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A Comparative Investigation of the Effects of Frequent Testing upon Achievement in Secondary Advanced Algebra

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A COMPARATIVE INVESTIGATION OF THE EFFECTS OF FREQUENT TESTING UPON ACHIEVEMENT IN SECONDARY ADVANCED ALGEBRA

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CHAPTER I

INTRODUCTION TO THE PROBLEM

Procedures concerning the administration of examinations with more than the normal frequency have attracted the attention of investigators and have provided a subject of controversy and discussion. Although modern education is viewed as a life and growth situation where curriculum exists as a body of experience, most educators seem to agree that pupils tend to accomplish more when confronted with the realization that a day of reckoning is at hand, when they are to give an account of their knowledge (13:65). Such a situation contains dynamic or motivating properties which aid learning (9:1).

THEORETICAL ATTRIBUTES

Literature, for the most part, contends that frequent testing elicits better performance when compared to infrequent testing. However, opinions vary on how this occurs. Following are some possible explanations:

1. Knowledge of results. That better knowledge of results is an asset of frequent testing is agreed upon to some extent by all investigators researched. Standlee has said that knowledge of results of performance on quizzes

provides the students with a greater opportunity to see their areas of strength and weakness in the subject matter. Students work toward eliminating areas of weakness, thus obtaining greater achievement (19:322). Besides informing the students, frequent testing creates success or failure situations where correct performance is rewarded while incorrect performance is punished. In terms of effectiveness, Curtus and Woods have shown that the degree to which students participate actively in the correction of examinations is important (3). Jones advocated that knowledge of test results should be given as soon after testing as possible in order to avoid incorrect responses from becoming "set" (13:67).

2. Distributed learning. An alternate explanation of increased learning following the use of frequent tests is that distributed rather than concentrated learning occurs (14:427). Fitch has said that, ". . . frequent measurement is expected to result in steadier application of the individual to the task at hand" (6:15). The theory, as explained by Hovland, says that distributed practice affords a time interval in which incorrect responses introduced by fatigue are partially eliminated by forgetting. Massed practice, on the other hand, results in more immediate material retained, but it also results in stronger bonds to erroneous information introduced by fatigue factors which are more difficult to forget, thus resulting in less learning (11:586).

3. Practice. Related to distributed learning, and a result of frequent testing, is practice. Jones viewed curriculum as a body of experience, and ideas as processes. He stated that ". . . to get an idea is not to stow something away in a mental compartment: it involves the active development of a functional habit, which can only be secured through substitution and exercise" (13:65). The active recall involved in examination gives the opportunity to strengthen those connections which are essential for effective learning and retention (13:66).

4. Extrinsic motivation. Perhaps one of the most obvious and agreed upon explanations of increased achievement following frequent testing is that this kind of testing provides more extrinsic motivation, i.e., students will work harder throughout the course because they want to get good grades on the tests and this yields higher achievement (19:322). Fitch has concluded that frequent testing of achievement ". . . may motivate such outside endeavor as will result in superior achievement" (6:34).

5. Enforced activity. Another reason that might explain increased achievement associated with frequent testing would be that tests ". . . may affect learning simply through [regular] enforced activity with respect to the subject matter during the test itself" (19:322). In a test situation the learner is called upon to work

under pressure to participate more actively in the learning situation (12:602).

6. Structuring. Another positive attribute invoked by frequent testing is better organization and class structuring. It aids the instructor to clarify his educational aims, compels a more careful organization of the course, and prevents random divergence (13:69). It also tells the student, "These are the facts and principles that I believe are important; remember them!" (19:323).

7. Homogeneous grouping. One investigator attributed increased performance associated with frequent testing to the fact that the control group became more homogeneous and therefore easier to teach (10:376).

8. Good attitude. Whatever the reason for explaining the beneficial effects of frequent testing, none is more critical than the development of a good attitude on the part of the learner. It was observed with only one exception (4), that students favored frequent testing in the learning situation.

It seems clear that the foregoing reasons explaining the advantages of frequent testing are so interrelated that to credit any one of them with the results would be unwise. In fact, most researchers attribute their findings to several reasons. For example, Standlee found that ". . . a combination of several possible dimensions of quizzes--

enforced activity with respect to subject matter, structuring, knowledge of results, and extrinsic motivation . . ." was necessary to elicit higher educational performance (19:324). Jones recommended ". . . not simply tests at the close of the class hour, but terminal reviews in the full sense of the word . . . tests (comprehensive) plus self correcting and discussion" (13:67).

STATEMENT OF THE PROBLEM

Relatively speaking, few studies have concerned themselves with the problem of frequent testing, and as Keys pointed out, empirical evidence, uncomplicated by differences in the amount of testing material employed, on the effects of frequent testing is, at best, scarce (14:427). Also many studies used tests and test results for direct instruction, thus introducing additional variables. Furthermore, the choice of subjects and disciplines has been limited, the better part being taken from college psychology and sociology classes or high school science classes.

This investigation was not an attempt to modify previous experiments, nor was it an attempt to identify which of the conjectured explanations of the beneficial effects of frequent testing best fits. It dealt with only one discipline and investigated the effect frequent testing had on that discipline.

Specifically, the present study was designed to test the principal hypothesis--that frequent testing is associated with increased learning performance in high school advanced algebra.

NULL HYPOTHESES TESTED

To test the principal hypothesis stated, a comparative group experiment was initiated in which the following null hypotheses were tested:

Hypothesis I. There is no difference in algebra ability between groups prior to the experiment (as measured by the Numerical Ability and Abstract Reasoning sections of Form L of the Differential Aptitude Tests (1), and one teacher-made test).

Hypothesis II. There is no difference in learned algebra skills between groups due to frequent testing during the experiment (as measured by the teacher-made periodic tests).

Hypothesis III. There is no difference in retained algebra skills between groups due to frequent testing following the experiment (as measured by the teacher-made final exam).

Hypothesis IV. There is no difference in total performance between groups during the semester of study (as measured by the sum of all the teacher-made test scores and homework scores).

ASSUMPTIONS

Assumptions related to the study were:

1. Algebra skill required mental ability.

2. Algebra skill was susceptible to direct instruction.

3. Frequent testing and correction invoked knowledge of results, distributed learning, practice, extrinsic motivation, enforced activity and structuring.

4. Uncontrolled outside influences affected groups equally during the experiment.

5. Teacher-made tests validly and reliably measured student performance in terms of course objectives.

6. The standardized numerical ability and abstract reasoning tests, together with the first teacher-made test, were valid and reliable indicators of pre-experimental algebra ability.

7. The teacher-made final exam was an unbiased indicator of student performance in terms of course objectives for groups.

8. Students were not selected for classes of advanced algebra with respect to age, sex or intelligence.

CONDITIONS OF THE STUDY

1. Time. The experiment took place during the second semester of the school year 1970 (January 26 - June 11). The actual experimental period lasted approximately eleven school weeks (March 9 - May 28). The classes which took part in the experiment met during fourth and fifth periods (11:45 A.M. - 12:40 P.M. and 12:45 P.M. - 1:40 P.M.) during the school day. Due to a rotating schedule, however, each class was missed every seven days.

2. Discipline. The course of the study used in the experiment was second semester advanced algebra. This course is an extension and continuation of first year algebra and is taken following the geometry course. The areas of study consisted of irrational numbers and quadratic equations, quadratic relations and systems, and exponential functions and logarithms. (See Appendix A)

3. Subjects. A total of forty-eight advanced algebra students were engaged in the experiment. They consisted of sophomore, junior and senior high school students ranging in age from 15 to 18 years. All students had successfully completed two semesters of first year algebra and two semesters of geometry (averaging a grade of C or better), and one semester of advanced algebra (averaging D or better).

