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## An Analysis of the Modern Mathematics Program in Grades Four Through Eight of the Cashmere School System

Janice Ferrill Irle  
*Central Washington University*

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AN ANALYSIS OF THE MODERN MATHEMATICS PROGRAM  
IN GRADES FOUR THROUGH EIGHT  
OF THE CASHMERE SCHOOL SYSTEM

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A Thesis  
Presented to  
the Graduate Faculty  
Central Washington State College

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In Partial Fulfillment  
of the Requirements for the Degree  
Master of Education

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by  
Janice Ferrill Irle  
August, 1969

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Donald M. Schliesman

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## CHAPTER I

### INTRODUCTION TO THE PROBLEM

#### Purpose of the Study

The purpose of this study was to analyze the modern mathematics program of the fourth through eighth grades of the Cashmere School District, Cashmere, Washington.

When the modern mathematics program was initiated in the District in 1964, the goal was to provide ". . . . better understanding of the basic concepts and structure of mathematics and provide a firmer foundation for understanding and use of mathematics in a scientific society" (18:i-iv). The student, as a result, should gain a broader concept of what mathematics is. The mathematics content should be taught with less emphasis on rote learning and more on developing models. More use should be made of symbols to represent ideas and relationships from which pupils can make mathematical generalizations.

This investigation attempted to determine whether or not the present mathematics program is meeting the goals of increased understanding of the number system and its operations.

#### Need for the Study

Every operational curriculum needs evaluating to see whether or not it meets its objectives. The modern

mathematics program at Cashmere was adopted because of recommendations by the secondary school faculty who felt that the facts available indicated that the students could achieve the same amount of traditional arithmetic plus modern concepts under the modern program; the adoption was supported by the administration which believed the program was consistent with the district's progressive philosophy. If elementary school teachers could be shown that the goals of the program were being met, they would have more confidence in the mathematics curriculum.

The literature shows considerable research has been done on the value of modern mathematics. Studies generally agree that students have equal skill at solving traditional problems whether they are in a traditional or modern program (14:623). In the area of increased understanding of the number system, the findings of the studies are inconclusive. For this reason the mathematics program of each school system needs to be evaluated to determine its value in the curriculum.

## CHAPTER II

### REVIEW OF THE LITERATURE

In the late 1950's, Sputnik served as a focal point for the concern over the scientific and mathematical education of students (11:798). The rapid advance of technology underscored the need for change (4:2). The result was numerous groups studying the content of mathematics, and the placement and organization of content, then proposing changes (5:14).

The proposed changes were labeled "modern mathematics", "new mathematics", and sometimes "contemporary mathematics". The programs in use at that time are referred to as "traditional" or "conventional" programs.

Although each modern program has its own unique characteristics, there are some themes which are common to all of the programs and might be used to define modern mathematics. Set theory in its elementary form is introduced in most programs. The study of structure, that is, the basic properties common to all mathematics, is part of each program. The idea of a standard unit of measurement is part of the elementary school curriculum. Improved understanding of our number system is developed by studying other number systems. Rote manipulations without understanding are discouraged. Number lines and other

graphs are used to help students see relationships, and programs are organized to move from the concrete to the abstract (1:27).

### Studies of Specific Concepts

In studying the student's ability to multiply, Grafft found that for average and above-average students, modern mathematics increases understanding and transfer of the operation although there is no increase in computational skill (8:163).

When studying the effects of teaching the commutative, associative, and distributive properties to fourth grade students, Schmidt found that the student's ability to apply fundamental processes to examples and problems was increased (20:4511).

Freibel chose California seventh graders with no modern mathematics background to use in his study of measurement understandings. Three classes were assigned to SMSG materials while three other classes used a traditional text. Both groups used the assigned materials for the entire school year. Test results showed a slight, though not significant, advantage in measurement understandings for the group using SMSG materials. Traditional achievement tests indicated comparable achievement for both groups (7:476-80).

Schlensoog found that sixth graders who studied numerals in bases other than 10 showed no better understanding of base 10 numerals than did students who studied no other bases (19:254).

In 1960 Moray studied the effect of having sixth graders who were enrolled in a traditional program spend one class period per week for four months studying sets, variables, and statistics. The results indicated that the students had gained knowledge in the areas studied with no decrease in traditional skills. In 1965 Moray tried the experiment again, this time with students in a modern program. He found the same differences between the experimental and control groups on the Elementary Math Concepts Test as he had in 1960; however, he found a significant decrease in ability to perform traditional skills. This decrease was significant for both the control and experimental groups (13:4538A).

### Studies of Programs

A study made by Simmons, indicated that fifth, sixth, and seventh grade students enrolled in either transitional, that is, a basically traditional course with some elements of modern mathematics incorporated into it, or modern programs will score as high or higher than those in traditional programs when tested by means of a traditional test. The conclusion reached by Simmons was that there was no

decline in reasoning or computational ability by students enrolled in a modern mathematics program (21:6566).

Simmon's results were supported by Woodall's findings when he compared the test scores of students in the SMSG program from 1960-65 with the scores of students in the traditional program. Woodall found no significant differences between students in either program when tested for mathematical ability by use of the Stanford Achievement Test. There was no apparent difference in attitude toward mathematics by students in either group. There was some indication that fourth grade students in the modern program for the first year did not do as well as those in the traditional approach (23:4040B).

Fourth and fifth grade students in a large southwestern school district were the subjects of a study by Tryon. The experimental group studied a modern program while the control group studied a conventional program. Tryon found no significant difference between the two groups in arithmetic, computational skills, problem solving ability, and creative ability. The attitude of average and fast students was better in the modern program while the slow learners in the traditional program had a more favorable attitude toward mathematics than those in the modern program (22:3792B).

On the other hand, when Hungerman tested sixth grade students in the Detroit schools she found that not only did the group in the SMSG program perform better on seven of ten tests which stressed the outcomes of contemporary programs, but their scores were significantly higher on five of ten traditional arithmetic tests than scores of students in a conventional program. Further, she found that, although there is a marked correlation between achievement and intelligence of students in conventional programs, this correlation is only moderate for students in contemporary programs. She found no significant difference in the attitude of students in either contemporary or conventional programs toward mathematics (12:30).

A comparison was made by Osborn of the achievement of students who had studied no modern mathematics with those students who had studied one, two, or three years of SMSG mathematics. He used seventh, eighth, and ninth grade students, then tested them in the tenth grade. There was no significant difference in the results for any of the groups when they were tested by traditional materials. The students who had studied SMSG mathematics did significantly better than the other group when tested on mathematics concepts. The test of mathematics concepts was devised by authors of the SMSG mathematics and therefore may have given biased results, although an impartial panel

of mathematicians had agreed that the test concepts were important to mathematics. An interesting result was that the positive attitude toward mathematics decreased the longer the students were in the program (16:7119).

Greathouse used three methods for teaching arithmetic to fifth and sixth grade students. The meaningful instruction sections were in programs which could be classified as modern mathematics because they emphasized understanding as well as computational skill. One section was given individualized-meaningful-instruction, a second section was given meaningful-instruction as a group, and a third section was given drill-computation instructions. Results of this study indicated that meaningful-instruction is as effective or, in the case of individualized-meaningful-instruction, more effective than drill-computation in the area of computational ability, quantitative reasoning, and mathematical understanding (10:5913).

When Peterson compared the achievement in terms of traditional goals by students using either modern, transitional, or traditional materials, he found no difference in achievement of students with low IQ, regardless of program. In the average and high IQ ranges, the students in modern and transitional programs had significantly higher achievement scores in the area of mechanical skills. Those in the transitional program tested



low in the area of applications, while those in the modern program did poorly in the area stressing traditional terms and definitions (17:2790B).

