td>55</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$F(3,52)(.05) = 2.78$
CHAPTER V

DISCUSSION

It appears from the results of this study that the purported superior effects of immediate knowledge of results seem to be questionable, at least for some programed mathematics tasks. This study found no significant difference between any of the treatments used. It seems to matter little whether the knowledge of results is delayed or immediate or whether delays between frames are used as the subject works through the program, at least for delays of 10 seconds.

There are several reasons which may possibly explain the outcome of this study. The first to be considered is the one which is implicit throughout this text. As was mentioned in the introduction of this study, the main support for the assumption of superiority of immediate knowledge of results is based primarily on the results of animal and human motor learning tasks. It seems quite possible that these results do not necessarily apply to the more complex forms of human learning, e.g., well developed cognitive skills.

The results reported here tend to support this reasoning—immediate knowledge of results for cognitive tasks may be too soon for the average person to become
fully "aware" of the process and result of his response. Although none of the treatment groups were found superior to any of the others, it may be noted that four out of the six comparisons between the immediate and delay groups showed that the delay group had greater mean post-test scores. Again it should be pointed out that these differences were not significant, but simply a trend.

Various methods may be used to foster a difference between these two primary treatment groups. A longer delay could be used. This would presumably intensify any possible differences—at least up to a point. Using this same design, a 5, 15, and a 20 second group could be added.

Another procedure would be to control the activity of the subjects during the delay period. It seems quite likely that the activities of the subjects during this delay would effect their learning rate. If mediating activities were varied during various temporal delay periods, the effect of all these variables could be studied at once. It should not be overlooked that this study did, to a limited extent, control the activities of the delay group since they had hour glasses which they were instructed to watch while the sand was draining to the opposite end. It was noted by the experimenter that the subjects were very much involved in this task.
during the delay period. Superiority of delay may have resulted if the delay period had been either a free period or a period in which stimulus related material was presented. The free period would be a time that would allow the subject to either review the question and response or a number of other free choice responses. In this study the apparent concentration on the draining sand may have "forced" the subject's mind off of the task at hand but did not necessarily let him relax or review.

Following Brackbill's (1964) findings, one of the assumptions of this study was that delay of knowledge of results would be superior to immediate knowledge of results for retention of material. Brackbill (1964) defined retention as the amount of saving upon relearning. Retention in this study, defined differently from Brackbill's definition, is essentially the same as learning. That is both retention and learning are the same sort of measure—post-test score after performing a specific learning task. Retention is measured by having half of each treatment group tested 24 hours later than the other half, which was tested 48 hours after completing the programmed task. As the results indicate, there was no significant superiority of the delay group over the immediate group. It seems then that under the conditions of this study, a delay does not cause the learner to
retain more of the material over longer periods of time. Noting the differences in the definitions of retention, these results do not necessarily run counter to what Brackbill (1964) has found. The delay group may or may not have been superior to the immediate group on relearning the math program. But since none of these sorts of measures were taken, no determinations can be made.

One possible fruitful measure which could have been taken but was not was item difficulty and its relationship to delay of knowledge of results. Brackbill (1964) analyzed the difficulty of each item in her learning task. Upon checking the results of her treatment groups, she found item difficulty during initial learning was equal in both groups. During relearning "It was found that for those subjects who learned under immediate feedback, degree of difficulty affected retention as much as it had affected learning." In fact these subjects had over ten times as many errors in relearning the three most difficult items as they did in relearning the three easiest ones. However, for the subjects who learned under the delayed feedback condition, the difficult items were retained just as well as the easier items.

Had this experimenter taken a measure of item difficulty, an analysis of retention between the immediate and delay groups for both 24 and 48 hour post-testing
could have been done. Since there was no difference in learning between the immediate and delay 48 hour post-test group, it would seem rather doubtful that performance on difficult items would show up significantly different between the two groups; yet, it is possible. It would be advisable to check this variable in future studies since any illuminations of the dynamics of learning at various levels of difficulty would obviously lead to more effective teaching practices.

Another possible reason for no difference between groups may be accounted for by special qualities of the program used. For example, had the program appeared to the subjects as being rather non-stimulating then the approach to it may have turned into a rote, mechanical operation rather than a contemplative cognitive operation. As a result, while delay of knowledge of results may have been superior, in this non-stimulating learning task the delay serves as a chance to get away from the task rather than a period in which the subject rehearses his problem and solution. Furthermore, the delay in this case may serve as a reward thus lessening the reinforcing value of the knowledge of results.

Another factor may have been that the subjects knew they would get no credit for the work they were doing on the math program. In this case reward could conceivably
come primarily from finishing the program while the knowledge of results was only a secondary reward. It is suggested that future studies take this into consideration and set up the task so that it appears to the subjects that the program is just another class assignment. Although it is probable that this factor effects the performance of all groups equally, it is possible that it may be unequal for various treatment groups. In the case of a delay of knowledge of results each successive delay may be viewed as an aversive stimuli, a period of time standing in the way of the reward. It was noted that many of the subjects who had hour glasses to watch tapped on the tops of them, presumably to make the sand drain sooner. This may be interpreted in at least two different ways. First, it could be used to support the above contention, being that the subjects were very anxious to finish a task which was in itself quite unrewarding. The objective in this case is completion of the entire task. A second interpretation might be that the program was very rewarding with the delays serving only as interruptions in what the subjects might have wished to be a continuous task.