4. Location. The experiment was conducted at Newport Senior High School, Bellevue, Washington.

5. Restrictions. Students participated in many learning experiences both inside and outside the school, in addition to the learning activities provided in the classes involved in this study, which may have affected their growth in algebra ability.

DEFINITIONS OF TERMS USED

1. Algebra ability. In the study, this refers to the process of acquiring and performing algebra skills as set forth by course objectives.

2. Frequent testing. In the study, this refers to the process of conducting short five-item, fifteen-minute tests approximately every five class sessions.

3. Infrequent testing. In the study, this refers to the process of conducting long, fifteen-item, forty-five-minute tests approximately every fifteen class sessions.

CHAPTER II

REVIEW OF THE LITERATURE

Supporting Literature

In 1917, Gates used classes of grammar school students to investigate the relative values of learning by recitation as compared to learning by reading (9). Both sense and nonsense materials were used. Subjects were allotted varying amounts of time to learn materials by pure reading, by reading followed by recitation, or by pure recitation. It was found that a combination of introductory reading followed by recitation was most beneficial. On a written exam children showed that 20 per cent reading followed by 80 per cent recitation on nonsense material, and 40 per cent reading followed by 60 per cent recitation on sense material were optimum time conbinations. Furthermore, these combinations resulted in nearly 100 per cent and 27 per cent more immediate recall of respective nonsense and sense materials than did pure reading. After a delay of three to four hours, these statistics were doubled. Gates concluded that no learning would take place from reading without some recitation (9:100).

> The function of recitation is similar to that of practice in any form of sensory-motor learning. Memorization consists of selecting essential bonds, eliminating the unfit, and exercising the former until the connections are so well formed that once initiated, the series of responses will occur in proper sequence (9:99).

In 1929, Curtus and Woods found that the degree to which students participate actively in the correction of examinations was reflected positively on identical unannounced retests (3). Subjects, seventh through eleventh grade science students, participated in each of the four parts of the test-correcting experiment as follows: Under Method I, students corrected their own tests and free discussion was allowed. Under Method II, the teacher first marked the tests, then the students filled in corrections and free discussion was allowed. Under Method III, the teacher made all corrections and the students were limited only to free discussion. Under Method IV, the teacher again made all corrections, but discussion was limited only to answering student questions. Results of delayed retests showed a significant superiority of Method I over all others. No significant difference was found between Methods II and III, although they were both significantly better than Method IV.

In 1929, Jersild, using college beginning psychology students, found that recall attributed to enforced activity during a testing situation was dependent upon test structure (12). In summary, five experiments, all of the equivalent-group design, were performed. In each, the experimental group was given a pre-test and later both the experimental and control groups took the same pre-test as a final examination. In the first two experiments the tests were constructed of true-false items and yielded no significant difference between the experimental and control groups. In experiments III and IV a modified multiple choice test was employed. (All responses were correct to a degree.) In experiment V a short-answer essay test was used. The student saw the questions but could only guess at the answers. In experiments III, IV and V results were significantly in favor of the experimental group which was pre-tested each time (12:607).

In 1940, Hovland used thirty-two Yale University students to investigate the relative effects of distributed as opposed to massed practice in memorization.

> To have each subject act as his own control, eight experimental programs were required. In four, the learning to the criterion of one perfect recitation was by massed practice (six seconds between repetitions); the other four by disturbuted practice (two minutes between repetitions). After the appropriate interval, the subjects relearned by massed practice to the original criterion of one perfect recitation. This procedure permitted both recall and relearning scores to be used. Each subject completed two cycles learning a total of sixteen lists in counter-balanced arrangement (11:569-570).

Serial learning of nonsense lists was employed. Relearning and retention after intervals of 6 seconds, 2 minutes, 10 minutes, and 24 hours showed a significant superiority in favor of distributed practice. Hovland implied that distributed learning was better because it resulted in less learning of erroneous material introduced by fatigue factors during each trial. Also, such material was partially forgotten between trials and reorganized so that a minimum amount would be relearned on successive trials resulting in faster memorization (12:586).

In 1964, Chapanis used sixteen male Johns Hopkins University students to study the effect that knowledge of results had on performance of the monotonous task of punching random numbers onto a computer tape (2). The students were recruited through the student employment bureau and paid for their services. The students were placed in four groups which differed only in knowledge of output. When the data was analyzed, absolutely no significant differences were found. In fact, F ratios were so small that there was ". . . no reason to suppose that there were any trends worth examining further" (2:265). Although the findings were negative, two aspects of the experiment may have had great influence. First, even though knowledge of results has been associated with better performance in other experiments, the sheer monotony of this experiment may have nulified its effect. Second, the main reward of money was not contingent upon performance and probably served to overshadow any rewarding effect occurring from knowledge of results. In short, knowledge of results was of no benefit to the students.

Literature on Test Frequency

In 1923, Jones using college juniors and seniors as subjects, found that written examinations given at the close of presentation increased retention. His procedure and results were as follows:

> Five series of experiments were conducted on paired associates, lecture samples, and hour lectures, to investigate the effect of examination on classroom learning, as measured after intervals from three days to eight weeks. In all experiments the retest scores were reliably higher than the control scores, the retest average ranging from 33 per cent to approximately 100 per cent above the control average (13:51).

In 1929, Deputy used freshman philosophy students to investigate the effect frequent testing had on performance (4). Subjects were matched on intelligence and placed in one of three sections of philosophy numbering 35 students each. Section I was given a ten-minute written check at every bi-weekly meeting of the class. Section II was the control for the first half semester and was given no written check. Section III was given a twenty-minute written check once a week over the previous lesson. Written checks constituted two-thirds of the grade. Results of the midterm exam showed Section I significantly superior to both Sections II and III. No other differences were significant. For the second part of the experiment, Sections I and III became controls and Section II became experimental, having bi-weekly written checks. Results of the final test showed no significant differences between the sections. Results of a questionnaire showed that the original experimental Sections I and III were in favor of daily written checks while Section II was not. Deputy credits this poor attitude to a change in the testing situation rather than to the types of tests taken. In conclusion, Deputy implies that knowledge of success as a positive attribute of frequent testing is dependent upon attitude.

> In three sections of freshman philosophy studied in this experiment, the mean score was significantly increased when written exercises to measure the students success were given each time the class met, provided the attitude on the part of the students toward the work was favorable (4:334).

In 1931, Turney investigated the effect frequent short tests had on the achievement of college students (18). Two sections of educational psychology numbering forty and twenty-eight were selected as the respective experimental and control groups tested. The control group was slightly more intelligent than the experimental group. Also, its mean pre-test score was significantly higher, 108.1 vs. 85.2. During the semester the experimental group was given twelve short quizzes (about one each week) over assigned material while the control group was not. All quiz scores were made available to students on the following class session. Despite the handicap on the initial scores, the experimental group caught up to the control group on the final; that is, no significant difference was found. Turney credits favorable results to increased motivation induced by quizzing. Also, according to a questionnaire, the experimental group's attitude toward the frequent testing was very favorable (18:762).

In 1932, Hertzberg et al. used college educational psychology students to investigate the effect objective tests had on learning (10). Two experiments were performed in which the experimental group was given short written quizzes every third class session while the control group was not. Quizzes were kept as study aids. In each experiment, the experimental group showed a significant superiority over the control group which amounted to 12 or 15 per cent on the midterm tests. When, however, a final examination was given to both sections, unpreceded by review with practice test materials, the experimental groups scored no higher than the control groups. Hertzberg credited his results partly to the observation that the experimental groups were more homogeneous than the control groups and, therefore, easier to teach. However, significance levels were not stated (10:376).