The Minnesota National Laboratory for the Improvement of Secondary School Mathematics studied experimental materials by randomly assigning them to Minnesota schools. The materials studied were those developed by the School Mathematics Program, University of Illinois Committee on School Mathematics, and University of Maryland Mathematics Project. Analyses made of test results so far have shown achievement differences favoring some experimental programs at some grade levels, but these differences were insignificant when compared to differences between students before the programs were initiated (15:327).

Greabell studied the effectiveness of the modern programs in some California schools as compared with the traditional programs. He classified the modern programs as either "crash modern" or "systematic modern". In the "crash modern" program, modern materials were given to teachers at all grade levels with little or no introductory preparation. The "systematic modern" program was characterized by much preparation and gradual adoption of the modern program. His testing results were not conclusive, but tended to suggest that a "crash modern" program was less effective in all areas than either a

"systematic modern" or a traditional program. The "systematic modern" did a better job of meeting modern goals than did either of the other programs (9:4).

### Summary

Thus far, the results of program evaluations would indicate that the reversal of public opinion, concerning modern mathematics, from enthusiasm in the early 1960's to doubt in 1967 is justified (14:623). Most of the experiments show at best no decrease in traditional mathematics skills. The results are inconclusive relative to the concepts which modern mathematics stresses. Fishman's feeling that the new programs were aimed at the college preparatory level (6:990A) is supported by the results of those experiments which show the slow learner accomplishing less and disliking math more in the modern program.

Perhaps that most reasonable conclusion about modern mathematics was reached by Davis. He assumed that these programs were simply trial balloons for the real revolution to come, that these programs did not begin to make use of the newest technological developments and psychological knowledge of learning. The real revolution will come with widespread use of programs presently being developed (4:1-4).

## CHAPTER III

### PROGRAM AND METHODS OF THE STUDY

#### Definitions

Modern mathematics. Any course which stresses understanding of the operations of arithmetic and the structure of the number system will be considered modern mathematics for the purposes of this study.

Traditional mathematics. Mathematics which is patterned after the mathematics taught in the elementary and secondary schools prior to 1957 which emphasized computational skill.

#### The Population

Community and School Environment. Cashmere is a small town, population approximately 2000, located at the eastern edge of the Cascades. The economy relies heavily on the orchards and supporting industries. The largest single employer is the saw mill. Thus, most of the population is working class, small-business owners, and a larger than average percentage of retired workers.

The school district draws from a 10 mile section of the Wenatchee River Valley with less than 100 of its students living more than two miles from school.

The first five grades, about 450 students, are presently housed in the elementary school; the sixth, seventh, and eighth grades, approximately 250 students, are in the middle school; the ninth through twelfth grades, around 400 students, are in the high school.

The district has a fairly progressive system having introduced team teaching in 1958, then having constructed a building designed for team teaching in 1961. The high school adopted the Chemical Education Materials Study (CHEMS) and Biological Sciences Curriculum Study (BSCS) shortly after they were introduced nationally. Sophomores take a humanities course combining English and history.

A modern mathematics program was introduced in the fourth through twelfth grades in 1964 and in grades one through three in 1965.

The Sample. The students used in this study were seventh and eighth graders enrolled in the Cashmere School District in June 1965 and seventh and eighth graders enrolled in the district in June 1968. Tables VIII through XI of Appendix A give complete statistics for each student involved in the study.

The seventh grade classes consisting of 45 and 52 students in 1965 and 1968, respectively, were about evenly composed of boys and girls. At the time of the test in 1965

the average age of the students was 13 years 2 months while in 1968 the average age was 13 years 1 month. The average intelligence quotient (IQ) was 114 and 111, respectively.

The 1965 eighth grade class of 49 students was predominantly girls with only 40% boys. On the other hand, in 1968 only 40% of the 60 eighth graders were girls. The average age of these students was 14 years 2 months and 14 years 3 months in 1965 and 1968, respectively. The average IQ for both groups was 110.

Table I gives more complete data for the classes. The age is given to the nearest month at the time the students were given the tests for this study. The IQ scores are the results of the California Test of Mental Maturity given to all the students in the seventh grade. The grade placement scores are measures of the student's achievement at the beginning of the eighth grade as measured by the Iowa Test of Basic Skills.

Group I is the seventh grade class of 1964-65, Group II is the seventh grade class of 1967-68, Group III is the eighth grade class of 1964-65, and Group IV is the eighth grade class of 1967-68. Table I provides high, low, and average statistics for age, achievement (GP), and intelligence (IQ) of each group.

The statistics of Table I are those for the subjects used in the final analysis. If students were not

TABLE I

## CHARACTERISTICS OF GROUPS USED IN THE STUDY

|                         | I*                         | II*                        | III*                        | IV*                        |
|-------------------------|----------------------------|----------------------------|-----------------------------|----------------------------|
| Number of students      | 45                         | 52                         | 49                          | 60                         |
| Number of boys          | 24                         | 25                         | 18                          | 37                         |
| Number of girls         | 21                         | 27                         | 31                          | 23                         |
| Age range               | 12yr 5mo<br>to<br>14yr 1mo | 12yr 6mo<br>to<br>14yr 3mo | 13yr 3mo<br>to<br>14yr 11mo | 13yr 9mo<br>to<br>15yr 5mo |
| Average age             | 13yr 2mo                   | 13yr 1mo                   | 14yr 2mo                    | 14yr 3mo                   |
| IQ range                | 88<br>to<br>135            | 68<br>to<br>135            | 77<br>to<br>130             | 80<br>to<br>132            |
| Average IQ              | 114                        | 111                        | 110                         | 110                        |
| GP range <sup>a</sup>   | 6.4<br>to<br>11.7          | 5.7<br>to<br>11.3          | 4.6<br>to<br>11.4           | 5.1<br>to<br>12.0          |
| Average GP <sup>a</sup> | 9.1                        | 8.5                        | 9.0                         | 9.2                        |

\* Group I is 1964-65 seventh grade, group II is 1967-68 seventh grade, group III is 1964-65 eighth graders, and group IV is 1967-68 eighth graders.

<sup>a</sup>GP refers to achievement grade placement at the beginning of the eighth grade.

continuously enrolled in Cashmere schools from 1964, their data were not used for the 1968 groups.

### The Program

The teacher's attitude and methods. The Cashmere school faculty had mixed emotions when the modern mathematics program was initiated. The impetus for the program was from the junior and senior high school mathematics teachers. The administration was also in favor of the more modern program and so with introductory faculty meetings the program was begun. Several extension courses were offered during the first two years for teachers who felt they needed a more modern mathematics background.

There were a variety of classroom situations under which the program was taught. Some classrooms were self-contained and the students were heterogeneously placed; at some grade levels mathematics was team taught; in some, the students were grouped by ability with each homeroom teacher teaching one section; and at some grade levels one faculty member taught all the mathematics.

The materials used. The program selected was the School Mathematics Study Group program. The SMSG text was in a paper-back developed for use by schools for experimentation and for text book companies as a model. It was first published in 1961.

The SMSG program was used in 1965 and 1966, then the mathematics series published in 1966 by Holt, Rinehart, and Winston was adopted for grades one through eight. This textbook series was authored by Eugene D. Nichols, Frances Flournoy, Robert Kalin, and Leonard Simon.

The modern program replaced a traditional program. Different texts had been used in various grades. The seventh grade text had been Using Mathematics by Kenneth B. Henderson and Robert E. Pingry, copyrighted in 1956 by the McGraw-Hill Book Company. A comparison of the objectives of the Henderson text as compared with the objectives of the present seventh grade text will be used to show the differences in the traditional and modern programs.