If the latter is the case, it would seem that a delay would be of little value since instead of resulting in a period where the subjects can reflect on what they have
done, it may, especially under high motivation, stand in the way and make each step slightly discontinuous. As was pointed out earlier, the delay mechanism itself tended to control the activities of the subjects during the delay since they had to manipulate and watch them during the delay period. If the delay were automatically controlled, it may interfere less with the task and provide for more continuity in that the subject would be freer to reflect on the completed problem.

Another variation for future studies which may prove valuable would be to use a program that requires more active participation of the subjects. The program used in this study required very little in the way of actual problem solving. If the subjects were required to compute problems, one may expect that different schedules of reinforcement might effect the rate of learning. It is possible that the less challenging tasks would not. As Deese (1958) points out this activity may increase the learner's motivation or help him eliminate errors early in practice, especially in the case of a short delay.

There seems to be a possibility that the particular program used may have handicapped the better students. A common sense assumption would be that the better students should do significantly better than poorer students within each treatment group. However, as was shown in
the results, there was no significant difference within or between groups in performance. This tends to show that the less capable subjects learned as much as the faster students. On the other hand this appears to offer evidence to support the contention that learning proceeds best when the learner is allowed to proceed at his own rate.

As was mentioned in the results section, achievement scores were "normalized" because two forms of achievement tests were used and two different testing times were involved. They were tested six months apart. Other similar achievement data was available for similar comparisons but due to the grossly unequal variable no further analysis along this line was done. Even though research may support an assumption of equivalence between achievement scores of these two tests (in this case, the ITBS and California Achievement Test) the manner of making scores from two separate testing times equal in time is highly questionable.

Few generalizations upon the results of this study can be made until future studies with other math programs and programs of entirely different content are similarly studied. The only safe assumption that can be drawn from these results is that under the conditions employed in this study, using this particular program, immediate
knowledge of results did not appear to be superior to delay of knowledge of results. The relationship of age to delay of reinforcement on a program also needs to be explored.

In that the main principles of the programed approach involved active participation, immediate feedback, and "good" arrangement of material, it appears that it is invalid to compare the programed approach to that of the more traditional method as Bierbaum (1965), Cohen (1962), Stone (1965), and Willis (1965) to cite only a few, have done when checking the effects of immediate versus delayed reinforcement. Furthermore the added use of monitory rewards, as Brackbill has used in all of her studies on delay of knowledge of results, is an invalid procedure if the effects of programed instruction are to be generalized to the classroom.
CHAPTER VI

SUMMARY

It was the purpose of this study to explore the effects of immediate and delayed reinforcement on a programed math task. Most studies in this area support the contention that immediate reinforcement is superior to that of delay. It seems that any delay over a few seconds will prevent any learning at all. A review of these studies shows that most have been animal studies, though more recently humans have been increasingly employed as subjects. A limitation of these studies with humans lies in the fact that they have focused primarily on motor performance tasks and very simple verbal learning. No studies were found in the literature comparing immediate and delayed reinforcement on a programed learning task. Brackbill's (1964) lastest study, though titled appropriately, came close but did not employ a legitimate programed approach. She did, however, bring out a critical point—that of retention and its relation to reinforcement. Her findings show that retention is best if knowledge of results is delayed.

This experimenter, following the work of Brackbill, hypothesized that a delay of knowledge of results would not cause a significant decrease in learning but that it
would result in a longer retention of the programmed material. The subjects were fifty-six 6th grade pupils. The program was a commercial mathematics program in ratios and proportions.

The results of the study showed that a 10 second delay of knowledge of results did not have a significant decremental effect on learning. Retention, measured by post-testing half of each group 24 hours later than the other half, was not found to be significantly greater for the delay group as was hypothesized.

Two other groups were employed in this study—an immediate reinforcement group that had a delay between frames and an immediate reinforcement group that operated the delay mechanism before receiving knowledge of results but did not delay going on to receive the results (or reward). In neither of the analyses of data mentioned above did either of these two groups differ from the former two. All groups were the same.

Subjects' reading ability, as measured by reading comprehension and reading vocabulary achievement tests scores, did not appear to be a variable effecting performance of either the immediate or delay group or even within the respective treatment groups.

The immediate group made more errors in working through the program than did any other group but the difference
was again nonsignificant.

From the results of this study it seems that there may be some reason to question the contention that immediate reinforcement is always best and for all types of tasks. Under the conditions of this study it was not. It seems possible that the higher forms of human learning may proceed better, or at least no worse, with short delays in feedback. A delay of longer than 10 seconds may show entirely different results—either facilitating or interfering. More complex types of material may require more time for assimilation into the human cognitive system.

To an extent the activities of the groups were controlled since the timing mechanism (an hour glass) required the subjects' undivided attention during the delay. It was suggested that free time during the delay may provide for a greater opportunity of the subjects to reflect on the unit of material to be learned.

The fact that the poorest readers did as well as the best readers may be interpreted in two ways. It may suggest that there is something about the programed approach that handicaps the better student, e.g. short choppy sentences which get in the learner's way. However, it seems more reasonable to assume that it is the individual nature of the program that is the cause of this lack of difference in performance since the level of performance
was generally good for all subjects. All subjects were allowed to proceed at their own rate.