In 1933, Kulp used a class of thirty-two college graduate sociology students to investigate the effect frequent short objective examinations had on achievement (15). The class was given a ten-minute objective test each week for the first half of the course. These tests were graded and results were placed on a chart for the students' information, but the actual tests were not returned. After seven weeks, those students who showed above-average standing on a midterm exam were excused from the weekly tests for the remainder of the course, while the rest continued as before. After fifteen weeks, the superiority of the upper half over the lower half, which had amounted to 39 per cent on the midterm exam, fell to 5 per cent on the final exam and was no longer significant. The midterm and final exams were identical with weekly tests which preceded them, differing only in the number of items. Kulp concluded that ". . . weekly tests do tend to increase the amount of learning even in a situation dealing with mature graduate students" (15:159).

In 1934, Keys found that both increased learning and retention resulted from ". . . frequent as contrasted to infrequent testing, apart from differences in volume of tests administered, or the use of the test materials as teaching aids" (14:435). Two sections of college educational psychology students were used in the experiment. They numbered 143 each and were matched both for sex ratio and previous knowledge of the subject. The experimental phase consisted of three monthly periods of twelve class sessions each. For the first period, the experimental group was given weekly reading assignments and announced weekly tests over the material; while the control group was given one lump assignment and one announced test at the end. For the second period, the control group received weekly assignments as did the experimental group but the testing remained the same. For the third period, both groups took one monthly exam, while only the control group was given weekly assignments. The content of the monthly exams taken by the control group was identical both in type and amount of material to those taken by the experimental group in weekly installments. Results of all tests were posted. However, no tests were returned, nor were quiz or review sessions conducted. The total mean gain of the experimental group over the control group on the periodic test material was 26.5 ± 4.1. The difference in gains on scores on an unannounced final examination after a lapse of five to fourteen weeks was 4.2 ± 1.4 in favor of the experimental group. However, on the regular end-term examination, where both groups had an equal opportunity to cram, scores were the same for both groups. Keys attributed his findings both to distributed learning and knowledge of results which were amplified by the frequent testing situation. Also, results of a questionnaire showed that subjects preferred short frequent tests (14:434).

In 1936, Gable used ninth grade biology students ". . . to determine the effect on pupil achievement of a system of anticipated daily check testing as compared with frequent unannounced unit tests and frequent announced unit tests" (8:29). Three groups, all of which were equated on intelligence, pre-test scores, and socio-economic status, were used in two experiments. The daily check group was given a short ten to twenty question objective test at the beginning of each lecture period covering the work of the previous day or days. The "sprung" group was given unannounced tests at various uneven intervals (not exceeding two weeks) on the material covered to the date of the test. The control group was given the same test as was the "sprung" group, only it was announced and given on the following day. The total testing for all groups was the same in both the amount and the type of material used. Classes corrected their own papers and the total testing time varied only slightly between groups. To measure gains, the teacher-made pre-test was given as an unannounced final to all groups. The results showed that the control group was significantly superior to the others in both experiments. A significant difference was found in favor of the "sprung" group over the daily check group in the first experiment but not in the second, namely 6.8 $\pm$ 1.2. Results of the same test given after a three month delay were similar (8:29).

In 1950, Fitch et al. used college students enrolled in government classes to investigate the effect of frequent testing on motivation and performance in large lecture classes (6). In summary, two classes numbering 97 and 198 students were engaged as control and experimental groups, respectively. Both groups received four one-hour tests during the semester over the work covered each month, and a final test covering the whole semester. The lecture method was employed over identical material for both classes and the same reading assignments were given. Also, each week six voluntary discussion groups were provided at times which accommodated both groups equally. In addition, the last half-hour of every third class meeting in each class was devoted to answering questions on the reading assignments. This was followed by a written ten-minute quiz over the reading material in the experimental group only. The groups were then compared by analysis of covariance techniques using previous government class grades and test grades determined by the Purdue grading system. It was found that the experimental group was significantly superior to the control group both in pooled test grades and discussion group attendance. Fitch concluded by saying that ". . . frequent testing of achievement in the college or university lecture classroom may motivate such outside endeavor as will result in superior achievement" (6:34). Also, polled students favored both weekly testing and discussion groups (6:17).

In 1960, Standlee and Popham used 104 undergraduate students in four sections of introductory educational psychology at Indiana University to investigate how extrinsic motivation, knowledge of results, course structuring and enforced activity are related to increased performance following frequent testing (19). Although students in the four sections were selected by conventional means, they were found to be statistically equivalent with respect to age, sex, intelligence (ACE) and class standing. In brief, three sections, A, B and C, were given the same weekly quizzes consisting of twenty true-false items, while section D, the control section, was not. In sections B and C conditions were varied so that it was judged that one or more of the previously mentioned control variables were eliminated. Analysis of covariance based on a 100 item multiple choice pre-test was used to measure significant gains determined by a midterm and final test of the same type. It was found that only section A, which employed all four control variables, was significantly better than the control group on the midterm exam. No significant difference occurred on the final exam (19:324).

In 1965, Pikunas and Mazzota used high school chemistry students to study the effect weekly testing had on performance (16). The semester was divided into three marking periods; the experiment took place during the first two. In this way, added motivation occurring at the end of the semester was avoided. Two groups, I and II, of two classes each took part in the experiment. During the first marking period both groups I and II acted as controls studying different materials but taking only one midterm test over respective materials. During the second marking period, groups I and II switched study materials and both acted as experimentals taking weekly tests besides a midterm test over respective materials. In this way the total study material was the same for both the experimental and control phases. Results showed that the weekly test group was significantly superior to the control group. Pikunas and Mazzota attributed their results to motivation elicited by the testing situation (16:375).

Summary

In summary, it appears that announced frequent tests have been associated with better performance when compared to either unannounced or infrequent tests. Authors credited their findings to a number of factors: knowledge of results, distributed learning, practice, extrinsic motivation, enforced activity, structuring, good attitude and even homogeneous grouping. It is also evident that most studies employed unequal amounts of testing materials, and some introduced extraneous variables by engaging extra discussion groups or quiz sessions. Also studies have been limited in their choice of subjects and disciplines, most employing college psychology and sociology classes or high school science classes.

The study which constitutes the subject of this paper is distinguished from all of the foregoing in that high school math classes were used. It also differs, with rare exceptions, in that tests administered to these classes were equal both in content and total amount, differing only in that the experimental group took these in brief five-day installments, while the control group was given longer tests approximately every fifteen days. Furthermore, test correction was uniform for both groups and no outside discussion sessions were held.

CHAPTER III

METHOD OF RESEARCH

GROUPS STUDIED

The subjects for the experiment consisted of the writer's fourth and fifth period advanced algebra classes at Newport Senior High School during the second semester of the school year 1969-1970. Although students were registered for classes through the usual counseling methods, it was believed that selection was not biased in terms of age, sex or intelligence. The period four class, which became the control group, totaled 28 students--4 seniors, 13 juniors, and 11 sophomores. The period five class, which became the experimental group, totaled 20 students--12 juniors and 8 sophomores. For the experiment, groups were equated on the basis of numerical ability, abstract reasoning and initial algebra skill.

APPARATUS

To test the stated null hypotheses the following apparatus were used:

1. To test Hypothesis I as a basis for equating groups, scores from two standardized tests of numerical ability and abstract reasoning from Form L of the

"Differential Aptitude Test" series (The Psychological Corporation, New York, New York 10017) (1) and one teacher-made test covering the first six weeks of course material were used.