Method of teaching is one of the major differences in the books. The traditional book, hereafter referred to as Henderson, showed how to solve problems, while the modern text, Nichols, gave many examples of problems and their solutions and expected the students to discover how to solve the problems and, hopefully, why the method works.

The two texts were similar in that both included place value, Roman numerals, divisibility, decimal equivalents of fractions, ratio, percent, line and angle measurement, areas of rectangles, triangles, and circles, volumes of rectangular solids, and graphs.



The Henderson text dealt with division of whole numbers and the four arithmetic operations with fractions and decimals. The Nichols text assumed the seventh grade students knew these operations and so they were simply reviewed.

In actual material taught, the Henderson text stressed practical applications such as banking, transportation costs and budgets, while the Nichols text emphasized structure of number systems through different bases and properties of fields.

The basic changes made by the modern math programs were three in number: (1) Students learn why, not just how, to use mathematics, (2) Students are introduced to most concepts at an earlier age, and (3) The underlying principles of mathematics are stressed rather than the practical values of mathematics.

### Hypotheses

The hypothesis to be tested by this study is that students being taught mathematics in a modern program will improve in their understanding of the structure of the number system and in their ability to understand arithmetic operations.

To test this hypothesis statistically, the following null hypotheses were formed;

- (1) There will be no difference in the ability to understand the structure of the number system between students who have studied modern mathematics for one year and students who have studied modern mathematics for four years.
- (2) There will be no difference in the ability to understand arithmetic operations between students who have studied modern mathematics for one year and those who have studied modern mathematics for four years.

### Assumptions

In order to test the null hypotheses certain assumptions for this study must be made.

- (1) The same testing procedures were used in administering and scoring each of the tests, California Test of Mental Maturity, Iowa Test of Basic Skills, Arithmetic, and the Structure of the Number System, to each of the students.
- (2) Only the factors of age, IQ, and achievement (grade placement) affected the student's scores.
- (3) The material in the SMSG series was comparable to that in the replacement series by Nichols.
- (4) Teachers in the program had similar mathematical backgrounds and used comparable methods of instruction.

### Limitations of the Study

This study was limited to the Cashmere School System and, in particular, the modern mathematics program of grades four through eight.

The goals of the program, increased understanding of arithmetic operations and knowledge of the properties common to number systems, were the only factors investigated.

Although the modern mathematics program of Cashmere included a deductive method of learning and new content areas, no attempt was made to test these.

Many analyses have been made of the traditional arithmetic skills of students in a modern program, therefore this investigation did not concern itself with manipulative skills.

### Procedure

The mathematics program was taught in a variety of ways beginning in 1964 using School Mathematics Study Group materials and continuing with Holt textbook series in 1966. There were no special controls placed on any group; the teachers were free to use the materials in whatever manner seemed best to them.

At the end of the first year of the modern mathematics program, two mathematics tests were administered to the seventh and eighth grade classes. At the end of the 1967-68 school year the seventh and eighth grade classes were given the same two tests which had been given to the 1964-65 classes.

Each year the tests were administered by the classroom teacher during the regular fifty minute class period. The tests were given on the last two full days of the school year.

The tests were hand scored by the classroom teacher. A single list of scores for each group was then compiled. Scores achieved by students in the study are indicated in Tables VIII through **XI** of Appendix A.

#### Instruments used in the Study

The tests, Arithmetic and The Structure of the Number System, were developed by the Educational Testing Service of Berkeley, California.

Arithmetic was used to test the students' understanding of the number system and its operations. According to Buros, "The test is directed toward the measurement of basic understanding. . . .not a test of manipulative skills" (2:607). It is a 50 item multiple-choice test. The reliability of the internal consistency of the test is .86 as determined by the Kuder-Richardson Formula 20 (2:607).

The Structure of the Number System was used to test the students' understanding of the properties which are common to many number systems. It is a 40 item test. The Kuder-Richardson Formula 20 test of internal consistency for this test gave a reliability of .82 (2:655).

Each test is a 40 minute multiple choice test. The student marks the one answer which he determines to be correct on a separate answer sheet. Form A of each test was used.

## CHAPTER IV

### TREATMENT OF DATA AND INTERPRETATION OF FINDINGS

After the data had been collected, the first measurement taken of the scores was the mean of Arithmetic and The Structure of the Number System scores for each group and for the males and females of each group. These means and their standard deviations are listed in Table II.

Table II also lists the correlations of the student's scores on each test. The correlation was significant in every group, which means that if a student scored high on Arithmetic, he probably scored high on The Structure of the Number System and, similarly, if the student had a low score on Arithmetic, he probably, had a low score on The Structure of the Number System.

Looking at the means for Arithmetic one sees that the mean, 27.22, of the 1964-65 seventh graders is above the mean, 23.87, of the 1967-68 seventh graders. The mean, 30.33, of the 1964-65 eighth graders is, however, below the mean, 31.61, of the 1967-68 eighth graders. The same pattern holds when comparing 1964-65 females' means with 1967-68 females' means and for seventh grade males. The 1967-68 eighth grade male average was lower, rather than higher, than the average of the 1964-65 eighth grade males.

TABLE II

MEANS, STANDARD DEVIATIONS, AND CORRELATIONS OF SCORES  
FOR THE ARITHMETIC AND STRUCTURE OF THE NUMBER SYSTEMS TESTS

| Group <sup>a</sup> | Number<br>in the<br>Sample | <u>Arithmetic</u> |                       | <u>Structure</u> |                       | Correlation<br>Coefficient |      |
|--------------------|----------------------------|-------------------|-----------------------|------------------|-----------------------|----------------------------|------|
|                    |                            | mean              | standard<br>deviation | mean             | standard<br>deviation |                            |      |
| I                  | all                        | 45                | 27.22                 | 7.58             | 14.71                 | 6.10                       | .80* |
|                    | males                      | 24                | 28.04                 | 7.54             | 15.00                 | 6.69                       | .77* |
|                    | females                    | 21                | 26.29                 | 7.70             | 14.38                 | 5.48                       | .85* |
| II                 | all                        | 52                | 23.87                 | 8.74             | 13.12                 | 5.54                       | .65* |
|                    | males                      | 25                | 22.28                 | 9.00             | 11.92                 | 5.57                       | .71* |
|                    | females                    | 27                | 25.33                 | 8.40             | 14.22                 | 5.39                       | .56* |
| III                | all                        | 49                | 30.33                 | 7.12             | 15.78                 | 4.58                       | .69* |
|                    | males                      | 18                | 33.50                 | 8.59             | 16.89                 | 5.40                       | .72* |
|                    | females                    | 31                | 28.48                 | 5.46             | 15.13                 | 3.98                       | .63* |
| IV                 | all                        | 60                | 31.61                 | 9.55             | 18.63                 | 7.57                       | .84* |
|                    | males                      | 37                | 31.38                 | 10.03            | 18.84                 | 7.92                       | .85* |
|                    | females                    | 23                | 32.00                 | 8.92             | 18.30                 | 7.13                       | .85* |

a Group I is 1964-65 seventh grade, group II is 1967-68 seventh grade, group III is 1964-65 eighth grade, and group IV is 1967-68 eighth grade.

\*Significant at the .05 level.