Among things suggested for future studies were the use of longer and shorter delays, different types of programed material, more "involving" material, greater control of delay interval, and automatic timing devices. Analysis of item difficulty may have revealed a relationship between delayed knowledge of results and retention of learning for material of varying degrees of difficulty.

One of the errors committed in the past which has given support to the immediate reinforcement position is that two basically unequal groups have typically been used to study the effects of delayed reinforcement. It is a valid procedure to compare the programed approach to the traditional approach but not to draw conclusions from such studies about effect of immediate reinforcement since many variables besides temporal ones are not the same. There seems to be little doubt that programing is just plain good teaching but there is some question that immediate reinforcement need always be an integral part of this.
BIBLIOGRAPHY
BIBLIOGRAPHY


Brackbill, Yvonne, Wagner, J. E. & Wilson, D. Feedback delay and the teaching machine. Psychol. in the Sch., 1964, 1(2), 149-156.


Dyal, J. A. Effect of delay of knowledge of results and subject response bias to extinction of a simple motor task. J. exp. Psychol., 1966, 71(4), 559-563.


APPENDIX
APPENDIX A

INSTRUCTIONS FOR IMMEDIATE GROUP

Ratios and Proportions is a program of study in a way that may be new to you. Instead of reading about ratios and then working problems, you will be reading and working problems from the very beginning.

You will need to pay close attention to each numbered unit, which is called a "frame." Each frame will tell you something, and then will ask you a question. Write your answer for each frame on a separate sheet of paper, and number your answer the same way the frame is numbered. You can then check the program where the correct answers appear, and find out whether your answer is right.

Checking the right answer immediately helps you to learn faster. As you can see, this program is different from other lessons that you do in class.

An important thing to remember in this program of study is to write your answer to each frame before you check the printed answer. Even if you are sure you have written the right answer, always check your answer with the printed answer to be sure.

Even though you will almost always write the correct answer to a frame, you might miss once in a while. If you miss, read the frame over again, and write the right
answer before you go on.

Here is what the first frame looks like:

1. 0 0
   0
Set 1

How many balls or elements are there in Set 1? 3

The answer to the question is "3" and your answer sheet should show that frame 1 is answered with "3."

After you have written your answer, then you can check the printed answer, which appears in the right margin, although you cannot see it when you are reading the frame. It is important to check your answer with the correct answer. You should do this immediately after answering the question.

Frame 2 looks like this:

2. 0 0
   0 0 0 0
Set 1 Set 2

How many more balls or elements does Set 1 have than Set 2? 2

The answer to the question is "2," and after writing your answer, you can check the right margin to see the printed answer. Remember to check immediately after you have answered the question.

Frame 4 asks for another kind of answer.
Does Set 1 have the same number of elements as Set 2? (yes/no) yes

The answer to the question is "yes" or "no" as the hint (yes/no) tells you.

Frame 11 asks you to select a word for a blank.

11. Subtraction or division may be used to compare the number of elements in two sets.

bets
sets
for
plus

After you read the frame, look at the words which might be used to make the statement in the frame correct if one of the words were put in the sentence where the blank shows. Choose one of the words, "sets," and you find that it correctly completes the sentence. This "fills" the blank. So your answer sheet for frame 11 should read: "sets" and when you check the printed answer, you find you have chosen the right word for the blank.

Sometimes, a frame will have a statement containing a blank, but without any choice of words given. In those frames, you have to remember the correct word and write it on your answer sheet.
Other frames ask a question and there is no blank, but some words from which to choose an answer will be given. Write your answer beside the frame number the same way you would if there had been a blank.

Still another kind of frame asks a question, and there is no choice of words given with which to answer the question. In such frames, you must remember the correct answer.

In some frames, there will be a blank in a statement, and the answers to choose from will be given with letters a), b), c), and so on, in front of the answer. A frame like this is frame 51.

51. Set 1 has 9 balls.
Set 2 has 6 balls.

Compare
the number of balls in
Set 1 with the number in
Set 2 by subtraction.

a) 9 - 6
b) 9 + 6

You can answer the instruction in the frame by choosing between the answers given as a) or b). Your answer sheet should look like this:

51. a) 9 - 6

You find when you check the right margin that you have chosen the correct answer. It is important to check the right answer immediately after you answer a question.
Some frames like 51, in which a choice of answers is given, will instruct you to write the letter of the correct answer. In these frames, you will not need to write the letter of the correct answer and the answer itself. Your answer sheet need only show the frame number and the letter a), b), c), or whatever the letter of the correct answer is.

There are other kinds of frames calling for other kinds of answers, but in all cases, if you read the frame very carefully before writing your answer, you will always know the right way to answer.

Remember, always look at the correct answer immediately after answering each question.

Are there any questions?

Now you may begin.
INSTRUCTIONS FOR DELAY GROUP

Ratios and Proportions is a program of study in a way that may be new to you. Instead of reading about ratios and then working problems, you will be reading and working problems from the very beginning.

You will need to pay close attention to each numbered unit, which is called a "frame." Each frame will tell you something, and then will ask you a question. Write your answer for each frame on a separate sheet of paper, and number your answer the same way the frame is numbered. After you turn your hour glass over and all the sand drains to the other end of the glass, you can then check in the program where the correct answer appears and find out whether your answer is right. This delay will help you learn faster.

An important thing to remember in this program of study is to write your answer to each frame before you check the printed answer. Even if you are sure you have written the right answer, always check your answer with the printed answer to be sure.