2. To test Hypothesis II, scores from nine short, fifteen-minute, five-item tests and three long, forty-five-minute, fifteen-item tests (all of the teacher-made type), were used. The items for each short test were selected by chance from a pool of ten items. A block of three short tests was paralleled by one long test made from the remaining fifteen items from three ten-item pools. For example, consider short test two. One unused or lap-back item was selected by chance from pool one for short test two. Four items were selected by chance from pool two for short test two, and one item was selected by chance from pool two for short test three. The remaining items were used for the long test as previously described. By using lap-back items, it was felt that students would tend to review more thoroughly. (See Appendix A)

3. To test Hypothesis III, scores from a single three-part final exam were used. Parts one and two each consisted of seven items testing the first and second halves of the class material during the experimental phase (twenty minutes in length each). Part three consisted of fourteen items testing the material for the whole experimental period (forty-five minutes in length). Items for the final were not identical to those previously used.

However, they paralleled questions selected at random from previous test pools. Actually, of the twenty-eight items on the final exam, thirteen items each paralleled items used on respective long and short tests, while two items were new.

4. To test Hypothesis IV, the sums of all the scores on the teacher-made tests and on the story problem homework assignments were used. The homework assignments were taken from the class text: Modern Algebra and Trigonometry, by Dolciani, Berman and Wooten (5).

All teacher-made tests were of the short answer and computation type. Items for these tests were either made up from or selected directly from the class text (5), from the test supplement (17), or from former tests. No true-false items were employed.

PROCEDURE

To insure maximum control over the experimental situation and to test the stated null hypotheses, the following instructional and testing procedures were engaged.

Instructional Procedures

1. All individual aspects of the investigation were kept confidential and subjects were at no time informed of the experiment during its duration.

2. Arrangements were made so that both fourth and fifth period classes were held consecutively in the same

room after lunch. Thus, both classes experienced the same physical surroundings and neither experienced a split math period. Also, there was no time for students to mingle between classes.

3. Every effort was made to teach the same material to both classes without bias on the part of the instructor. A lecture and discussion teaching method was employed. Lesson plans were made and followed, and the same material was emphasized in both classes. However, since both classes did not meet daily due to scheduling, identical material was not always covered on the same day. Also, since more difficult material took extra class time, the number of topics covered by each respective periodic test varied.

4. Homework assignments were identical for both classes. However, only story-problem assignments were graded. Also, no other tests were given other than those specified.

Testing Procedure

1. Testing and correction was uniform for both groups on all tests. Tests were given, corrected by the teacher, and returned (except for the two standardized pre-tests and the final exam) the following class session when they were discussed and again collected. (See Appendix B)

2. During the first six-week period, both classes proceeded as normal. At the end of two weeks, both classes were tested for numerical ability and abstract reasoning. At the end of six weeks, both were tested for newly-acquired algebra skills. It was felt that by delaying the first teacher-made algebra skills test that outside influences biasing the learning situation would be sensed.

3. The experimental period lasted approximately eleven school weeks. During this time, the fifth period class, the experimental group, took a short fifteen-minute, five-item test approximately every five class sessions. At the same time, the fourth period class, the control group, took long forty-five minute, fifteen-item tests approximately every fifteen class sessions. Due to scheduling and other conflicts, it was recognized that it was impossible to give tests precisely at the end of five and fifteen class days respectively, but the test ratio of three short tests to one long test was strictly maintained. Each third short test was given on the same day as the corresponding long test.

4. The final exam was given over a three-day period (June 1-3, 1970). The two short parts were given on the first two days followed by the last part on the third day. It was judged that in this way neither group would be favored.

5. To sample student test attitude after the final exam, both groups were asked if they preferred

shorter, more frequent tests or longer, less frequent tests, and to comment why.

ANALYSIS TECHNIQUES

To evaluate the experimental data and to test the stated null hypotheses, the following statistical tools were used.

1. Treatment of Missing Data

For those students who were absent from the rolls during one or more tests, a dummy score was substituted. The formula used to provide this score was given by Fryer (7:378). The formula for the dummy score, d, is as follows:

$$d = \frac{kT + nR - S}{(k - 1)(n - 1)}, \text{ where}$$

k = number of subjects in the class

n = number of tests given to the class

T = sum of the n - 1 test scores for the absent student

R = sum of the k - 1 scores on the test for which the student was absent

S = sum of the kn - 1 scores (7:378).

By utilizing dummy scores, the maximum sample size was maintained, thus making possible a more meaningful analysis of the data.

2. Tests of Significance

To test significant differences between group means and, therefore, to test the stated null hypotheses, Fisher's t-test for unpaired variates was employed at the .05 level of confidence, i.e.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sum x_1^2 + \sum x_2^2}{N_1 + N_2 - 2}\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}} \quad (7:177)$$

The larger the value of $|t|$, the less likely that the difference between the means is due to chance.

To test significant differences between group test variances and, therefore, to add to the description of the data, F ratios were employed at the .05 level of confidence, i.e.

$$F\ (n_1 - 1,\ n_2 - 1) = \frac{S_1^2}{S_2^2}$$

where S_1^2 and S_2^2 represent respective group variances, while $n_1 - 1$ and $n_2 - 1$ represent relative degrees of freedom in terms of the numbers of group scores n_1 and n_2 (7:168).

All data were processed on the computer at Central Washington State College in Ellensburg, Washington.

CHAPTER IV

RESULTS AND DISCUSSION

PRE-EXPERIMENTAL TEST RESULTS

Results of pre-test scores for both groups may be seen in Table I. It was found that in no case was there a significant difference between group means as measured by "t" at the .05 level in the pooled or on any of the individual pre-test scores. On this basis, Hypothesis I--there is no difference in algebra ability between groups prior to the experiment--was accepted.

EXPERIMENTAL TESTING RESULTS

Results and scores of the two groups on various periodic tests given during the experimental period can be seen in Table I. Figures shown for the experimental group on tests five, six and seven are, in each case, the sum of scores on three short, five-item tests. The score of the control group is that made on the long, fifteen-item test composed of equivalent items. It was observed that the experimental group scored significantly higher than the control group on test six; however, other differences were not significant (.05 level). Since testing was not the same between groups and test items

TABLE I

MEANS, VARIANCES, STANDARD DEVIATIONS AND t SCORES
OF EXPERIMENTAL AND CONTROL GROUPS

Test Description	Test Number	Mean		Standard Deviation	
		Control Group ($\bar{X}_1$)	Experimental Group ($\bar{X}_2$)	Control Group (S_1)	Experimental Group (S_2)
Numerical Ability	1	33.535	33.650	3.382	4.944
Abstract Reasoning	2	41.892	43.200	4.331	4.323
Teacher-made pre-test	3	22.107	24.650	8.047	10.297
Total of Tests 1, 2 and 3	4	97.535	101.500	12.512	14.580
Periodic	5	39.392	42.750	15.234	13.873
Periodic	6	28.392	35.600	12.356	11.762
Periodic	7	36.214	36.250	11.731	11.417
Final Exam	8	58.071	78.100	28.823	29.159
Total Performance	9	241.000	277.450	83.198	84.846

TABLE I

MEANS, VARIANCES, STANDARD DEVIATIONS AND t SCORES
OF EXPERIMENTAL AND CONTROL GROUPS

Test Description	Test Number	Variance		Results of t Test Based on $(\bar{X}_2 - \bar{X}_1)$	
		Control Group (S_1^2)	Experimental Group (S_2^2)	t score	d.f.
Numerical Ability	1	11.443	24.450	.089	46
Abstract Reasoning	2	18.765	18.694	1.032	46
Teacher-made pre-test	3	64.765	106.028	.921	46
Total of Tests 1, 2 and 3	4	156.554	212.578	.984	46
Periodic	5	232.099	192.460	.793	46
Periodic	6	152.691	138.357	2.049*	46
Periodic	7	137.619	130.355	.011	46
Final Exam	8	830.772	850.305	2.357*	46
Total Performance	9	6921.925	7198.997	1.479	46

*Significant at .05 level

were not actually identical, making a valid comparison was difficult. Certainly, there was not enough evidence to reject the null hypothesis. Therefore, Hypothesis II--there is no difference in learned algebra skills between groups due to frequent testing during the experiment--was accepted. This conclusion should be held in reservation pending more investigation. In fact, it is in direct contradiction to findings by Keys in a similar experiment (14). However, Keys' study differed from this one in that items were identical between tests; four short, weekly tests were compared to one monthly test; subjects were of college age; and the discipline was educational psychology.