TABLE II (continued)

| Group                | Number<br>in the<br>Sample | <u>Arithmetic</u> |                       | <u>Structure</u> |                       | Correlation<br>Coefficient |
|----------------------|----------------------------|-------------------|-----------------------|------------------|-----------------------|----------------------------|
|                      |                            | mean              | standard<br>deviation | mean             | standard<br>deviation |                            |
| seventh grade<br>all | 97                         | 25.42             | 8.35                  | 13.86            | 5.83                  | .72*                       |
| males                | 49                         | 25.10             | 8.73                  | 13.43            | 6.27                  | .75*                       |
| females              | 48                         | 25.75             | 8.03                  | 14.29            | 5.37                  | .68*                       |
| eighth grade<br>all  | 109                        | 31.04             | 8.53                  | 17.35            | 6.53                  | .79*                       |
| males                | 55                         | 32.07             | 9.55                  | 18.20            | 7.20                  | .79*                       |
| females              | 54                         | 29.98             | 7.28                  | 16.48            | 5.70                  | .79*                       |
| all males            | 104                        | 28.79             | 9.78                  | 15.95            | 7.16                  | .80*                       |
| all females          | 102                        | 27.99             | 7.89                  | 15.45            | 5.63                  | .75*                       |

\*Significant at the .05 level.

The scores on The Structure of the Number System showed similar results. The mean, 14.71, of the 1964-65 seventh graders was higher than the mean, 13.11, of the 1967-68 seventh graders. The 1964-65 eighth grade mean, 15.78, was lower than the mean, 18.63, of the 1967-68 eighth grade. The means of males and females followed the same pattern.

For both tests the eighth grade groups had higher means than the seventh grade groups. The means for all eighth graders, 31.04, on Arithmetic was greater than the mean, 25.43, for all seventh graders. On The Structure of the Number System the mean for all eighth graders, 17.35, was higher than the mean, 13.43, for all seventh graders.

Taken as a whole the males' average, 28.78, on Arithmetic was greater than the females' average, 27.99. The same was true on The Structure of the Number System where males averaged 15.95 and females averaged 15.45. The males did not, however, have the greater mean in each group.

Having noted the differences in the means between the 1964-65 and the 1967-68 students, and the fact that although the scores had increased for one group, the scores for the other group had decreased, the next question was "Are these differences significant or can they be attributed to the normal variations within groups of students?" In order to answer this question statistically a t-test was used to test the null hypotheses.



### t-test

The results of the t-test, shown in Table III, indicate that the difference in means between all seventh and eighth graders and between 1967-68 seventh and eighth graders was significant in every instance. On Arithmetic the differences were significant between 1964-65 seventh and eighth grade students as a whole and males, but not females.

The difference in means between the 1964-65 and 1967-68 seventh graders was significant on Arithmetic and the difference in the means of the two eighth grade classes was significant on The Structure of the Number System.

Since the t-test indicated a significant decrease in means on one test and a significant increase in means on the other, but only for some groups, it seemed reasonable to consider that factors other than time in the program might be affecting the scores. Therefore, it was decided to control the factors of age, intelligence, and achievement by means of an analysis of covariance.

### Analysis of Covariance

The control factor, age, was the student's age at the time of the test; the control factor, intelligence, was IQ at the beginning of the seventh grade; the control factor, achievement, was measured by grade placement (GP)

TABLE III

ARITHMETIC AND THE STRUCTURE OF THE NUMBER SYSTEM  
t-SCORES FOR DIFFERENCES IN THE MEANS  
 BETWEEN THE SUBGROUPS

|            | GROUPS <sup>a</sup>        |                 |                |                 |                 |
|------------|----------------------------|-----------------|----------------|-----------------|-----------------|
|            | I---II                     | III---IV        | I---III        | II---IV         | 7th---8th       |
| Arithmetic |                            |                 |                |                 |                 |
| all        | 2.03*<br>(95) <sup>b</sup> | -.81<br>(107)   | -2.04*<br>(92) | -4.48*<br>(110) | -4.77*<br>(204) |
| males      | 2.43*<br>(47)              | .81<br>(53)     | -2.15*<br>(40) | -3.73*<br>(60)  | -3.89*<br>(102) |
| females    | .41<br>(46)                | -1.67<br>(52)   | -1.13<br>(50)  | -2.70*<br>(48)  | -2.78*<br>(100) |
| Structure  |                            |                 |                |                 |                 |
| all        | 1.34<br>(95)               | -2.43*<br>(107) | -.95<br>(92)   | -4.44*<br>(110) | -4.06*<br>(204) |
| males      | 1.75<br>(47)               | -1.07<br>(53)   | -1.01<br>(40)  | -4.04*<br>(60)  | -3.61*<br>(102) |
| females    | .10<br>(46)                | -1.93<br>(52)   | -.54<br>(50)   | -2.25*<br>(48)  | -2.00*<br>(100) |

\* Significant at the .05 level.

<sup>a</sup> Group I 1964-65 seventh graders, Group II 1967-68 seventh graders, Group III 1964-65 eighth graders, Group IV 1967-68 eighth graders.

<sup>b</sup> Degrees of freedom.

on an achievement test taken early in the eighth grade. Each of the control factors was used singly and in all possible pairs for the analysis of covariance.

Although the two tests, Arithmetic and The Structure of the Number System, were designed to test two different hypotheses, the results for the same groups were similar on both tests, as might be expected from the high correlation between students' scores on the two tests. The analysis of covariance results for both tests, therefore, are given on the same table, Table IV.

Looking at Table IV it is noted that the difference in means between the 1964-65 seventh graders and the 1967-68 seventh graders is significant on Arithmetic, only when the achievement factor, GP, is controlled and when IQ and GP are both controlled. On The Structure of the Number System the only significant differences occur when GP is controlled. This implies that, although the mean of the 1967-68 group is lower than the mean of the 1964-65 group, this difference is probably caused by a normal variation within the groups and not by a decrease in the mathematical abilities of the students. The decrease in the means is significant when achievement levels are controlled.

Table V shows that for the difference in Arithmetic means between the 1964-65 eighth grade and the 1967-68

TABLE IV

ANALYSIS OF COVARIANCE FOR DIFFERENCES BETWEEN THE 1964-65  
AND 1967-68 SEVENTH GRADE MEANS ON ARITHMETIC AND  
THE STRUCTURE OF THE NUMBER SYSTEM TESTS

| Control         | Arithmetic                |               |             | Structure of the<br>Number System |              |             |
|-----------------|---------------------------|---------------|-------------|-----------------------------------|--------------|-------------|
|                 | all                       | males         | females     | all                               | males        | females     |
| Age             | 1.65<br>(91) <sup>a</sup> | 2.70<br>(43)  | .03<br>(42) | .75<br>(91)                       | 1.44<br>(43) | .00<br>(42) |
| IQ              | 3.15<br>(91)              | 3.04<br>(43)  | .13<br>(42) | 1.50<br>(91)                      | 1.42<br>(43) | .01<br>(42) |
| GP <sup>b</sup> | 11.77*<br>(91)            | 3.94<br>(43)  | .04<br>(42) | 9.45*<br>(91)                     | 2.09<br>(43) | .00<br>(42) |
| IQ-GP           | 5.26*<br>(90)             | 4.07*<br>(42) | .06<br>(41) | 2.20<br>(90)                      | 1.20<br>(42) | .00<br>(41) |
| Age-GP          | .12<br>(90)               | .70<br>(42)   | .37<br>(41) | .09<br>(90)                       | .00<br>(42)  | .47<br>(41) |
| Age-IQ          | 3.29<br>(90)              | 2.62<br>(42)  | .34<br>(41) | .93<br>(90)                       | .83<br>(42)  | .03<br>(41) |

\* Significant at the .05 level.

<sup>a</sup> Degrees of freedom within the subgroups; one degree of freedom between the subgroups.

<sup>b</sup> GP is the grade placement factor.