Even though you will almost always write the correct answer to a frame, you might miss once in a while. If you miss, read the frame over again, and write the right answer beside the miss, before going on to the next frame. Even if you did not get the right answer first, it is
important to get the right answer before you go on.

Here is what the first frame looks like:

1. 0 0
   0
Set 1

How many balls or elements are there in Set 1?

The answer to the question is "3" and your answer sheet should show that frame 1 is answered with "3." After you have written your answer, and have turned your hour glass over and let all the sand drain to the other end then you can check the printed answer. It appears in the right margin, although you cannot see it when you are reading the frame.

Frame 2 looks like this:

2. 0 0
   0 0 0 0
Set 1 Set 2

How many more balls or elements does Set 1 have than Set 2?

The answer to the question is "2" and after writing your answer and then letting the sand in the hour glass drain to the other end, you can check the right margin to see the printed answer.

Frame 4 asks for another kind of answer.

4. 0
   0
   0 0
   0 0 0 0
   0 0 0 0 0
Set 1 Set 2
Does Set 1 have the same number of elements as Set 2? (yes/no) yes

The answer to the question is "yes" or "no" as the hint (yes/no) tells you.

Frame 11 asks you to select a word for a blank.

11. Subtraction or division may be used to compare the number of elements in two _________.

bets
sets
for
plus

After you read the frame, look at the words which might be used to make the statement in the frame correct if one of the words were put in the sentence where the blank shows. Choose one of the words, "sets," and you find that it correctly completes the sentence. This "fills" the blank. So your answer sheet for frame 11 should read: "sets" and when it is time for you to check the printed answer, you find you have chosen the right word for the blank.

Sometimes, a frame will have a statement containing a blank, but without any choice of words given. In those frames, you have to remember the correct word and write it on your answer sheet.

Other frames ask a question and there is no blank, but some words from which to choose an answer will be
given. Write your answer beside the frame number the same way you would if there had been a blank.

Still another kind of frame asks a question and there is no choice of words given with which to answer the question. In such frames, you must remember the correct answer.

In some frames, there will be a blank in a statement, and the answers to choose from will be given with letters a), b), c), and so on, in front of the answer. A frame like this is frame 51.

51. Set 1 has 9 balls.
    Set 2 has 6 balls.

Compare
the number of balls in
Set 1 with the number in
Set 2 by subtraction.

a) 9 - 6
b) 9 + 6

You can answer the instruction in the frame by choosing between the answers given as a) or b). Your answer sheet should look like this:

51. a) 9 - 6

After you have turned the hour glass over and waited for the sand to drain to the other end, you find when you check the right margin that you have chosen the correct answer.

Some frames like 51, in which a choice of answers
is given, will instruct you to write the letter of the correct answer. In these frames, you will not need to write the letter of the correct answer and the answer itself. Your answer sheet need only show the frame number and the letter a), b), c), or whatever the letter of the correct answer is.

There are other kinds of frames calling for other kinds of answers, but in all cases, if you read the frame very carefully before writing your answer, you will always know the right way to answer.

Now, you are ready to go to frame 1 of the program and begin your study. Remember before checking to see what the right answer is, you should turn your hour glass over and wait for the sand to drain to the other end.

Are there any questions?

Now you may begin.
Ratios and Proportions is a program of study in a way that may be new to you. Instead of reading about ratios and then working problems, you will be reading and working problems from the very beginning.

You will need to pay close attention to each numbered unit, which is called a "frame." Each frame will tell you something, and then will ask you a question. Write your answer for each frame on a separate sheet of paper, and number your answer the same way the frame is numbered. You can then check in the program where the correct answer appears, and find out whether your answer is right. After you turn your hour glass over and wait for all the sand to drain out, you may then go to the next frame. This delay will help you learn much faster.

An important thing to remember in this program of study is to write your answer to each frame before you check the printed answer. Even if you are sure you have written the right answer, always check your answer with the printed answer to be sure.

Even though you will almost always write the correct answer to a frame, you might miss once in a while. If you miss, read the frame over again and write the right answer beside the miss, before going on to the next frame.
Even if you did not get the right answer first, it is important to get the right answer before you go on.

Here is what the first frame looks like:

1. 0 0
   0
Set 1

How many balls or elements are there in Set 1? 3

The answer to the question is "3," and your answer sheet should show that frame 1 is answered with "3."

After you have written your answer, then you can check the printed answer, which appears in the right margin, although you cannot see it when you are reading the frame. After checking the answer, you should then turn your hour glass over and wait for all the sand to drain out before going on to the next frame.

Frame 2 looks like this:

2. 0 0
   0 0 0 0
Set 1 Set 2

How many more balls or elements does Set 1 have than Set 2? 2

The answer to the question is "2," and after writing your answer, you can check the right margin to see the printed answer. Then turn the hour glass over and wait for the sand to drain down before going on.
Frame 4 asks for another kind of answer.

4. 

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Set 1  Set 2

Does Set 1 have the same number of elements as Set 2? (yes/no) yes

The answer to the question is "yes" or "no" as the hint (yes/no) tells you.

Frame 11 asks you to select a word for a blank.