POST-EXPERIMENTAL TESTING RESULTS

Results of the final exam indicated in Table I showed a significant difference between group means in favor of the experimental group (.05 level). Thus, the null Hypothesis III was rejected in favor of the principal hypothesis--that frequent testing is associated with increased learning performance in high school advanced algebra. In this case, the increased learning performance was in terms of retained algebra skills following frequent testing.

RESULTS OF TOTAL PERFORMANCE

Scores of total performance as noted in Table I were obtained by summing the scores on all the teacher-made tests and story problem assignments. It was found that group means for total performance were not significantly different at the .05 level as measured by "t". On this basis, Hypothesis IV--there is no difference in total performance between groups during the semester of study--was accepted. Since all parts of total performance were not under direct experimental control, the preceding reasoning is contingent on the previously-noted assumption that uncontrolled outside influences affected both groups equally during the experiment.

RESULTS OF STUDENT TEST ATTITUDE

The experimental group was asked if they preferred testing "the way it was done", or if they would have rather had longer, less frequent tests, and to comment. Also, the control group was asked if they preferred testing "the way it was done", or if they would have rather had shorter, more frequent tests, and to comment. Results are indicated in Table II.

TABLE II

RESULTS OF STUDENT ATTITUDE SURVEY

	Favored Short Frequent Testing	Favored Longer Less-Frequent Testing	Abstained
Control Group	19 (68%)	3 (11%)	6 (21%)
Experimental Group	17 (85%)	3 (15%)	0

Comments varied. The following four statements would summarize common comments as interpreted by the writer:

1. Short, frequent tests are easier to study for, since there is less material to review. (16 students)

2. Short, frequent tests count less and if one does poorly, it can easily be made up in the next test. (12 students)

3. Short, frequent tests give sooner knowledge of learned skills. (5 students)

4. Short, frequent tests force regular study. (2 students)

In the control group, 68 per cent favored short, frequent testing, 11 per cent favored long, less frequent testing, and 21 per cent abstained. In the experimental group, 85 per cent favored short, frequent testing, while the rest favored long, less frequent testing. Although the overall attitude was in favor of short, frequent testing, it should not be construed that the control group necessarily favored the testing method used by the experimental group.

DISCUSSION

The Principal Hypothesis

The acceptance of the Principal Hypothesis on the basis of the experimental group's superior performance on the final examination needs interpretation. It is impossible to know which, if any, of the assumed attributes of frequent testing--knowledge of results, distributed learning, practice, enforced activity, extrinsic motivation or structuring--may have been responsible. Surely most were present in both periodic testing situations. The best conjecture, however, would be that the more frequent testing situation was catalytic in amplifying several. For example, knowledge of results through test correction occurred more often, learning materials were distributed onto more short tests, and practice of skills on tests was more frequent. On the other hand, enforced activity, extrinsic motivation, and structuring were probably least affected since, for both groups, the total testing time was the same, all tests were graded, and equivalent test items and homework assignments were based on the same course objectives.

That favorable attitude toward the frequent testing situation, in the experimental group, had a positive effect on learning, is almost certain. In all research reviewed, when frequent testing was associated with good testing attitude, higher significant results occurred. In view of student comments, and the fact that the attitude sample was taken before the results of the final exam were known, it would seem that this favorable attitude was actually elicited by the testing situation itself. In fact, since the control group did not favor their testing situation, it is quite possible that attitude was solely responsible for the experimental group's superior performance on the final exam.

One conjectured positive attribute of frequent testing, however, did not affect results. Homogeneous grouping, as stated by Hertzberg (10:376), was not a factor favoring learning in the experimental group. F ratios showed no significant difference between group variances on either summed pre-test scores [$F(19, 27) = 1.36$] or on the final exam scores [$F(19, 27) = 1.02$] at the .05 level.

Even though factors elicited by frequent testing produced better performance on the final exam, they were not able to consistently produce reliably higher scores on periodic tests, nor were they able to produce a reliably higher score of total performance. The first consideration would be that outside influences may have biased results.

Certainly these parts of the experiment were not conducted under the same stringent controls as was the final exam. For example, periodic tests differed in date, duration, and test items. The total performance score consisted of not only these periodic test scores, but also homework scores which were completely susceptible to uncontrolled factors. (At least, null findings on the first teacher-made test, lent some support to the experimental assumption that uncontrolled outside influences affected both groups equally.)

Looking beyond the possibility of bias, another explanation can be offered. According to distributed learning theory, performance on the final exam was not necessarily contingent to the periodic test results. As interpreted from Hovland (11), massed practice, here represented by each long testing situation, may result in higher immediate performance than in the case of distributed practice, here represented by each set of three corresponding short test situations. However, massed practice, through fatigue factors, induces stronger learning of "conflicting habits" which are more difficult to reorganize and forget (11:586). Therefore, distributed learning results in greater overall retention. Since the total performance score consisted both of teacher-made pre-test and periodic test scores, it would seem that their effect was simply to outweigh any influence frequent testing had on homework and the final exam. This should surely be investigated further.

OTHER ASPECTS AND COMPARISONS

Although general results parallel similar studies in other fields, several previously unmentioned aspects should be stated. First, unlike Hertzberg et al. (10), it was found that increased learning on the final examination occurred without the use of tests as extra study aids. Second, the increased retention on the final exam was independent of the amount of test materials employed between groups. In virtually all studies investigated, with the exception of Keys (14), Gabel (8), and Deputy (4), increased performance was associated with both frequent testing and a greater amount of testing materials employed. Certainly the possibility that increased learning was a result of more testing rather than more frequent testing was not considered. Third, unlike Fitch's study (6), increased learning as measured by the final exam was independent of outside variables introduced by special discussion groups.

GENERALIZATION AND RECOMMENDED INVESTIGATION

Certainly any generalization is restricted to the scope and limitations of the study. To predict results outside the specific experimental realm for different subjects, disciplines, instructors, etc., would be pure speculation. This study should be viewed as a small indicator in the growing knowledge that frequent testing, in some instances, may invoke factors that initiate increased learning.

To add to this knowledge more investigation in all fields is needed. For example, this experiment should be repeated. Other possible follow-up studies are as follows:

1. A study should be conducted to investigate the optimum frequency of examination.

2. A study should be conducted to investigate the optimum length of examination.

3. A study should be conducted to investigate the effect of frequent testing on homework performance.

CHAPTER V

SUMMARY AND CONCLUSIONS

Summary

This study attempted to determine the influence on learning performance of a system of frequent testing as compared to infrequent testing, apart from differences in the amount of testing materials employed, the implementation of outside discussion groups, or the use of tests as teaching aids. It was based on the performance of two classes of high school advanced algebra students numbering 20 in the experimental group and 28 in the control group. Classes were equated on the basis of numerical ability, abstract reasoning and initial algebra performance.