TABLE V

ANALYSIS OF COVARIANCE FOR DIFFERENCES BETWEEN THE 1964-65  
AND 1967-68 EIGHTH GRADE MEANS ON ARITHMETIC AND  
THE STRUCTURE OF THE NUMBER SYSTEM TESTS

| Control         | Arithmetic                |              |              | Structure of the<br>Number System |              |               |
|-----------------|---------------------------|--------------|--------------|-----------------------------------|--------------|---------------|
|                 | all                       | males        | females      | all                               | males        | females       |
| Age             | .17<br>(103) <sup>a</sup> | .22<br>(49)  | 1.16<br>(48) | 2.30<br>(103)                     | .44<br>(49)  | 1.83<br>(48)  |
| IQ              | .33<br>(103)              | .59<br>(49)  | 1.90<br>(48) | 2.79<br>(103)                     | .40<br>(49)  | 2.48<br>(48)  |
| GP <sup>b</sup> | .04<br>(103)              | 1.20<br>(49) | 2.45<br>(48) | 4.79*<br>(103)                    | .02<br>(49)  | 3.06<br>(48)  |
| IQ-GP           | .20<br>(102)              | 1.51<br>(48) | 2.81<br>(47) | 5.36*<br>(102)                    | 1.14<br>(48) | 3.78<br>(47)  |
| Age-GP          | .41<br>(102)              | 1.06<br>(48) | 2.84<br>(47) | 8.04*<br>(102)                    | 2.19<br>(48) | 3.86<br>(47)  |
| Age-IQ          | 1.37<br>(102)             | .65<br>(48)  | 3.90<br>(47) | 8.74*<br>(102)                    | 1.96<br>(48) | 5.18*<br>(47) |

\* Significant at the .05 level.

<sup>a</sup> Degrees of freedom within the subgroups; one degree of freedom between the subgroups.

<sup>b</sup> GP is the grade placement factor.

eighth grade, the analysis of covariance indicates no significant differences. For The Structure of the Number System the difference in the means is significant when achievement level (GP) is controlled and also when two controls are used, that is, when age and IQ, age and GP, or IQ and GP are controlled.

Table VI, which gives the results of the analysis of covariance comparing seventh and eighth graders, does not list results for the control factor grade placement. The grade placement for all students was determined by a test given at the eighth grade level and therefore was given to the seventh graders after they had taken the Arithmetic and The Structure of the Number System tests, while eighth grade students' grade placement was determined before they took the tests. This might lead to conclusions which are invalid and therefore this data was not considered in this investigation.

The difference in the means on Arithmetic between the seventh and eighth grade classes, whether comparing those classes the same year or all seventh graders with all eighth graders, is significant when IQ or IQ and age is controlled, but not when age alone is controlled. The results for The Structure of the Number System indicate significant results for all seventh and eighth grade groups when IQ and age are controlled, but not consistently

TABLE VI

ANALYSIS OF COVARIANCE FOR DIFFERENCES BETWEEN THE SEVENTH AND EIGHTH GRADE MEANS ON ARITHMETIC AND THE STRUCTURE OF THE NUMBER SYSTEM TESTS

| Control       | Arithmetic               |                 |                 | Structure of the Number System |                 |                 |
|---------------|--------------------------|-----------------|-----------------|--------------------------------|-----------------|-----------------|
|               | 1964-65                  | 1967-68         | all             | 1964-65                        | 1967-68         | all             |
| <u>Age</u>    |                          |                 |                 |                                |                 |                 |
| all           | .21<br>(88) <sup>a</sup> | 3.88<br>(106)   | 3.25<br>(200)   | .00<br>(88)                    | 4.93*<br>(106)  | 3.21<br>(200)   |
| males         | .61<br>(36)              | 3.09<br>(56)    | 2.62<br>(90)    | .04<br>(36)                    | 4.20*<br>(56)   | 3.03<br>(98)    |
| females       | .03<br>(46)              | 1.29<br>(44)    | .90<br>(96)     | .00<br>(46)                    | 1.08<br>(44)    | .44<br>(96)     |
| <u>IQ</u>     |                          |                 |                 |                                |                 |                 |
| all           | 4.05*<br>(88)            | 8.27*<br>(106)  | 6.61*<br>(200)  | .67<br>(88)                    | 7.48*<br>(106)  | 8.04*<br>(200)  |
| males         | 4.54*<br>(36)            | 8.17*<br>(56)   | 7.62*<br>(98)   | .85<br>(36)                    | 7.80*<br>(56)   | 5.70*<br>(98)   |
| females       | 1.95<br>(46)             | 6.08*<br>(44)   | 5.57*<br>(96)   | .43<br>(46)                    | 3.66<br>(44)    | 2.13<br>(96)    |
| <u>Age-IQ</u> |                          |                 |                 |                                |                 |                 |
| all           | 15.84*<br>(87)           | 76.63*<br>(105) | 92.82*<br>(199) | 7.29*<br>(87)                  | 50.56*<br>(105) | 56.91*<br>(199) |
| males         | 12.34*<br>(35)           | 51.56*<br>(55)  | 61.88*<br>(97)  | 4.04<br>(35)                   | 35.06*<br>(55)  | 38.54*<br>(97)  |
| females       | 6.49*<br>(45)            | 21.48*<br>(43)  | 27.05*<br>(95)  | 3.36<br>(45)                   | 12.09*<br>(43)  | 14.41*<br>(95)  |

\* Significant at the .05 level.

<sup>a</sup> Degree of freedom within the subgroups; one degree of freedom between the subgroups.

for age or IQ alone. Controlling the age factor gives significant differences only when the 1967-68 seventh and eighth grade groups are considered. Controlling IQ yields significant results when comparing 1967-68 seventh and eighth grades, and when comparing all seventh with all eighth graders.

Table VII which lists results for the analysis of covariance of differences between males and females does not give data for analyses in which grade placement is a control factor and all males and all females scores are being considered. When all males and females scores are being used, there is a mixture of seventh and eighth grade scores and so as noted previously the results are apt to be invalid.

Comparing differences for males and females, one sees that for both Arithmetic and The Structure of the Number System these differences are significant between all males and females when IQ is controlled and between eighth grade males and females when age and achievement (GP) are both controlled.

The differences between scores of eighth grade males and females on Structure of the Number System, when achievement (GP) or age and GP together are controlled, are significant. Between seventh grade males and females, the controlling of GP yields significant results.



TABLE VII

ANALYSIS OF COVARIANCE FOR DIFFERENCES BETWEEN  
 THE MALE AND FEMALE MEANS ON ARITHMETIC AND  
THE STRUCTURE OF THE NUMBER SYSTEM TESTS

| Control         | Arithmetic                |              |                | Structure of the Number System |               |                |
|-----------------|---------------------------|--------------|----------------|--------------------------------|---------------|----------------|
|                 | all                       | 7th          | 8th            | all                            | 7th           | 8th            |
| Age             | .14<br>(200) <sup>a</sup> | .06<br>(91)  | .55<br>(103)   | .12<br>(200)                   | .25<br>(91)   | .74<br>(103)   |
| IQ              | 7.83*<br>(200)            | .17<br>(91)  | 1.07<br>(103)  | 4.81*<br>(200)                 | .44<br>(91)   | 1.04<br>(103)  |
| GP <sup>b</sup> |                           | 3.88<br>(91) | 3.29<br>(103)  |                                | 5.35*<br>(91) | 4.32*<br>(103) |
| IQ-GP           |                           | .10<br>(90)  | 3.59<br>(102)  |                                | .41<br>(90)   | 3.30<br>(102)  |
| Age-GP          |                           | 3.29<br>(90) | 4.99*<br>(102) |                                | .87<br>(90)   | 4.64*<br>(102) |
| Age-IQ          | 1.29<br>(199)             | .02<br>(90)  | 3.68<br>(102)  | .86<br>(199)                   | .16<br>(90)   | 3.40<br>(102)  |

\* Significant at the .05 level.