11. Subtraction or division may be used to compare the number of elements in two ________.

bets
sets
for
plus

After you read the frame, look at the words which might be used to make the statement in the frame correct if one of the words were put in the sentence where the blank shows. Choose one of the words, "sets," and you find that it correctly completes the sentence. This "fills" the blank. So your answer sheet for frame 11 should read: "sets" and when you check the printed answer, you find you have chosen the right word for the blank.

Sometimes, a frame will have a statement containing a blank, but without any choice of words given. In those frames, you have to remember the correct word and write
it on your answer sheet.

Other frames ask a question and there is no blank, but some words from which to choose an answer will be given. Write your answer beside the frame number the same way you would if there had been a blank.

Still another kind of frame asks a question, and there is no choice of words given with which to answer the question. In such frames, you must remember the correct answer.

In some frames, there will be a blank in the statement, and the answers to choose from will be given with letters a), b), c), and so on, in front of the answer. A frame like this is frame 51.

51. Set 1 has 9 balls.  
Set 2 has 6 balls.

Compare the number of balls in  
Set 1 with the number in  
Set 2 by subtraction.

a) 9 - 6  
b) 9 + 6  
a) 9 - 6

You can answer the instruction in the frame by choosing between the answers given as a) or b). Your answer sheet should look like this:

51. a) 9 - 6

You find when you check the right margin that you have chosen the correct answer. After turning your hour glass
over and waiting for the sand to drain to the other end, you may go to the next frame.

Some frames like 51, in which a choice of answers is given, will instruct you to write the letter of the correct answer. In these frames, you will not need to write the letter of the correct answer and the answer itself. Your answer sheet need only show the frame number and the letter a), b), c), or whatever the letter of the correct answer is.

There are other kinds of frames calling for other kinds of answers, but in all cases, if you read the frame very carefully before writing your answer, you will always know the right way to answer.

Remember, always turn your hour glass before going on to the next frame.

Are there any questions?

You may now begin.
Ratios and Proportions is a program of study in a way that may be new to you. Instead of reading about ratios and then working problems, you will be reading and working problems from the very beginning.

You will need to pay close attention to each numbered unit, which is called a "frame." Each frame will tell you something, and then will ask you a question. Write your answer for each frame on a separate sheet of paper, and number your answer the same way the frame is numbered. After you turn your hour glass over, you can then check in the program where the answer appears, and find out if your answer is right. Do not wait for the sand to drain out. Check your answer immediately. This will help you learn faster. An important thing to remember in this program of study is to write your answer to each frame before you check the printed answer. Even if you are sure you have written the right answer, always check your answer with the printed answer to be sure.

Even though you will almost always write the correct answer to a frame, you might miss once in a while. If you miss, read the frame over again, and write the right answer beside the miss, before going on to the next frame. Even if you did not get the right answer right first, it
is important to get the right answer before you go on.

Here is what the first frame looks like:

1. 0 0
   0
   Set 1

How many balls or elements are there in Set 1?

The answer to the question is "3," and your answer sheet should show that frame 1 is answered with "3."

After you have written your answer, you should turn your hour glass over but do not wait for the sand to drain out. You should check the correct answer immediately. It appears in the right margin, although you cannot see it when you are reading the frame.

Frame 2 looks like this:

2. 0 0
   0 0 0 0
   Set 1  Set 2

How many more balls or elements does Set 1 have than Set 2?

The answer to the question is "2," and after writing your answer you should turn your hour glass over. But do not wait for the sand to drain down. Check your answer immediately after turning the hour glass. The answer is in the right margin.
Frame 4 asks for another kind of answer.

4. 

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Set 1  Set 2

Does Set 1 have the same number of elements as Set 2?

(Yes/No) Yes

The answer to the question is "yes" or "no" as the hint (yes/no) tells you.

Frame 11 asks you to select a word for a blank.

11. Subtraction or division may be used to compare the number of elements in two _________ sets

bets
sets
for
plus

After you read the frame, look at the words which might be used to make the statement in the frame correct if one of the words were put in the sentence where the blank shows. Choose one of the words, "sets," and you find that it correctly completes the sentence. This "fills" the blank. So your answer sheet for frame 11 should read: "sets" and when you check the printed answer, you find you have chosen the right word for the blank.

Sometimes, a frame will have a statement containing a blank, but without any choice of words given. In those frames, you have to remember the correct word and write it on your answer sheet.
Other frames ask a question and there is no blank, but some words from which to choose an answer will be given. Write your answer beside the frame number the same way you would if there had been a blank.

Still another kind of frame asks a question, and there is no choice of words given with which to answer the question. In such frames, you must remember the correct answer.

In some frames, there will be a blank in a statement, and the answers to choose from will be given with letters a), b), c), and so on, in front of the answer. A frame like this is frame 51.

51. Set 1 has 9 balls.
    Set 2 has 6 balls.

Compare the number of balls in
Set 1 with the number in
Set 2 by subtraction.

   a) 9 - 6
   b) 9 + 6

You can answer the instruction in the frame by choosing between the answers given as a) or b). Your answer sheet should look like this:

   51. a) 9 - 6

You find when you check the right margin that you have chosen the correct answer. Remember to turn your hour glass before checking your answer but do not wait. Go
on to the next question immediately.

Some frames like 51, in which a choice of answers is given, will instruct you to write the letter of the correct answer and the answer itself. Your answer sheet need only show the frame number and the letter a), b), c), or whatever the letter of the correct answer is.