Results indicated that under the conditions of the experiment:

1. The frequent testing group showed no consistent significant gains over the infrequent testing group in terms of learned algebra skills on the periodic tests (.05 level).

2. Retention of algebra skills by the frequent testing group was some 34 per cent superior to that of the

infrequent testing group as measured by the final exam. This difference was significant (.05 level).

3. There were no significant gains in total performance between groups as measured by the sum of all teacher-made tests and story problem assignments (.05 level).

4. The frequent test group favored their testing situation by some 85 per cent.

5. The infrequent test group did not favor their testing situation; some 68 per cent implied that they would have preferred shorter, more frequent tests.

In view of the experimental group's significant superiority in terms of retention on the final exam, it would seem that most of the learning gains found by such investigators as Hertzberg, Kulp, Fitch, Turney and others, could have been had without engaging increased testing materials, but by merely giving these tests in smaller, more frequent installments.

Conclusion

In conclusion, we are reminded that education gets its meaning from human nature itself. A teaching method may facilitate learning, but at the same time elicit an attitude which destroys the overall purpose of instruction. At least in this study, students favored the experimental method employed. One student summed it up this way, "I like short tests because they are easier to study for, and if you have a bad day, you're not out so many points."

In view of the previous evidence, it would seem that, in this case, the easy way was the best way.

As a final warning it is appropriate to quote Jones:

> . . . examination must not be allowed to mechanize our procedure. In a growing subject we may well regard with suspicion any influence which tends toward a premature standardization of methods and results. A test should not merely review facts, it should stimulate experience; for the sake of convenience in objective marking, our tests must not be allowed to stress mere barren data at the expense of developing organized habits of dealing with data. A classroom imparting the propositions which we are to employ in thought, neglects its chief task unless it also incites us to think . . . (13:70)

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APPENDIX

APPENDIX A

COURSE TOPICS AND TEST CONSTRUCTION

Pool	Course Topics Covered by Pool Items	Pool Items per Test	Topics for Final Tests
Pre-Test	Quadratic Functions and Variation Quadratic Functions: $y = a(x-h)^2 + k$ Quadratic Functions: $y = ax^2 + bx + c$ Using Graphs of Quadratic Functions Using Radicals to Express Roots Rational and Irrational Roots Operations with Rational and Irrational Numbers Decimals for Real Numbers	11 Items for Pre-Test	None

APPENDIX A (continued)

Pool	Course Topics Covered by Pool Items	Pool Items per Test	Topics for Final Tests
1	Properties of Radicals Simplifying Sums of Radicals Products of Sums Containing Radicals Using Radicals to Solve Quadratic Equations Roots and Coefficients of a Quadratic Equation	5-Short Test 1 1-Short Test 2 5-Long Test A	Final Test I
2	The Nature of the Roots of a Quadratic Equation Solving Quadratic Inequalities Irrational Equations	4-Short Test 2 1-Short Test 3 5-Long Test A	Final Test I
3	Distance Between Points Perpendicular Lines Circles	4-Short Test 3 1-Short Test 4 5-Long Test A	Final Test I

Pool	Course Topics Covered by Pool Items	Pool Items per Test	Topics for Final Tests	
4	Circles Parabolas Ellipses Hyperbolas	4-Short Test 4 1-Short Test 5 5-Long Test B	Final Test I	Final Test III
5	Hyperbolas Inverse Variation	4-Short Test 5 1-Short Test 6 5-Long Test B		
6	Graphic Solutions Linear Quadratic Systems: Substitution Quadratic Quadratic Systems	4-Short Test 6 1-Short Test 7 5-Long Test B	Final Test II	

APPENDIX A (continued)

Pool	Course Topics Covered by Pool Items	Pool Items per Test	Topics for Final Tests
7	Rational Numbers as Exponents Real Numbers as Exponents Exponential and Logarithmic Functions	4-Short Test 7 1-Short Test 8 5-Long Test C	
8	Common Logarithms Interpolation Products and Quotients	4-Short Test 8 1-Short Test 9 5-Long Test C	Final Test II ↓
9	Powers and Roots Combined Operations Logarithms to Solve Equations	4-Short Test 9 1-Extra 5-Long Test C	Final Test III ↓

APPENDIX B

TESTING PROCEDURES

a. Students supplied their own writing instruments.

b. Tests and scratch paper were distributed to all students.

c. All tests remained covered and final directions were given before the start of the test.

d. All blackboards were erased and other materials deemed helpful in a test situation were removed from the bulletin boards.

e. All tests began from five to ten minutes into the period with the command "go".

f. Time for all tests was kept on a stop watch.

g. All tests ended at the allotted time with the command "stop".

h. All tests and scratch paper were collected at the end of each test.

i. All tests were marked (names covered) by the teacher and the number of points correct was tabulated and written on each test.

j. When possible, as in the cases of the pre-tests and the final exam, papers were shuffled between groups before correcting to eliminate any possibility of bias.

k. All tests, except the two standardized pre-tests and final, were returned at the following class meeting where the correct answers were read and where all items were reworked and discussed.

l. All tests were collected after the review and never again returned to the students.

APPENDIX C

TESTING SCHEDULE

Test	Items	Point Total	Time(Min.)	Date
Numerical Ability	40	40	30	2- 9-70
Abstract Reasoning	50	50	25	2-10-70
Pre-Algebra-Test	11	50	45	3- 9-70
Short Test 1	5	25	15	3-19-70
Short Test 2	5	25	15	3-27-70
Short Test 3	5	25	15	4-10-70
Long Test A	15	75	45	4-10-70
Short Test 4	5	25	15	4-20-70
Short Test 5	5	25	15	4-27-70
Short Test 6	5	25	15	5- 4-70
Long Test B	15	75	45	5- 4-70
Short Test 7	5	25	15	5-13-70
Short Test 8	5	25	15	5-21-70
Short Test 9	5	25	15	5-27-70
Long Test C	15	75	45	5-27-70
Final Test I	7	40	20	6- 1-70
Final Test II	7	40	20	6- 2-70
Final Test III	14	80	40	6- 3-70

APPENDIX D

CONTROL GROUP RAW SCORES

Subjects	by Number	1	2	3	4	5	6	7	8	9	10	11	12
	by Class	11	11	11	11	11	12	10	10	10	11	11	11
Test Description	Test Number												
Numerical Ability	1	26	34	31	30	37	29	32	35	34	30	30	32
Abstract Reasoning	2	36	35	40	40	48	47	42	45	47	37	35	47
Teacher-made pre-test	3	21	20	13	30	27	25	20	32	18	14	18	12
Total of Tests 1, 2 and 3	4	83	89	84	100	112	101	94	112	99	81	83	91
Periodic	5	12	39	19	44	56	50	46	36	25	11	26	38
Periodic	6	19	29	12	44	35	17	34	41	22	11	16	18
Periodic	7	25	28	23	45	53	37	28	44	26	33	37	27
Final Exam	8	41	50	15	52	99	80	60	68	44	23	43	28
Total Performance	9	151	224	149	301	342	276	260	299	206	134	197	174

APPENDIX D (continued)