<sup>a</sup> Degree of freedom within the subgroups; one degree of freedom between the subgroups.

<sup>b</sup> GP is the grade placement factor.

## CHAPTER V

### SUMMARY AND CONCLUSIONS

This investigation was undertaken in order to analyze the modern mathematics program of grades four through eight of the Cashmere School District, Cashmere, Washington.

Its specific purpose was to determine whether the students had increased their understanding of arithmetic operations and had become more knowledgeable concerning the structure of the number system.

The seventh and eighth grade students were given two standardized tests. One test, Arithmetic, was designed to test their understanding of arithmetic operations; the other, The Structure of the Number System, was to test their knowledge of the structure of the number system.

The tests were administered to 97 seventh and eighth graders at the end of the first year of the program; the same tests were administered four years later to 109 seventh and eighth graders.

These scores were treated statistically in an attempt to determine whether or not any change in the students' knowledge of modern mathematics had taken place. The means of the scores for the various classes were found and then subjected to a t-test. No conclusive results were apparent and therefore an analysis of covariance was used.

The factors of age, intelligence and achievement were controlled singly and in pairs to determine whether these factors might be significantly affecting the students' scores.

### Conclusions

There is little evidence to support the hypothesis that the Cashmere students are gaining increased understanding of the structure of the number system or arithmetic operations by being in the modern program as interpreted by the Cashmere School District.

The first null hypothesis for this investigation was that there will be no difference in the ability to understand the structure of the number system between students who have studied modern mathematics for one year and students who have studied modern mathematics for four years. The instrument used to test this hypothesis was The Structure of the Number System. A comparison of the scores of the 1964-65 seventh graders with the scores of the 1967-68 seventh graders and the 1964-65 eighth graders with the 1967-68 eighth graders shows that although the scores of the 1967-68 seventh graders were lower than the scores of the 1964-65 seventh graders, the scores of the 1967-68 eighth graders were higher than the scores of the 1964-65 eighth graders. Tests of significance showed that the

difference in seventh grade scores was not significant, but the difference in eighth grade scores was significant. Analyses of covariance showed that the difference in eighth grade means was significant if grade placement was controlled and when pairs of control factors were used. Although the eighth grade scores yielded significant results, the fact that the results were not significant when age or IQ was controlled and that differences in the seventh grade means was not significant would preclude rejection of the null hypothesis and therefore the first hypothesis was accepted.

The second null hypothesis was that there will be no difference in the ability to understand arithmetic operations between students who have studied modern mathematics for one year and those who have studied modern mathematics for four years. Arithmetic was used to test this hypothesis. As with The Structure of the Number System, the 1967-68 seventh grade mean was lower than the 1964-65 seventh grade mean, while the 1967-68 eighth grade mean was greater than the 1964-65 eighth grade mean. Tests of significance showed that the difference in seventh grade scores was significant, but the difference in eighth grade means was not significant. Analyses of covariance showed seventh grade differences were not significant when age or IQ was controlled and eighth grade differences were not significant for any of the control factors. The second null hypothesis was also accepted.

As students were not tested prior to the instigation of the modern mathematics program, this study does not mean to report that the program has not been wholly beneficial, but that the acquired knowledge has stabilized.

Though not of primary importance in this study, other comments need to be made concerning the knowledge of seventh graders as opposed to eighth graders and males as opposed to females.

The t-test indicated significant differences on both Arithmetic and The Structure of the Number System when all seventh graders scores were compared with all eighth graders scores and when 1967-68 seventh and eighth grade scores were compared; the differences between 1964-65 seventh and eighth graders were significant only on Arithmetic. The analysis of covariance showed the differences were significant if IQ, or age and IQ were controlled. Therefore, it would seem reasonable to conclude that eighth graders have a better understanding of arithmetic operations and structure of the number system than do seventh graders.

The results of the analyses of covariance of males' and females' scores indicate a few significant differences; in some instances the males had the higher mean, in others the females had the higher mean. The conclusion that there is no difference in the ability of males and females

to understand the structure of the number system and the operations of arithmetic follows from these data.

### Recommendations

As a large number of differences were significant when grade placement was controlled, further investigation of the causes and effects of achievement might be fruitful.

Comparing the content of the tests with the content of the eighth grade course, the writer did not feel that the differences in knowledge between seventh and eighth graders could be attributed to the material taught in the eighth grade; also, the fact that the differences were not significant if age was controlled would leave room for the investigation of other factors which affect test scores.

If the results of this study are considered valid, then the Cashmere School District should begin to formulate goals and investigate new programs to meet these goals.

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**APPENDIX A**

TABLE VIII

SCORES ON ARITHMETIC AND THE STRUCTURE OF THE NUMBER SYSTEM  
TESTS, SEX, AGE, INTELLIGENCE (IQ), AND ACHIEVEMENT (GP)  
FOR 1964-65 SEVENTH GRADE

| Student | Sex | Age | IQ  | GP   | Arithmetic | Structure |
|---------|-----|-----|-----|------|------------|-----------|
| 1       | M   | 161 | 118 | 11.7 | 47         | 30        |
| 2       | F   | 153 | 106 | 9.3  | 24         | 14        |
| 3       | F   | 153 | 135 | 10.7 | 39         | 30        |
| 4       | M   | 154 | 114 | 8.9  | 25         | 13        |
| 5       | M   | 156 | 114 | 7.9  | 22         | 14        |
| 6       | M   | 156 | 100 | 7.9  | 18         | 11        |
| 7       | M   | 162 | 116 | 9.0  | 29         | 18        |
| 8       | F   | 159 | 121 | 9.1  | 31         | 17        |
| 9       | M   | 158 | 116 | 8.2  | 26         | 8         |
| 10      | F   | 163 | 106 | 7.8  | 14         | 8         |
| 11      | F   | 157 | 122 | 10.4 | 40         | 19        |
| 12      | M   | 154 | 124 | 9.1  | 31         | 17        |
| 13      | M   | 156 | 130 | 9.1  | 31         | 21        |
| 14      | M   | 149 | 123 | 9.1  | 40         | 20        |
| 15      | F   | 155 | 118 | 8.9  | 25         | 13        |
| 16      | F   | 153 | 118 | 8.6  | 17         | 6         |
| 17      | F   | 163 | 98  | 9.1  | 19         | 13        |
| 18      | F   | 154 | 134 | 10.2 | 42         | 24        |
| 19      | M   | 154 | 128 | 10.0 | 26         | 19        |
| 20      | M   | 157 | 115 | 9.1  | 25         | 19        |
| 21      | F   | 163 | 117 | 10.6 | 23         | 9         |
| 22      | F   | 158 | 132 | 9.8  | 33         | 20        |
| 23      | F   | 163 | 124 | 10.7 | 34         | 17        |
| 24      | F   | 161 | 108 | 9.1  | 18         | 12        |
| 25      | M   | 154 | 102 | 7.4  | 17         | 5         |
| 26      | F   | 161 | 111 | 9.1  | 26         | 12        |
| 27      | F   | 159 | 115 | 9.1  | 26         | 11        |
| 28      | F   | 155 | 103 | 9.1  | 24         | 10        |
| 29      | M   | 155 | 115 | 9.1  | 16         | 6         |
| 30      | M   | 163 | 117 | 9.5  | 33         | 28        |
| 31      | F   | 169 | 88  | 9.1  | 24         | 11        |
| 32      | M   | 160 | 114 | 8.4  | 32         | 14        |
| 33      | M   | 156 | 101 | 9.1  | 19         | 6         |
| 34      | M   | 159 | 115 | 8.3  | 30         | 13        |
| 35      | M   | 163 | 127 | 10.7 | 36         | 25        |
| 36      | F   | 158 | 93  | 9.3  | 25         | 14        |
| 37      | M   | 154 | 103 | 9.1  | 32         | 16        |
| 38      | M   | 154 | 108 | 9.1  | 21         | 10        |