There are other kinds of frames calling for other kinds of answers, but in all cases, if you read the frame very carefully before writing your answer, you will always know the right way to answer.

Remember, always turn your hour glass before checking your answer.

Are there any questions?

You may now begin.
PROGRAMMED LESSON

1. 0 0
   0
   Set 1

   How many balls or elements are there in
   Set 1? 3

2. 0 0
   0 0 0 0
   Set 1   Set 2

   How many more balls or elements does
   Set 1 have than Set 2? 2

3. 0
   0 0

   Which set below has the same number of
   elements as the set above? Set 1

      0
      0 0 0 0
      0 0 0 0
      Set 1   Set 2   Set 3

4. 0
   0 0 0 0
   0 0 0 0
   Set 1   Set 2

   Does Set 1 have the same number of
   elements as Set 2? (yes/no) yes

5. 0 0 0
   0 0 0

   Which set does not have the same number
   of elements as the set above? Set 3
Set 1 | 0 0 | Set 2
---|---|---
Set 3 | 0 0

6. When we say compare Set 1 with Set 2, we mean compare the number of elements in Set 1 with the number of elements in Set 2.

0 0 0 0 0 0

Compare Set 1 with Set 2.
Set 1 has ___ balls and Set 2 has ___ balls.

7. 0 0 0 0 0 0

Compare Set 1 with Set 2.
Set 1 has ___ balls and Set 2 has ___ balls.

8. The symbol ÷ means "divided by."
Copy the symbol which means "divided by."

÷

9. The symbol ÷ means ____________.

multiplied by
divided by
added to
subtracted from

divided by

10. Set 1 can be compared with Set 2 by subtraction or by division.
Copy the symbols which express the two ways numbers can be compared.

+ 

- 

x
11. Subtraction or division may be used to compare the number of elements in two _______.

bets
sets
for
plus

12. Make the symbol that means "divided by." +

13. 0 0 0 0 0
    Set 1  Set 2

Set 1 has 1 more ball than Set 2; that is 3 - 2 = 1. Set 1 has been compared with Set 2 by _______.

subtraction
addition

14. 0 0 0
    0 0 0 0
    Set 1      Set 2

Set 1 has 2 more balls than Set 2. Set 1 has been compared with Set 2 by _______.

subtraction
multiplication

15. 0 0
    0 0 0
    Set 1       Set 2

How many more balls does Set 2 have than Set 1? 1

16. When numbers are compared by division, the word "to" can take the place of the + sign. Which word can be used for the + sign? to

but
an
happy
to
17. When we say compare Set 2 to Set 1, we mean compare the number of elements in Set 2 with the number of elements in Set 1. Compare Set 2 to Set 1 by division.

\[
\begin{array}{cc}
\text{Set 1} & \text{Set 2} \\
0 & 0 \\
0 & 0
\end{array}
\]

to

3 to 2

18. To compare Set A to Set B by division, write

\[
\frac{4}{2}
\]

Compare by division.

\[
\begin{array}{cc}
\text{Set 1} & \text{to} & \text{Set 2} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}
\]

2 \div 3

19. Compare Set 1 to Set 2 by division.

\[
\begin{array}{cc}
\text{Set 1} & \text{Set 2} \\
0 & 0 \\
0 & 0
\end{array}
\]

to

\[
\begin{array}{cc}
0 & 0 \\
0 & 0
\end{array}
\]

\[
\frac{4}{5}
\]

20. Compare Set 1 to Set 2 by division.

\[
\begin{array}{cc}
\text{Set 1} & \text{to} & \text{Set 2} \\
0 & 0 \\
0 & 0
\end{array}
\]

5 \div 2

21. When sets are compared by division, the word _____ can be used for the + sign.

but
to
and
equals
22. 0 0 0 0 0 0
    0 0 0 0
    Set 1 0
    Set 2

Compare Set 1 to Set 2.

______ to ________

23. 0 0 0 0
    0 0 0 0
    0 0
    Set 1  Set 2

Compare Set 2 to Set 1.

______ to ________

24. 0 0 0 0 0
    0 0 0 0
    Set 1  Set 2

Compare Set 1 to Set 2 by division.

______ to ________

25. 0 0 0
    0 0
    Set 1 0
    Set 2

Compare Set 1 to Set 2 by division.

______ to ________

26. 0 0 0 0 0 0 0 0 0 0
    Set 1  Set 2

Compare Set 1 to Set 2 by division.

______ ÷ ________

27. 0 0 0
    0 0 0
    Set 1  Set 2

Compare Set 2 to Set 1 by division.

______ ÷ ________
28. 

\[
\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

Set 1 \hspace{1cm} Set 2

Compare Set 1 to Set 2 by division.

\[
\begin{array}{cccc}
\text{to} & \text{to} \\
\end{array}
\]

29. 

\[
\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

Set 1 \hspace{1cm} to \hspace{1cm} Set 2

Compare Set 1 to Set 2

a) with the word to. 

b) with the division sign.

30. Compare Set 2 to Set 1 by division.

\[
\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

to

\[
\begin{array}{cccc}
\text{1} & \text{to} & \text{5} \\
\end{array}
\]

31. Compare Set 1 to Set 2 by division.

\[
\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

Use the word to.

\[
\begin{array}{cccc}
\text{4} & \text{to} & \text{3} \\
\end{array}
\]

32. Compare Set 1 to Set 2 by division.

\[
\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

Use the word to.

\[
\begin{array}{cccc}
\text{4} & \text{to} & \text{8} \\
\end{array}
\]

33. Now compare Set 2 to Set 1 by division.

\[
\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

Use the word to.

\[
\begin{array}{cccc}
\text{3} & \text{to} & \text{7} \\
\end{array}
\]
34. Compare Set 1 to Set 2 by division.