13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
12	10	10	10	10	10	11	11	11	10	12	12	10	11	11	10
32	30	36	37	36	38	31	36	35	34	38	31	39	31	37	38
38	40	44	41	47	43	41	44	43	46	43	34	44	35	44	47
2	21	29	36	28	18	14	26	33	18	37	13	19	25	23	27
72	91	109	114	111	99	86	106	111	98	118	78	102	91	104	112
29	37	60	64	31	46	28	33	58	48	37	26	58	28	50	68
4	38	37	43	43	13	20	36	46	29	38	21	40	17	27	45
27	32	54	50	57	38	17	37	43	45	27	19	46	21	41	55
9	45	87	109	109	52	34	46	90	62	81	34	64	55	33	114
127	254	348	386	341	226	163	208	354	264	298	118	288	192	106	362

APPENDIX E

EXPERIMENTAL GROUP RAW SCORES

Subjects	by Number	1	2	3	4	5	6	7	8	9
	by Class	10	11	10	10	11	11	10	10	11
Test Description	Test Number									
Numerical Ability	1	22	39	38	37	32	38	29	32	23
Abstract Reasoning	2	44	49	47	35	41	45	46	42	41
Teacher-made pre-test	3	23	42	36	32	19	28	25	31	25
Total of Tests 1, 2 and 3	4	89	130	121	104	92	111	100	105	89
Periodic	5	39	62	60	56	35	40	55	38	22
Periodic	6	35	53	52	47	26	38	53	31	17
Periodic	7	26	68	27	37	33	42	49	36	28
Final Exam	8	53	148	118	86	54	128	93	65	42
Total Performance	9	217	463	385	337	244	348	346	259	188

APPENDIX E (continued)

10	11	12	13	14	15	16	17	18	19	20
11	11	11	10	11	10	10	11	11	11	11
37	30	38	33	35	31	35	35	40	37	32
46	39	48	39	46	42	48	48	40	44	34
13	28	21	21	10	7	23	42	36	24	7
96	97	107	93	91	80	106	125	116	105	73
26	48	36	37	23	49	52	67	51	39	20
26	29	32	28	30	31	22	58	37	43	24
27	34	42	25	36	32	36	53	44	32	18
49	74	81	45	53	67	82	113	75	73	63
212	287	267	222	162	154	290	401	317	279	171

APPENDIX F

ADVANCED ALGEBRA

FINAL TEST I
(40 points) (20 minutes)

1. Rationalize the denominators of the following:
(6 points)

   a. $\dfrac{1}{1 - \sqrt{3}}$

   b. $\dfrac{1}{\sqrt[3]{2} + 1}$

2. Find the length of the diameter (d) of a sphere whose volume (V) is 11/21 cubic feet:
(5 points)

   where $V = \frac{4}{3}\pi r^3$, $\pi \fallingdotseq \frac{22}{7}$ and r = radius.

   Answer ⇒ d = ____________

3. Solve for h in the following equation:
(5 points)

   $\sqrt{h - 5} = h - 7$ Answer ⇒ h ∈ ____________

4. If there is just one root of the equation $x^2 - 3x + k = 0$, what is it:
(6 points)

   Answer ⇒ x ∈ ____________

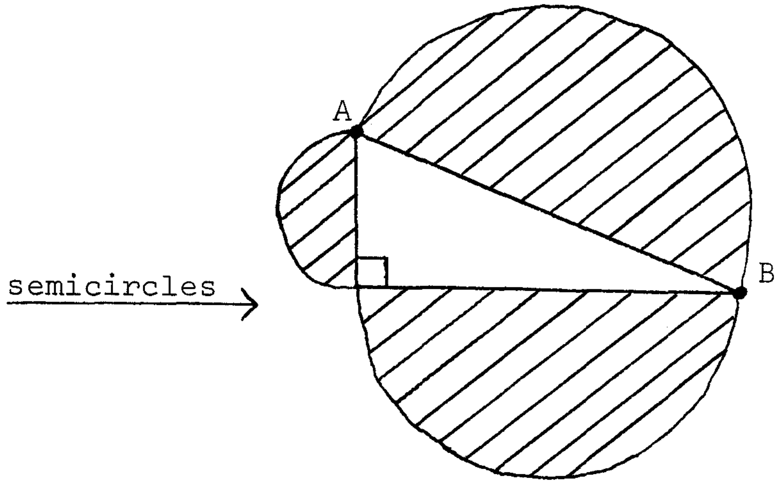
5. Find the midpoint and length of the segment joining points (-2, 3) and (10, 8):
(6 points)

   a. midpoint ⇒

   b. length ⇒

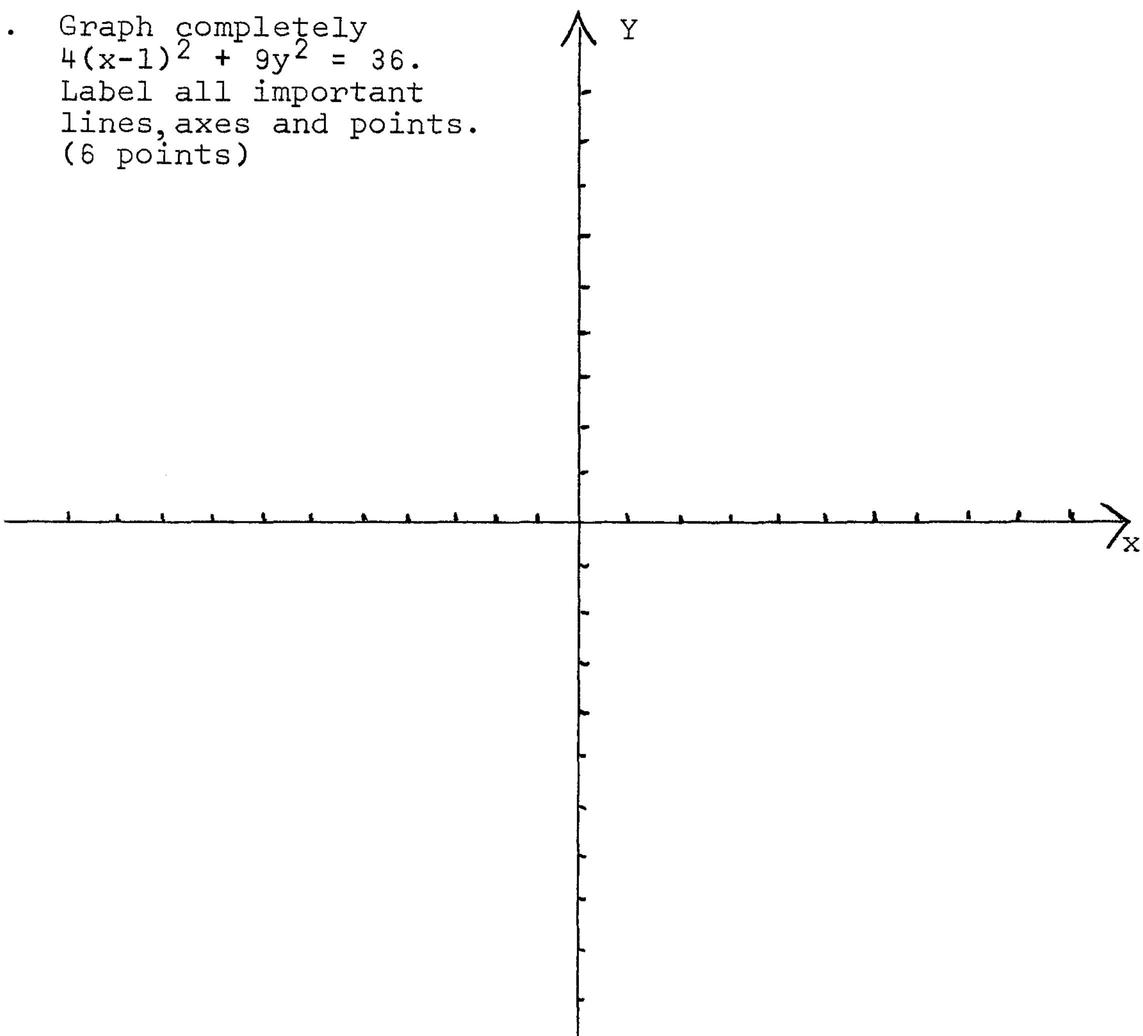
FINAL EXAM I (continued)