TABLE VIII (continued)

| Student | Sex | Age | IQ  | GP   | Arithmetic | Structure |
|---------|-----|-----|-----|------|------------|-----------|
| 39      | M   | 155 | 123 | 8.5  | 28         | 10        |
| 40      | F   | 160 | 127 | 8.7  | 27         | 14        |
| 41      | M   | 158 | 123 | 9.0  | 32         | 11        |
| 42      | M   | 160 | 116 | 10.4 | 34         | 16        |
| 43      | M   | 158 | 99  | 8.6  | 19         | 14        |
| 44      | F   | 158 | 101 | 8.1  | 19         | 14        |
| 45      | M   | 168 | 106 | 6.4  | 26         | 10        |

TABLE IX

SCORES ON ARITHMETIC AND THE STRUCTURE OF THE NUMBER SYSTEM  
 TESTS, SEX, AGE, INTELLIGENCE (IQ), AND ACHIEVEMENT (GP)  
 FOR 1967-68 SEVENTH GRADE

| Student | Sex | Age | IQ  | GP   | Arithmetic | Structure |
|---------|-----|-----|-----|------|------------|-----------|
| 1       | M   | 162 | 109 | 8.4  | 25         | 9         |
| 2       | F   | 171 | 68  | 6.8  | 7          | 7         |
| 3       | M   | 156 | 95  | 7.2  | 8          | 5         |
| 4       | M   | 162 | 103 | 5.8  | 14         | 5         |
| 5       | F   | 154 | 108 | 9.1  | 31         | 12        |
| 6       | M   | 154 | 113 | 8.9  | 18         | 12        |
| 7       | F   | 156 | 102 | 7.9  | 20         | 16        |
| 8       | F   | 160 | 134 | 11.3 | 39         | 27        |
| 9       | M   | 164 | 86  | 5.7  | 8          | 7         |
| 10      | F   | 158 | 113 | 9.3  | 26         | 21        |
| 11      | M   | 155 | 118 | 8.1  | 36         | 11        |
| 12      | F   | 154 | 109 | 8.9  | 22         | 9         |
| 13      | F   | 154 | 100 | 6.8  | 14         | 11        |
| 14      | F   | 159 | 109 | 8.3  | 16         | 8         |
| 15      | F   | 156 | 112 | 7.8  | 18         | 14        |
| 16      | M   | 155 | 83  | 7.4  | 20         | 10        |
| 17      | M   | 155 | 120 | 9.1  | 32         | 15        |
| 18      | M   | 153 | 133 | 10.2 | 37         | 22        |
| 19      | F   | 154 | 135 | 10.6 | 40         | 24        |
| 20      | F   | 154 | 124 | 10.9 | 26         | 20        |
| 21      | F   | 154 | 109 | 9.0  | 31         | 18        |
| 22      | M   | 151 | 118 | 7.9  | 25         | 19        |
| 23      | F   | 160 | 107 | 7.1  | 25         | 12        |
| 24      | M   | 155 | 122 | 9.6  | 34         | 20        |
| 25      | M   | 156 | 117 | 9.3  | 35         | 15        |
| 26      | F   | 154 | 121 | 9.7  | 29         | 17        |
| 27      | M   | 158 | 111 | 8.6  | 24         | 10        |
| 28      | M   | 152 | 106 | 7.6  | 18         | 9         |
| 29      | F   | 150 | 133 | 9.4  | 22         | 17        |
| 30      | M   | 155 | 106 | 8.3  | 22         | 12        |
| 31      | F   | 156 | 131 | 9.4  | 19         | 16        |
| 32      | M   | 163 | 114 | 7.8  | 17         | 12        |
| 33      | F   | 160 | 128 | 10.4 | 37         | 14        |
| 34      | F   | 152 | 95  | 7.2  | 21         | 12        |
| 35      | M   | 161 | 103 | 7.9  | 19         | 8         |
| 36      | F   | 162 | 104 | 8.4  | 18         | 13        |
| 37      | M   | 163 | 102 | 9.2  | 20         | 21        |
| 38      | M   | 151 | 113 | 8.5  | 22         | 13        |

TABLE IX (continued)

| Student | Sex | Age | IQ  | GP   | Arithmetic | Structure |
|---------|-----|-----|-----|------|------------|-----------|
| 39      | M   | 151 | 119 | 8.1  | 22         | 10        |
| 40      | F   | 154 | 132 | 9.1  | 27         | 10        |
| 41      | M   | 166 | 99  | 6.4  | 9          | 8         |
| 42      | F   | 151 | 121 | 9.4  | 34         | 8         |
| 43      | M   | 154 | 131 | 10.6 | 38         | 24        |
| 44      | F   | 155 | 108 | 9.5  | 28         | 15        |
| 45      | F   | 158 | 131 | 10.1 | 37         | 21        |
| 46      | M   | 155 | 94  | 7.7  | 12         | 4         |
| 47      | F   | 157 | 99  | 7.6  | 19         | 3         |
| 48      | M   | 157 | 100 | 7.3  | 17         | 11        |
| 49      | F   | 157 | 111 | 7.9  | 22         | 12        |
| 50      | M   | 168 | 106 | 7.9  | 25         | 6         |
| 51      | F   | 160 | 114 | 10.0 | 37         | 13        |
| 52      | F   | 156 | 106 | 8.1  | 19         | 14        |

TABLE X

SCORES ON ARITHMETIC AND THE STRUCTURE OF THE NUMBER SYSTEM TESTS, SEX, AGE, INTELLIGENCE (IQ) AND ACHIEVEMENT (GP) FOR 1964-65 EIGHTH GRADE

| Student | Sex | Age | IQ  | GP   | Arithmetic | Structure |
|---------|-----|-----|-----|------|------------|-----------|
| 1       | F   | 174 | 100 | 6.9  | 28         | 15        |
| 2       | M   | 178 | 77  | 4.6  | 29         | 16        |
| 3       | M   | 167 | 101 | 9.0  | 14         | 11        |
| 4       | F   | 164 | 124 | 10.2 | 34         | 17        |
| 5       | M   | 174 | 129 | 11.4 | 42         | 20        |
| 6       | F   | 165 | 118 | 8.3  | 30         | 25        |
| 7       | F   | 165 | 106 | 9.9  | 33         | 14        |
| 8       | F   | 174 | 106 | 10.8 | 41         | 21        |
| 9       | M   | 172 | 112 | 9.6  | 32         | 18        |
| 10      | F   | 171 | 102 | 8.2  | 22         | 12        |
| 11      | F   | 166 | 113 | 8.3  | 25         | 15        |
| 12      | F   | 164 | 118 | 9.1  | 27         | 11        |
| 13      | F   | 166 | 105 | 6.9  | 23         | 6         |
| 14      | M   | 174 | 117 | 10.3 | 37         | 13        |
| 15      | F   | 164 | 120 | 7.4  | 24         | 13        |
| 16      | M   | 168 | 100 | 8.1  | 22         | 11        |
| 17      | F   | 168 | 96  | 8.4  | 24         | 12        |
| 18      | F   | 169 | 109 | 9.2  | 25         | 15        |
| 19      | F   | 167 | 125 | 10.9 | 33         | 15        |
| 20      | M   | 170 | 120 | 10.1 | 35         | 16        |
| 21      | F   | 176 | 101 | 8.8  | 29         | 16        |
| 22      | F   | 168 | 104 | 9.9  | 33         | 15        |
| 23      | F   | 168 | 105 | 8.3  | 30         | 14        |
| 24      | F   | 171 | 110 | 9.9  | 28         | 15        |
| 25      | M   | 164 | 126 | 10.2 | 44         | 26        |
| 26      | F   | 172 | 121 | 10.5 | 34         | 23        |
| 27      | F   | 171 | 114 | 9.5  | 33         | 17        |
| 28      | F   | 172 | 122 | 11.0 | 29         | 21        |
| 29      | F   | 159 | 103 | 6.4  | 23         | 12        |
| 30      | M   | 176 | 102 | 8.7  | 23         | 12        |
| 31      | M   | 179 | 106 | 9.6  | 34         | 15        |
| 32      | F   | 170 | 119 | 10.1 | 34         | 17        |
| 33      | F   | 168 | 120 | 10.3 | 39         | 17        |
| 34      | F   | 169 | 116 | 8.9  | 26         | 14        |
| 35      | M   | 173 | 117 | 9.9  | 43         | 19        |
| 36      | M   | 168 | 130 | 10.4 | 38         | 23        |
| 37      | M   | 176 | 121 | 9.5  | 36         | 15        |
| 38      | F   | 166 | 119 | 10.9 | 36         | 17        |