Set 1    Set 2
0 0      0 0 0
0         0

Use the division sign.

35. Compare Set 2 to Set 1 by division.

Set 1    Set 2
0 0      0 0 0
0 0      0

Use the division sign.

36. If you compare Set 1 to Set 2 by division, the result is 3 to 5.
Now compare Set 2 to Set 1 by division.

Set 1    Set 2
0 0 0    0 0
0 0 0    0 0

Use the word to.

37. When we compare Set 2 to Set 1 by division, the result is 2 to 4.
Compare Set 1 to Set 2 by division.

Set 1    Set 2
0 0 0 0  0
0 0          0

Use the word to.

38. Set 2 compared to Set 1 by division is 4 to 7. Now compare Set 1 to Set 2 by division.

Set 1    Set 2
0 0 0 0  0 0
0 0          0
0 0          0

Use the division sign.
39. Set 1 0 0 0 0 0
    0 0 0 0

    a) Compare Set 2 to Set 1 by division.
    b) Compare Set 1 to Set 2 by division.

Use the word to.

a) 7 to 4
   b) 4 to 7

40. 6 divided by 3 can mean 3 divided into 6.
    6 ÷ 3 can mean 3 6
    10 ÷ 5 can be written 5 ____

5 10

41. You can write 14 ÷ 2 as 2 _____.

2 14

42. 4 divided into 8 can mean 8 divided by 4.
    4 3 can mean 8 ÷ 4

You can write 3 9
    ____ ÷ ____

9 ÷ 3

43. You can write 7 21 as
    ____ ÷ ____

21 ÷ 7

44. Set 2 has 15 balls. Set 1 has 5 balls. Which is the correct comparison of Set 2 to Set 1 by division?

   a) 5 ÷ 15
   b) 15 ÷ 5

b) 15 ÷ 5

45. 0
    0 0 0 0 0
    0 0

Set 1 0 0 0 0 0
    0 0

Set 2

To compare the number of balls in Set 1 with the number of balls in Set 2 by division, write 6 ÷ 2 = 3

This means that Set 1 has ____ times as many balls as Set 2.

3
46. 0 0 0
    0 0 0 0
Set 1    Set 2

To compare the number of balls in
Set 1 with the number of balls in
Set 2 by subtraction, write 6 - 2 = 4

This means that Set 1 has _______ more balls than Set 2.

47. Set 1 0 0 0 0    Set 2 0 0
    0 0 0 0    0 0

Which is the comparison by subtraction of the number of balls
in Set 1 with the number in Set 2?
   a) 8 + 4 = 2
   b) 8 - 4 = 4

b) 8 - 4 = 4

48. Set 1 0 0 0 0    Set 2 0 0 0
    0 0 0 0    0 0

Which is the comparison of the number of balls in Set 1 with the
number in Set 2 by division?
   a) 8 ÷ 5 = 2
   b) 8 - 4 = 4

a) 8 ÷ 4 = 2

49. Set 1 has 6 blocks. Set 2 has 2 blocks. Which of the following
compares the number of blocks in
Set 1 with the number in Set 2 by
subtraction?
   a) 6 + 2
   b) 6 ÷ 2
   c) 6 - 2

   c) 6 - 2

50. Set 1 has 12 triangles. Set 2 has 24 triangles. Which of the follow-
ing compares the number of triangles
in Set 1 with the number in Set 2
by division?

   a) 12 + 12
   b) 12 ÷ 2
   c) 12 - 12

   a) 12 + 12
51. Set 1 has 9 balls. Set 2 has 6 balls. Compare the number of balls in Set 1 with the number in Set 2 by subtraction.
   a) 9 - 6
   b) 9 + 6

52. 12 + 4 can be written _______.
   (Show the division using __________)

53. Set 1 0 0 0 0
    Set 2 0 0

    Set 2 has 1 less ball than Set 1.
    This comparison was made by ______.

    subtraction
    division

54. 0 0 0 0 0 0 0
    0 0 0 0 0 0
    Set 1 0 0 0 0 0 0
    Set 2 0 0 0 0 0 0

    Set 2 has ______ more balls than Set 1.
    a) 9
    b) 5
    c) 4

55. 0 0
    0 0
    0 0
    Set 1 0 0
    Set 2 0 0

    Set 1 has ______ more balls than Set 2.
    a) 2
    b) 5
    c) 3
56. 

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>Set 2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Set 2 has ____ fewer balls than Set 1.

a) 2  
b) 4  

57. 

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Set 1</td>
<td>Set 2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Set 1 has ____ times as many balls as Set 2.

a) 10  
b) 5  
c) 2  

58. Set 1 has 5 balls. Set 2 has 12 balls. Set 2 has ____ more balls than Set 1.

59. Which of the following compares Set 1's 12 balls to Set 2's 5 balls?

a) 5 + 12  
b) 12 + 5  

(Write the letter which is in front of the answer.)