6. Determine the area of the shaded region in the figure if AB = 6 cm. and $\pi \approx 3.14$: (6 points)



Answer ⇒ Area = __________

7. Graph completely $4(x-1)^2 + 9y^2 = 36$. Label all important lines, axes and points. (6 points)



APPENDIX F

ADVANCED ALGEBRA

FINAL TEST II
(40 points) (20 minutes)

1. Solve for c if both ordered pairs belong to the same inverse variation.

   (-4, 6); (8, c) c = ____________

2. Solve

   $a^2 - b^2 = 21$ $(a, b) \in \{ (\quad); (\quad); (\quad) \cdots \}$

   $2a - b = -1$ $(a, b) \in \{ (\quad); (\quad); (\quad) \cdots \}$

3. Solve for x

   i) $\log_5 (x + 1) = 1$

   ii) $\log_x (x + 12) = 2$

   iii) $x > 0$, $\log_x x^{(x + 1)} = 2$

4. Solve for n

   $\log_2 (n + 1) = 1/2 \log_2 9 + 4 \log_2 2 - 1/3 \log_2 64$

5. Evaluate antilog (.7091 + 2) if antilog .7084 = 5.11 and antilog .7093 = 5.12

x	log x

FINAL TEST II (continued)

6. Write the logarithmic equation you would use to compute

$$R = r\sqrt[3]{\frac{V}{v}}$$

log R = ____________

7. If $\log_5 b = 4$, $\log_5 A = 8$ and $\log_5 B = 12$, evaluate

$\log_b \frac{A}{B}$

$\log_b \frac{A}{B} =$ ____________

APPENDIX F

ADVANCED ALGEBRA

FINAL TEST III
(80 points) (40 minutes)

1. Write the first three digits of the infinite decimal representing $\sqrt{7}$.

   $\sqrt{7} \approx$ ____________

2. Solve for x

   $\sqrt{x} + \sqrt{x-3} = \dfrac{3}{\sqrt{x-3}}$ $x \in$ ____________

3. Solve for x

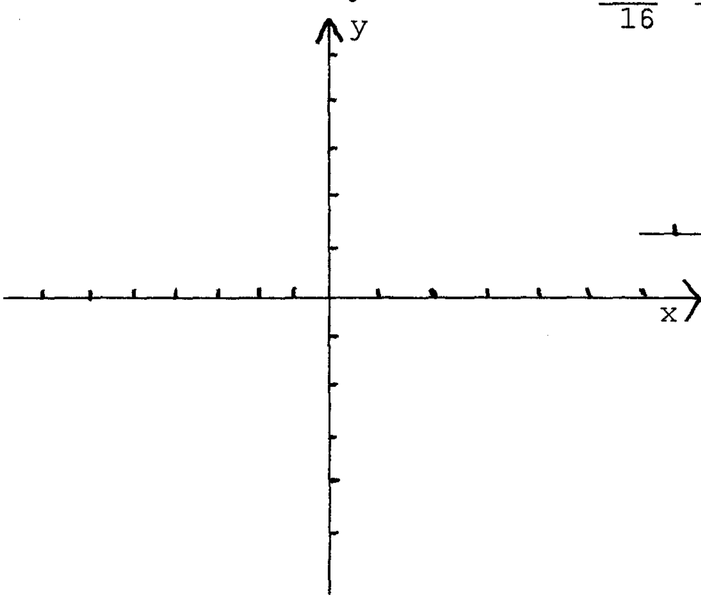
   $\sqrt{x^2 - 6x + 9} = (x-3)$ $x \in$ ____________

4. Find the equation of the perpendicular bisector of the segment joining points (0, 12) and (-8, -4).

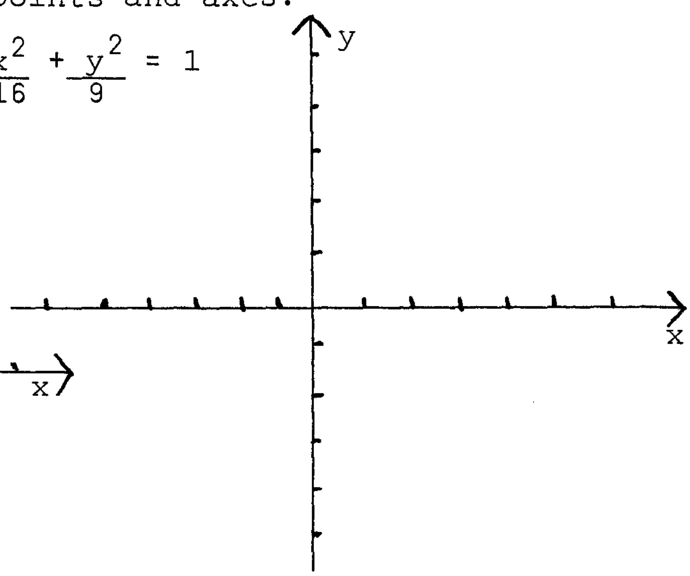
   Answer $\Rightarrow$ ____________

5 and 6. Graph completely each of the following.
Label all important lines, points and axes.

$(x-1)^2 = -4\ (y+1)$

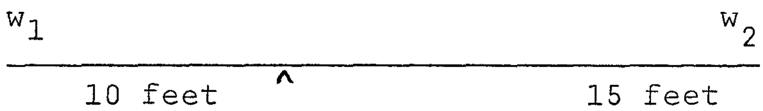


$\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$



FINAL EXAM III (continued)

7. The arms of a lever are 10 feet and 15 feet respectively. What weights summing to 75 pounds will balance the lever?



w_1 = ___________

w_2 = ___________

8. Solve for (m, n) in the following system.

$m^2 - n^2 = 15$
$2m - n = -2$

Answer ⇒ ___________

9. Solve for x

i) $10^{2x+1} = 10^{x-1}$

ii) $8^{x^2} = 2^{1-2x}$

iii) $\log (x^2 + x) = \log 12$

10. When a favorable wind caused an increase of 30 miles per hour over the usual speed of the plane, the pilot made a 630 mile trip between two cities in 6 minutes less time. Find the usual speed of the plane.

Answer ⇒ ___________

11. Evaluate antilog (.7723 + 3) if antilog (1.7723) = 59.2

Answer ⇒ ___________

12. If log A = 12 and log B = 4, evaluate $\log \frac{A}{B}$.

Answer ⇒ $\log \frac{A}{B}$ = ___________

FINAL EXAM III (continued)

13. Write the logarithmic equation you would use to compute:

$$\sqrt[3]{\frac{(4)\ (2.1)^2}{(5)}}$$

$$\log \sqrt[3]{\frac{(4)\ (2.1)^2}{(5)}} = \_\_\_\_\_\_\_\_\_\_$$

14. Evaluate n in each of the following:

i) 3 log n = 27

n ∈ ____________

ii) 4 log (n+1) = log 16

n ∈ ____________

iii) n log 27 = 2 log 3

n ∈ ____________