TABLE X (continued)

| Student | Sex | Age | IQ  | GP   | Arithmetic | Structure |
|---------|-----|-----|-----|------|------------|-----------|
| 39      | F   | 174 | 119 | 10.4 | 27         | 20        |
| 40      | M   | 166 | 114 | 7.2  | 32         | 9         |
| 41      | M   | 169 | 123 | 10.2 | 38         | 24        |
| 42      | F   | 165 | 107 | 8.5  | 24         | 11        |
| 43      | F   | 170 | 97  | 6.6  | 19         | 12        |
| 44      | M   | 176 | 96  | 7.4  | 27         | 10        |
| 45      | M   | 167 | 115 | 10.0 | 48         | 21        |
| 46      | F   | 170 | 100 | 7.8  | 27         | 14        |
| 47      | M   | 174 | 96  | 8.2  | 29         | 15        |
| 48      | F   | 171 | 87  | 6.7  | 20         | 9         |
| 49      | F   | 175 | 100 | 8.7  | 23         | 14        |

TABLE XI

SCORES ON ARITHMETIC AND THE STRUCTURE OF THE NUMBER SYSTEM  
TESTS, SEX, AGE, INTELLIGENCE (IQ), AND ACHIEVEMENT (GP)  
FOR 1967-68 EIGHTH GRADE

| Student | Sex | Age | IQ  | GP   | Arithmetic | Structure |
|---------|-----|-----|-----|------|------------|-----------|
| 1       | F   | 165 | 103 | 9.2  | 28         | 16        |
| 2       | F   | 165 | 116 | 7.6  | 22         | 7         |
| 3       | M   | 172 | 118 | 10.9 | 44         | 31        |
| 4       | M   | 174 | 100 | 7.6  | 17         | 7         |
| 5       | M   | 166 | 114 | 8.2  | 28         | 11        |
| 6       | M   | 175 | 91  | 8.5  | 32         | 12        |
| 7       | F   | 173 | 114 | 9.9  | 37         | 18        |
| 8       | M   | 173 | 98  | 7.6  | 12         | 8         |
| 9       | M   | 170 | 123 | 10.7 | 43         | 29        |
| 10      | F   | 166 | 127 | 10.4 | 41         | 27        |
| 11      | F   | 173 | 107 | 9.5  | 32         | 21        |
| 12      | F   | 175 | 118 | 10.7 | 44         | 28        |
| 13      | M   | 168 | 102 | 8.7  | 26         | 13        |
| 14      | M   | 165 | 128 | 9.6  | 43         | 23        |
| 15      | M   | 176 | 117 | 9.9  | 38         | 21        |
| 16      | M   | 178 | 95  | 7.8  | 24         | 7         |
| 17      | M   | 172 | 105 | 8.6  | 24         | 10        |
| 18      | F   | 171 | 106 | 8.4  | 18         | 8         |
| 19      | F   | 172 | 131 | 11.8 | 44         | 30        |
| 20      | M   | 167 | 127 | 10.4 | 35         | 26        |
| 21      | F   | 166 | 112 | 8.2  | 26         | 11        |
| 22      | M   | 172 | 114 | 7.7  | 34         | 20        |
| 23      | M   | 165 | 100 | 10.4 | 38         | 26        |
| 24      | M   | 170 | 106 | 7.3  | 24         | 9         |
| 25      | M   | 168 | 118 | 10.9 | 46         | 26        |
| 26      | M   | 173 | 97  | 7.7  | 17         | 13        |
| 27      | F   | 169 | 114 | 8.8  | 28         | 14        |
| 28      | M   | 168 | 104 | 8.7  | 32         | 16        |
| 29      | F   | 176 | 88  | 6.5  | 7          | 5         |
| 30      | M   | 166 | 120 | 10.1 | 47         | 31        |
| 31      | M   | 170 | 121 | 10.6 | 33         | 16        |
| 32      | F   | 170 | 121 | 10.0 | 37         | 21        |
| 33      | F   | 176 | 112 | 10.1 | 43         | 23        |
| 34      | F   | 171 | 98  | 9.0  | 34         | 14        |
| 35      | M   | 165 | 121 | 9.5  | 30         | 18        |
| 36      | F   | 173 | 103 | 8.6  | 30         | 24        |
| 37      | M   | 185 | 80  | 5.1  | 13         | 13        |
| 38      | M   | 173 | 127 | 10.8 | 38         | 28        |

TABLE XI (continued)

| Student | Sex | Age | IQ  | GP   | Arithmetic | Structure |
|---------|-----|-----|-----|------|------------|-----------|
| 39      | M   | 176 | 112 | 8.2  | 22         | 16        |
| 40      | M   | 168 | 123 | 11.7 | 45         | 34        |
| 41      | M   | 175 | 120 | 10.6 | 33         | 25        |
| 42      | M   | 169 | 105 | 8.5  | 28         | 13        |
| 43      | M   | 176 | 106 | 8.4  | 30         | 16        |
| 44      | M   | 181 | 85  | 7.6  | 19         | 16        |
| 45      | F   | 168 | 107 | 8.6  | 26         | 14        |
| 46      | M   | 165 | 109 | 8.4  | 40         | 19        |
| 47      | M   | 178 | 85  | 6.4  | 18         | 12        |
| 48      | M   | 170 | 106 | 7.9  | 27         | 14        |
| 49      | F   | 171 | 112 | 9.8  | 35         | 19        |
| 50      | M   | 173 | 132 | 12.0 | 47         | 31        |
| 51      | F   | 175 | 116 | 10.5 | 36         | 15        |
| 52      | F   | 168 | 108 | 9.2  | 30         | 24        |
| 53      | F   | 165 | 107 | 10.3 | 37         | 18        |
| 54      | F   | 175 | 112 | 9.3  | 32         | 18        |
| 55      | M   | 170 | 110 | 11.1 | 38         | 33        |
| 56      | M   | 171 | 131 | 11.4 | 43         | 23        |
| 57      | F   | 184 | 104 | 7.3  | 27         | 15        |
| 58      | M   | 171 | 107 | 7.2  | 21         | 13        |
| 59      | F   | 176 | 122 | 11.2 | 42         | 31        |
| 60      | M   | 170 | 113 | 9.2  | 32         | 18        |