60. Set 1 has 16 marbles. Set 2 has 8 marbles. Set 2 has $\frac{1}{2}$ as many marbles as Set 1. This comparison was made by ____.

- addition
- multiplication
- division
- subtraction

division
61. Set 1 has 6 blocks. Set 2 has 4 blocks. Set 1 has 2 more blocks than Set 2. This comparison was made by _______.
   addition
   subtraction
   multiplication
   division

62. We can compare the number of elements in Set 1 with the number of elements in Set 2 by either _____ or _____.
   addition
   division
   subtraction
   multiplication

63. 20 ÷ 5 can be written _____ ÷ _____

   5 ÷ 20

64. Which of the following shows a comparison by division?
   a) 4 ÷ 3
   b) 4 - 3
   (Write the letter which is in front of the answer.)

   a)

65. Which of the following shows a comparison by subtraction?
   a) 3 ÷ 2
   b) 3 - 2
   (Write the letter.)

   b)

66. When numbers are compared by subtraction, we state the difference between numbers. Which expression states a difference?
   a) 5 ÷ 3
   b) 5 - 3

   b)

67. Set 1 has 8 balls. Set 2 has 6 balls. Set 1 is compared to Set 2 by division; that is, 8 ÷ 6 or 8 to 6.
Set 3 has 4 elements compared to
Set 4's 2 elements. 4 to 2 means
that Set 3 is compared to Set 4
by _______.
- addition
- division
- multiplication
- subtraction

division

68. Set 1 has 9 balls to Set 2's 3 balls
means that Set 1 is compared to Set 2
by division. Which shows the com-
parison of these sets by division?
\[
\begin{align*}
9 \div 3 \\
9 - 3
\end{align*}
\]

\[
9 \div 3
\]

69. Set 1 has 5 balls to Set 2's 2 balls.
Which shows the comparison of Set 1
to Set 2?
\[
\begin{align*}
5 \div 2 \\
2 \div 5
\end{align*}
\]

\[
5 \div 2
\]

70. Set 1 has 11 elements and Set 2 has
3 elements. Which two of the follow-
ing show the comparison of Set 1 to
Set 2?
\[
\begin{align*}
11 - 3 \\
11 \div 3 \\
11 \times 3
\end{align*}
\]

\[
11 - 3 \\
11 \div 3
\]

71. Numbers can be compared either by
subtraction or by _______.
- addition
- division
- multiplication

division

72. Set 1 has 12 elements to Set 2's 6
elements. This means that Set 1 has
2 times as many elements as Set 2.
Which shows this comparison?
\[
\begin{align*}
12 \div 6 \\
6 \div 12
\end{align*}
\]

\[
12 \div 6
\]
73. We state the difference between numbers when numbers are compared by______.
   a) division
   b) subtraction
(Write the letter.)

74. Set 1 has 6 elements to Set 2's 3 elements means that Set 1 is compared to Set 2 by______.
   a) division
   b) subtraction
(Write the letter.)

75. Is 12 \div 6 the same as 6 \div 12? (Write "yes" or "no" to answer the question.) no

76. 0 0 0 0 0 0
    0 0 0 0 0 0
    0 0 0 0 0 0
Set 1    Set 2

Compare Set 1 to Set 2 by division.
   a) 10 \div 5
   b) 5 \div 10
(Write the letter.)

77. Set 1 has 24 elements to Set 2's 6 elements can be written 24 to 6.
    Set 1 has 5 elements to Set 2's 15 elements can be written______.
    15 to 5
    5 to 15

78. Set 1 has 5 balls to Set 2's 3 balls can be written______. Use the word to.
(Write the word.)

79. Set 1 has 13 elements to Set 2's 7 elements also means compare Set 1 to Set 2 by______.
(Write the word.) division
80. Which compares Set 1's 12 balls to Set 2's 5 balls?
   a) $5 \div 12$
   b) $12 \div 5$

81.  

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Set 1</td>
<td>Set 2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the comparison of Set 1 to Set 2 by division. Use the word to.

12 to 7

82.  

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Set 1</td>
<td>Set 2</td>
</tr>
</tbody>
</table>

A comparison of Set 1 to Set 2 can be written $7 \div 3$ or _______.

7 to 3

83. Compare Set 1's 4 balls to Set 2's 12 balls by division. Write the comparison both ways.

   ______ to _______
   ______ $\div$ _______

4 to 12

4 $\div$ 12

84. $6 \div 5$ can be written _______.

   ______ to _______

6 to 5

85. 14 to 7 can be written _______.

   ______ $\div$ _______

14 $\div$ 7

86. One of the meanings of $4 \div 7$ and 4 to 7 is compare Set 1's 4 balls to Set 2's 7 balls. $4 \div 7$ and 4 to 7 mean compare:
   Set 1's ______ balls to Set 2's ______ balls.

4 7
87. Compare Set 1 to Set 2.
   a) Set 1 has ___ balls to Set 2's ___ balls.
   b) ___ to ___
   c) ___ + ___

88. Compare Set 1 to Set 2.
   a) ___ to ___
   b) Set 1 has ___ balls to Set 2's ___ balls.
   c) ___ + ___

89. Set 1 has 18 balls. Set 2 has 6 balls. Compare Set 1 to Set 2 using the division sign.
   18 \div 6

90. Set 1 has 13 elements. Set 2 has 26 elements. Compare Set 1 to Set 2 using the word to.
   13 to 26